Enriched Space-Time FEM Method for Large Motion of Thin Flexible Structures Immersed in a Fluid

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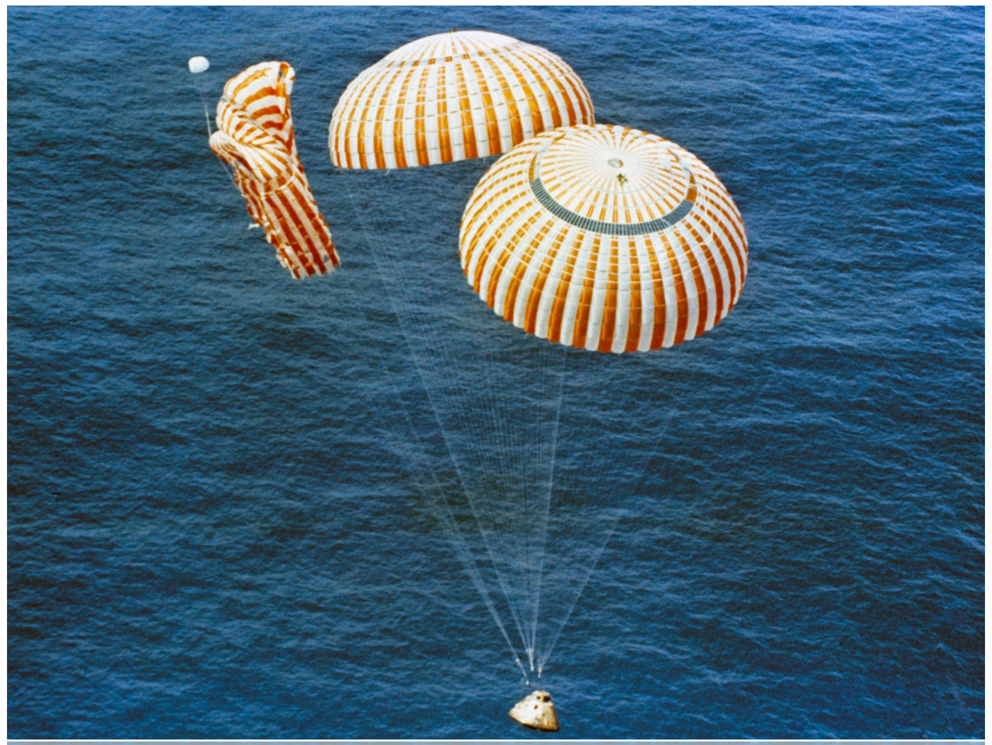
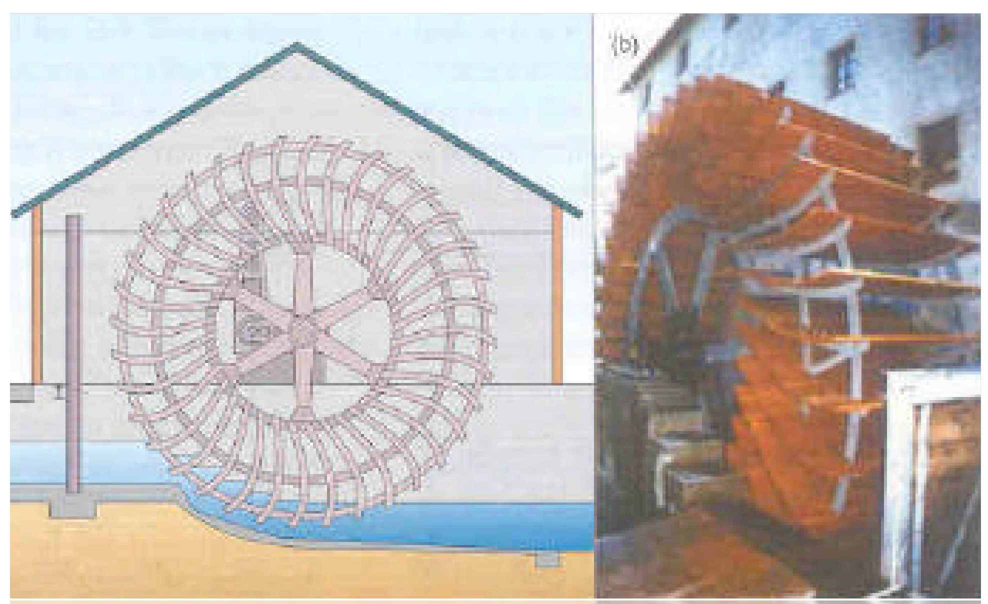
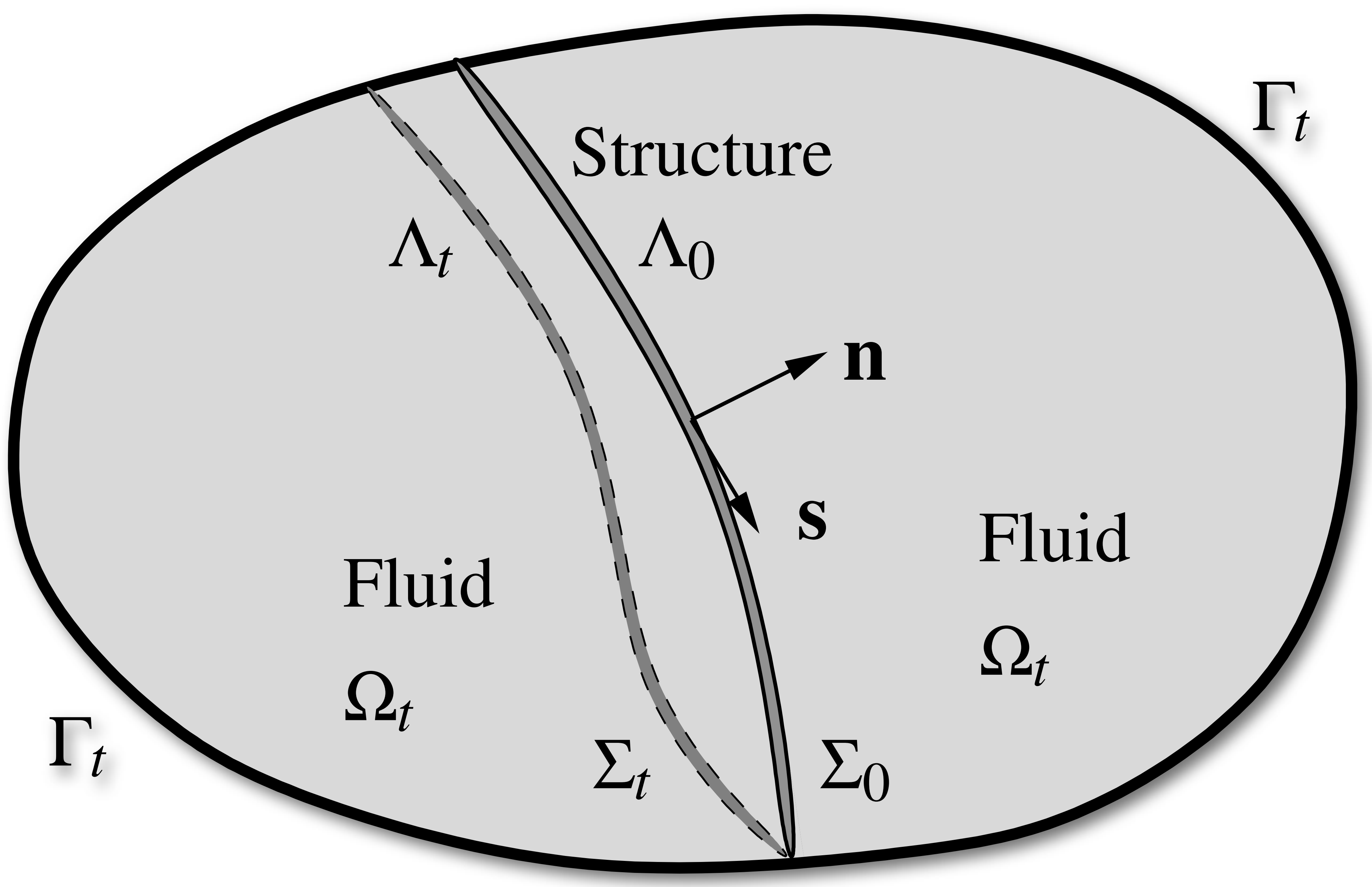
Interaction of Flow and Structure





The Surface Coupled Problem

- flexible thin-walled structure immersed in viscous fluid flow
- large translations, rotations and deformations of the thin structure



Governing Equations: Strong Form

<u>Fluid</u>: incompressible, Newtonian, velocity-pressure based

$$\begin{aligned} \rho \mathbf{v}_{,t} + \rho \mathbf{v} \cdot \nabla \mathbf{v} - \nabla \cdot \mathbf{T} - \rho \mathbf{b} &= \mathbf{0} \quad \text{in} \quad Q_t \\ \nabla \cdot \mathbf{v} &= 0 \quad \text{in} \quad Q_t \end{aligned}$$

$$\begin{aligned} \mathbf{T} &= -p\mathbf{I} + 2\mu\mathbf{D} \quad \text{in} \quad Q_t \\ \mathbf{D} &= \frac{1}{2}(\nabla \mathbf{v} + (\nabla \mathbf{v})^T) \quad \text{in} \quad Q_t. \end{aligned}$$

<u>Structure</u>: thin-walled, large deformations/strains, velocity-stress based

$$\mathbf{w} = [v_X \quad v_Y \quad \omega]^T$$

$$\mathbf{s} = [N \quad Q \quad M]^T$$

$$\begin{aligned} \rho \mathbf{G} \dot{\mathbf{w}} - (\mathbf{F}\mathbf{s})_{,X} - (\mathbf{H}\mathbf{s}) - \rho \mathbf{G} \mathbf{b} = \mathbf{0} \quad \text{in} \quad L_0 \\ \mathbf{C}^{-1} \dot{\mathbf{s}} - \dot{\mathbf{e}} = \mathbf{0} \quad \text{in} \quad L_0. \end{aligned}$$

<u>Interface Conditions</u>: velocity-traction based

$$\begin{aligned} \mathbf{v}^{\mathsf{F}} - \mathbf{v}^{\mathsf{S}} = \mathbf{0} \quad \text{on} \quad R_t \\ \mathbf{t}^{\mathsf{F}} + \frac{\mathrm{d}\Sigma_0}{\mathrm{d}\Sigma_t} \mathbf{t}^{\mathsf{S}} = \mathbf{0} \quad \text{on} \quad R_t \end{aligned}$$

Governing Equations: Weak Form

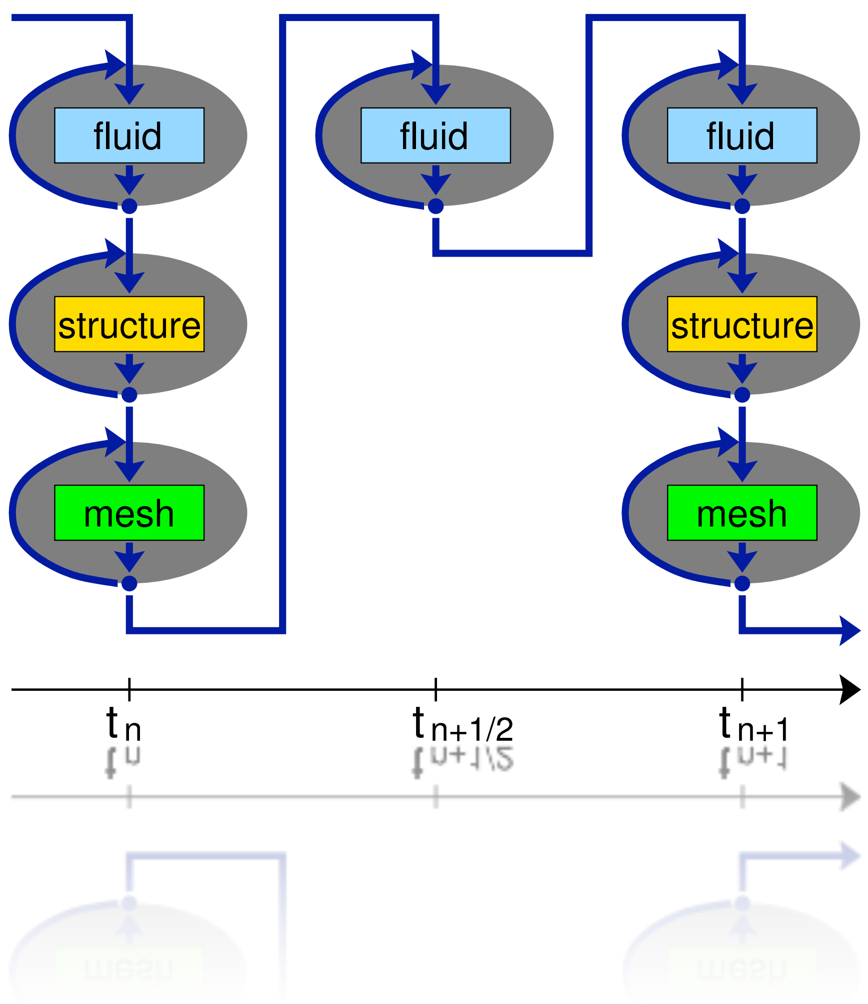
- Weighted residual method on the space-time domain
- Lagrange multiplier technique for coupling conditions

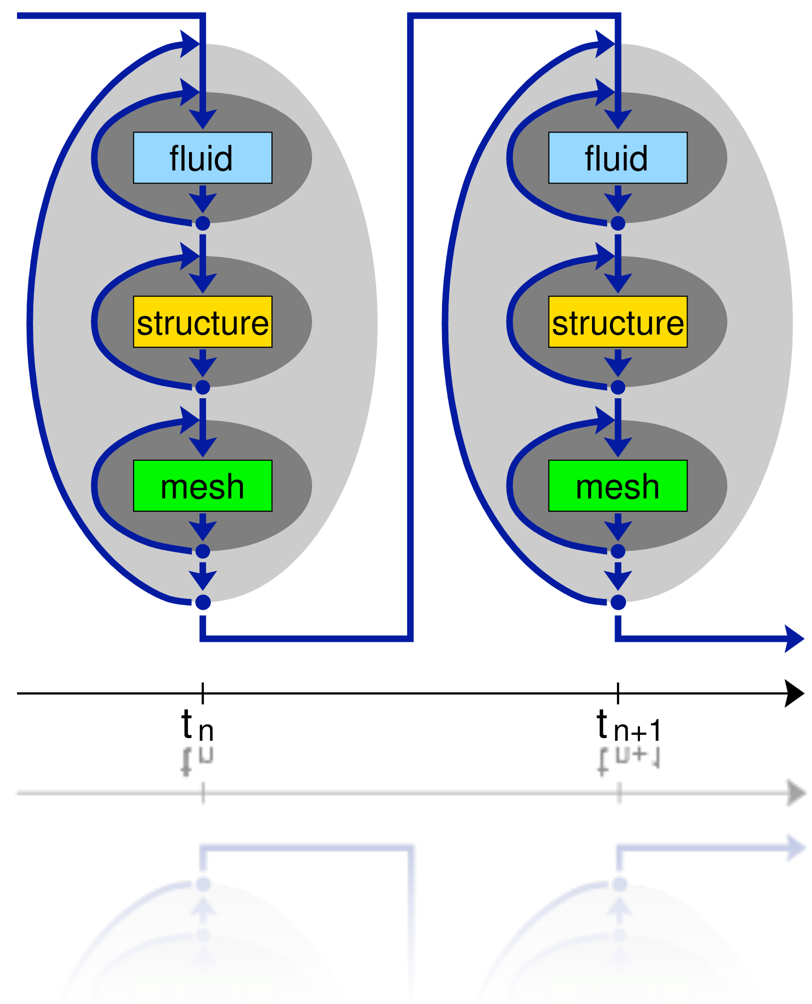
$$
\begin{aligned}
&\int_{Q_t^n} \delta\mathbf{v}\cdot\rho(\mathbf{v}_{,t}+\mathbf{v}\cdot\nabla\mathbf{v})\,\mathrm{d}Q_t+\int_{Q_t^n}\delta\mathbf{D}:2\mu\mathbf{D}\,\mathrm{d}Q_t-\int_{Q_t^n}\nabla\cdot(\delta\mathbf{v})\,p\,\mathrm{d}Q_t-\int_{Q_t^n}\delta\mathbf{v}\cdot\rho\mathbf{b}\,\mathrm{d}Q_t\\
&+\int_{Q_t^n}\delta p\,\nabla\cdot\mathbf{v}\,\mathrm{d}Q_t\\
&-\int_{P_t^{n,g}}\delta\mathbf{v}\cdot\mathbf{t}\,\mathrm{d}P_t+\int_{P_t^{n,g}}\delta\mathbf{t}\cdot(\mathbf{v}-\bar{\mathbf{v}})\,\mathrm{d}P_t-\int_{P_t^{n,h}}\delta\mathbf{v}\cdot\bar{\mathbf{t}}\,\mathrm{d}P_t\\
&+\int_{\Omega_t^n}\delta\mathbf{v}(t_n^+)\cdot\rho(\mathbf{v}(t_n^+)-\mathbf{v}(t_n^-))\,\mathrm{d}\Omega_t\\
&+\sum_e\int_{\overset{e}{Q}{}_t^n}(\rho\delta\mathbf{v}_{,t}+\rho\mathbf{v}\cdot\nabla(\delta\mathbf{v})-\nabla\cdot(\delta\mathbf{T}))\cdot\tau_m\frac{1}{\rho}(\rho\mathbf{v}_{,t}+\rho\mathbf{v}\cdot\nabla\mathbf{v}-\nabla\cdot\mathbf{T}-\rho\mathbf{b})\,\mathrm{d}Q_t\\
&+\sum_e\int_{\overset{e}{Q}{}_t^n}\nabla\cdot(\delta\mathbf{v})\cdot\tau_c\,\rho\,\nabla\cdot\mathbf{v}\,\mathrm{d}Q_t=0\qquad\forall\delta\mathbf{v},\delta p,\delta\mathbf{t}.
\end{aligned}
$$

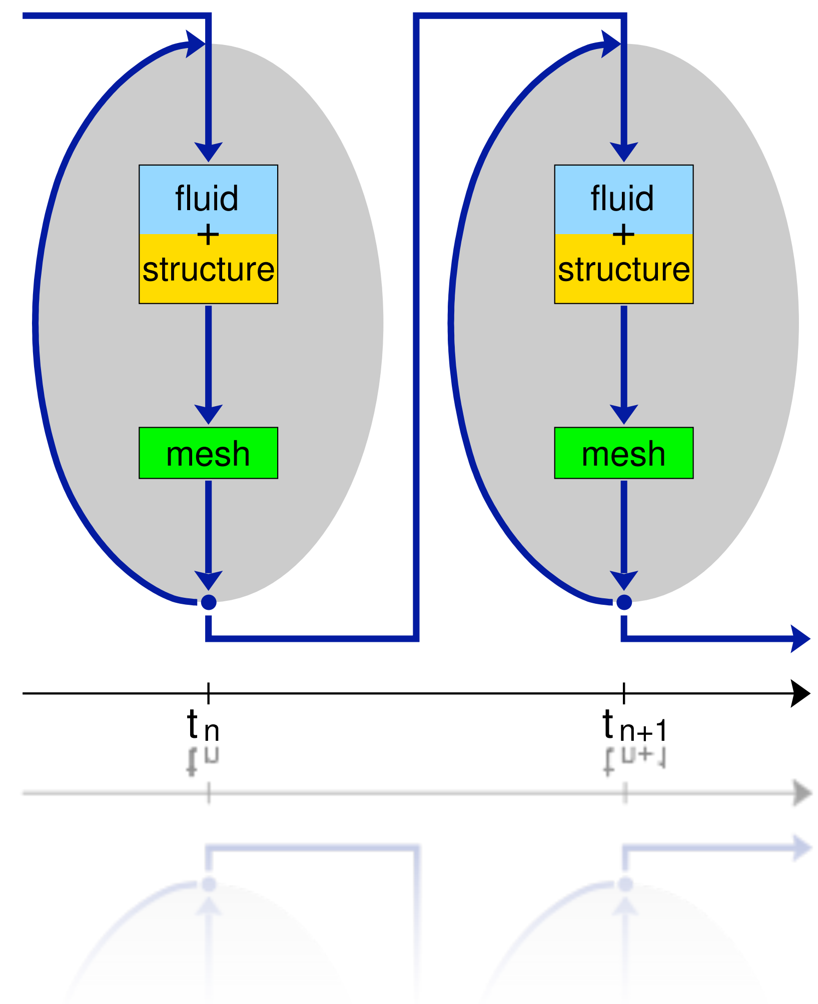
$$
\begin{aligned}
&\int_{L_0^n}\delta\mathbf{w}\cdot\rho\mathbf{G}\,\dot{\mathbf{w}}\,\mathrm{d}X+\int_{L_0^n}\delta\dot{\mathbf{e}}\cdot\mathbf{s}\,\mathrm{d}X-\int_{L_0^n}\delta\mathbf{w}\cdot\rho\mathbf{G}\,\mathbf{b}\,\mathrm{d}X\\
&+\sum_e\int_{L_0^n}\delta\mathbf{s}\cdot(\mathbf{C}^{-1}\dot{\mathbf{s}}-\dot{\mathbf{e}})\,\mathrm{d}X\\
&-[\delta\mathbf{w}\cdot\bar{\mathbf{s}}]_{\breve{P}_0^{n,h}}\\
&-\int_{\bar{P}_0^{n,h}}\delta\mathbf{w}\cdot\bar{\mathbf{t}}\,\mathrm{d}X\\
&+\int_{L_0^n}\delta\mathbf{w}(t_n^+)\cdot\rho\mathbf{G}\,(\mathbf{w}(t_n^+)-\mathbf{w}(t_n^-))\,\mathrm{d}X\\
&+\int_{L_0^n}\delta\mathbf{s}(t_n^+)\cdot\mathbf{C}^{-1}\,(\mathbf{s}(t_n^+)-\mathbf{s}(t_n^-))\,\mathrm{d}X=0\qquad\forall\delta\mathbf{w},\delta\mathbf{s}.
\end{aligned}
$$

$$
+\int_{R_t^n}\delta\mathbf{t}^{\mathsf{F}}\cdot(\mathbf{v}^{\mathsf{F}}-\mathbf{v}^{\mathsf{S}})\,\mathrm{d}R-\int_{R_t^n}\delta\mathbf{v}^{\mathsf{F}}\cdot\mathbf{t}^{\mathsf{F}}\,\mathrm{d}R+\int_{R_t^n}\delta\mathbf{v}^{\mathsf{S}}\cdot\mathbf{t}^{\mathsf{F}}\,\mathrm{d}R
$$

Coupling Schemes



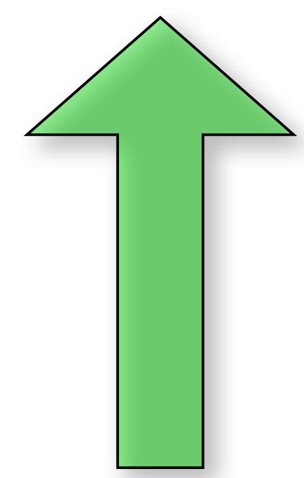




weak coupling

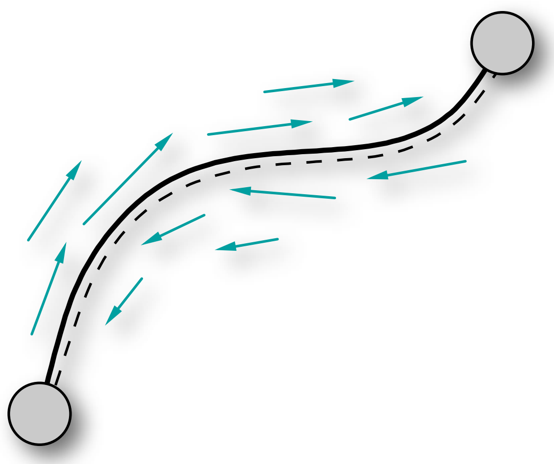
strong coupling (partitioned)

strong coupling (monolithic, simultaneous)

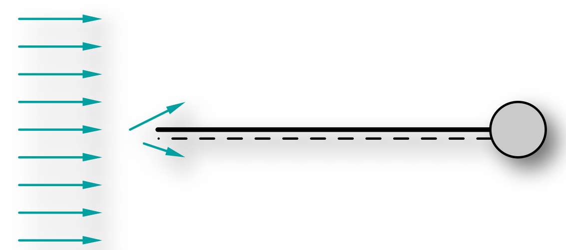


Appearance of Non-smooth Fluid Solutions

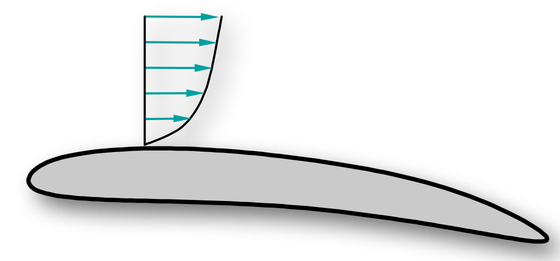
- flow-immersed thin structures cause discontinuous flow solutions !



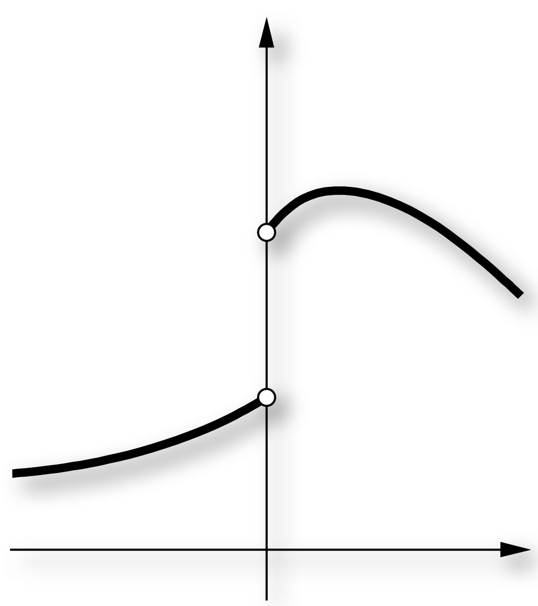
Navier-Stokes fluid (v-p) &
slip interface conditions

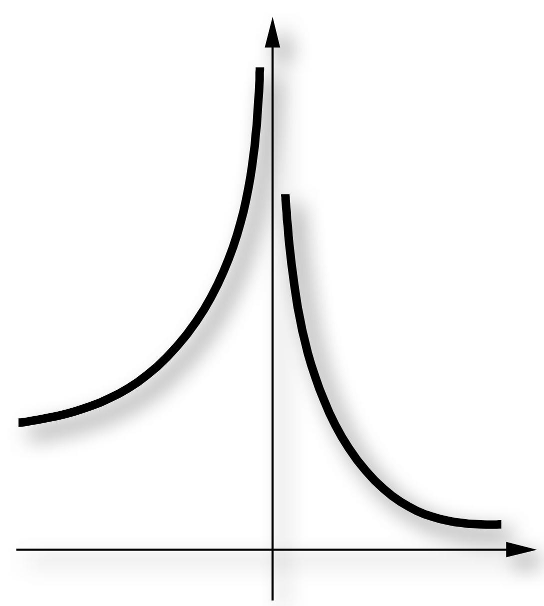


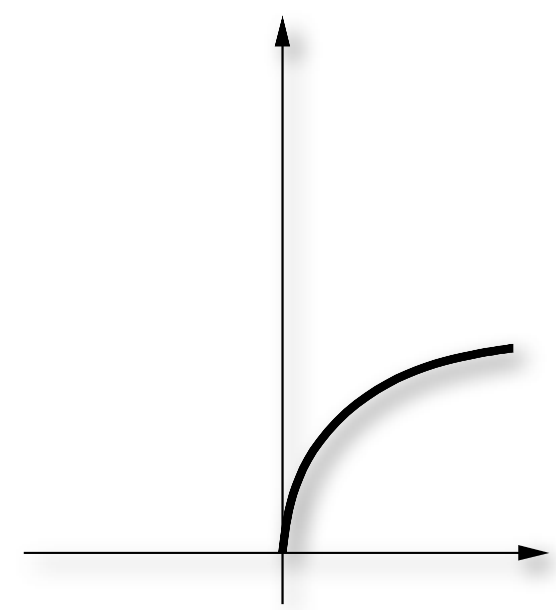
Navier-Stokes fluid (v-p) &
tip singularity



Navier-Stokes fluid (v-p) &
shear layer at interface





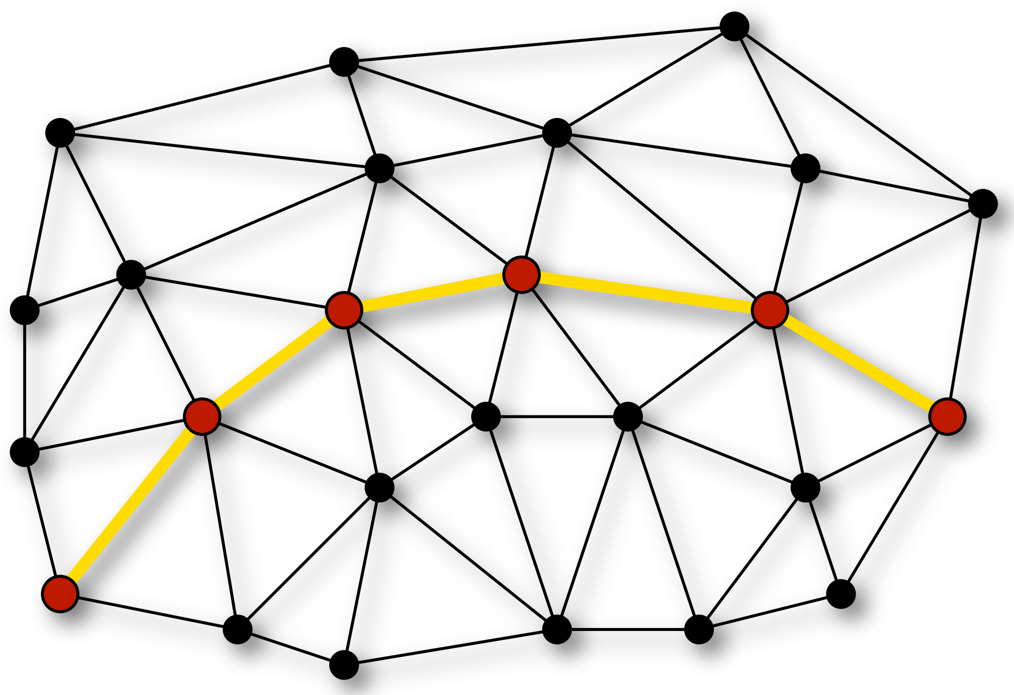


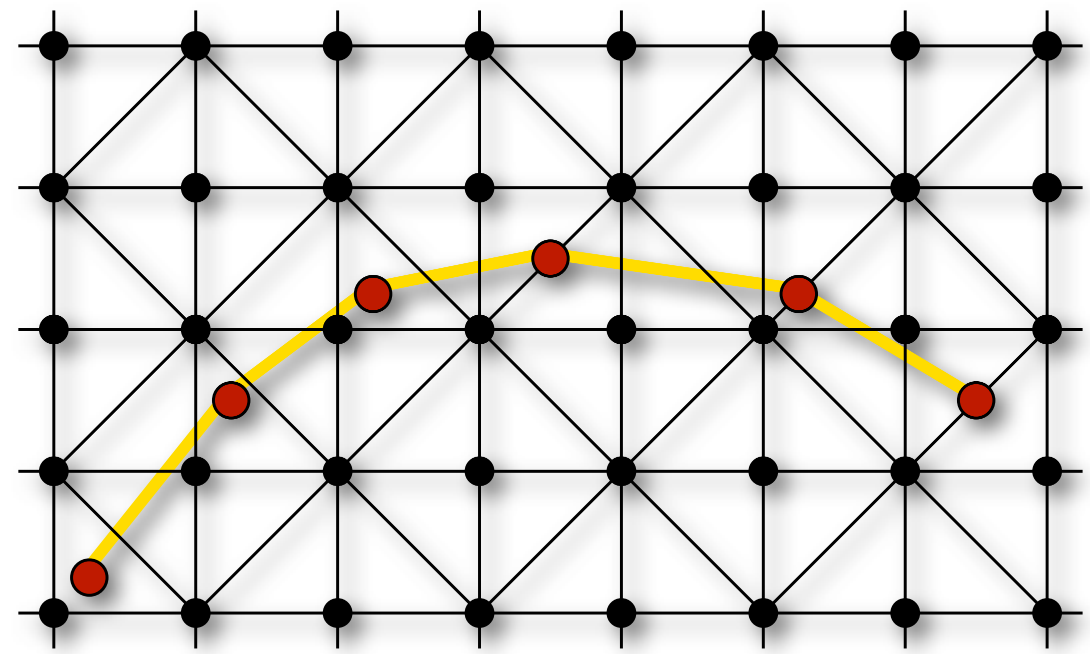
no-slip interface

- strongly discontinuous pressure
- weakly discontinuous velocity

Approximation of Non-smooth Solutions

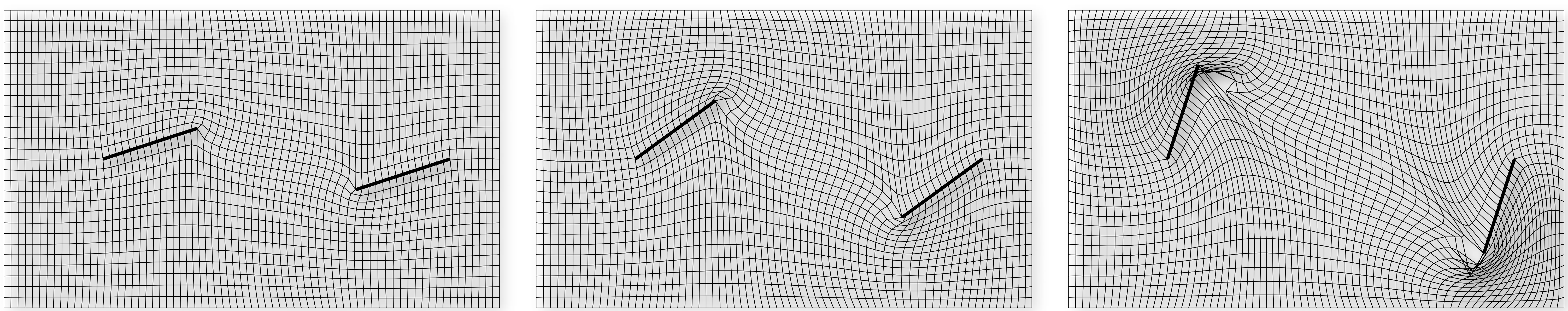
- Explicit Methods
- Implicit Methods





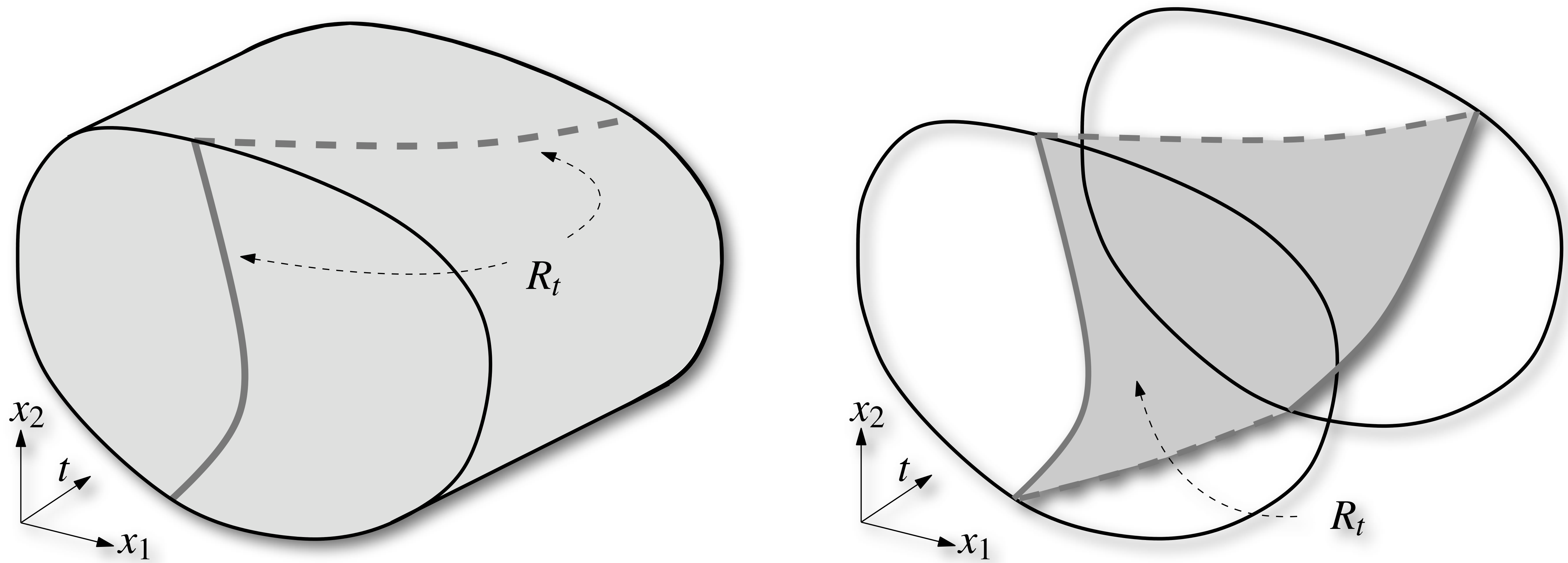
- fixed connectivity
- mesh moving / remeshing

- changing connectivity
- update of indicator function



Evolving Non-smooth Solutions

• The stated problem involves discontinuities at moving interfaces



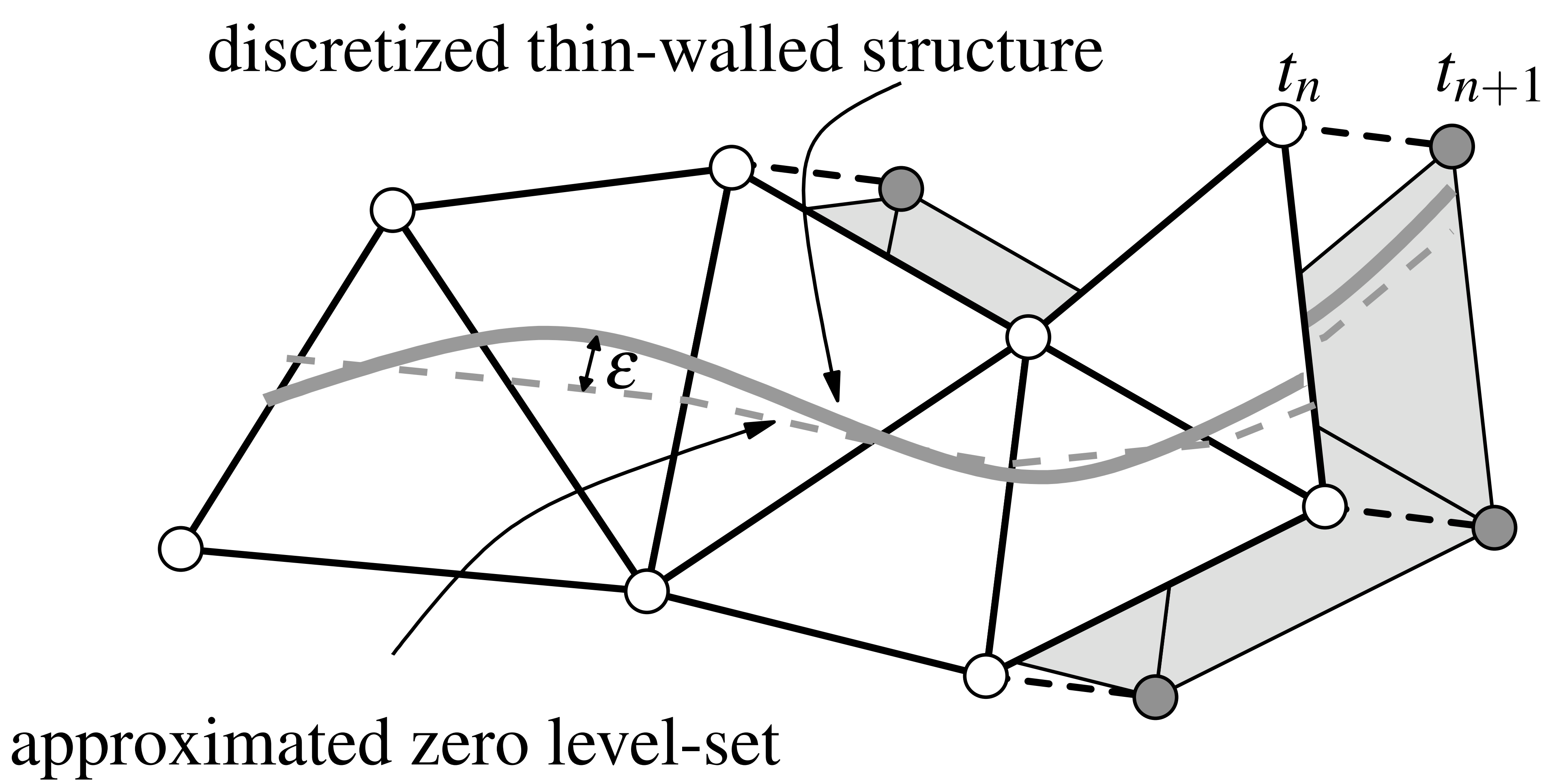
‣ Localization of non-smooth solution by implicit function (Level Sets)

‣ Incorporation of a priori knowledge on solution characteristics (XFEM)

Level Set Representation of Thin Structure

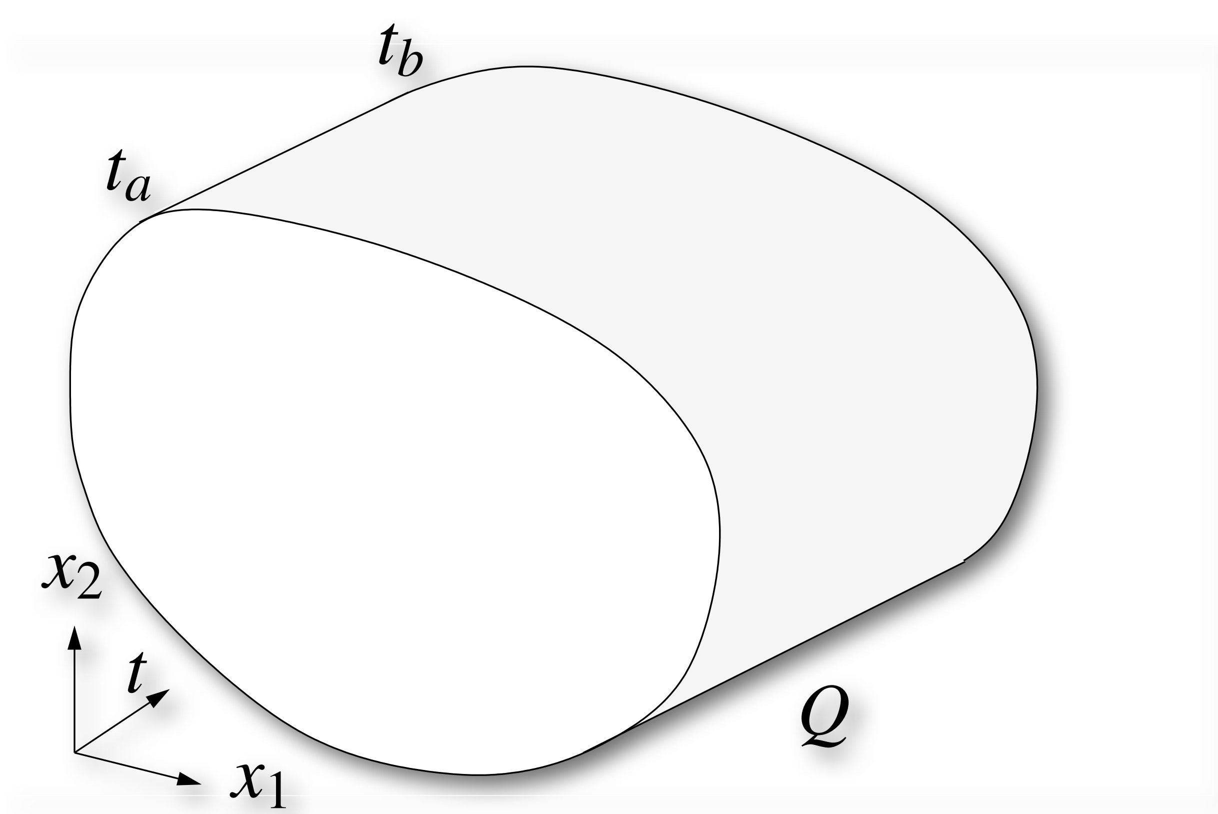
Construction of an updated space-time level set function from the current configuration of the thin structure using

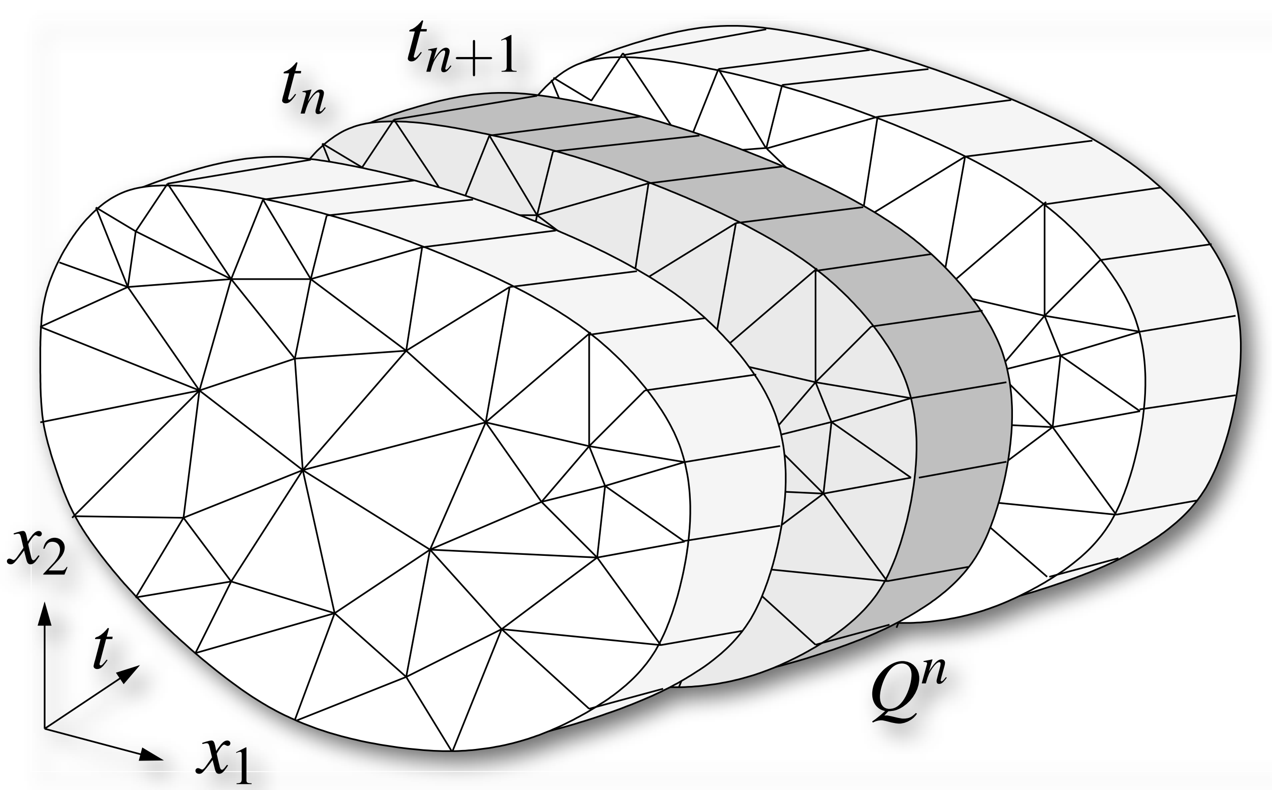
$$\phi(\mathbf{x},t) = \pm \min_{\mathbf{x}\in\Sigma}(\|\mathbf{x}(t) - \mathbf{x}^{\Sigma}(t)\|_2)$$



Space-Time Finite Element Approach

• Uniform discretization in space and time using finite elements





- ▸ Straightforward applicability of XFEM technology for evolving non-smooth solution in the space-time domain
- ▸ Enriched space-time finite elements enable the capturing of non-smooth solutions propagating through the domain

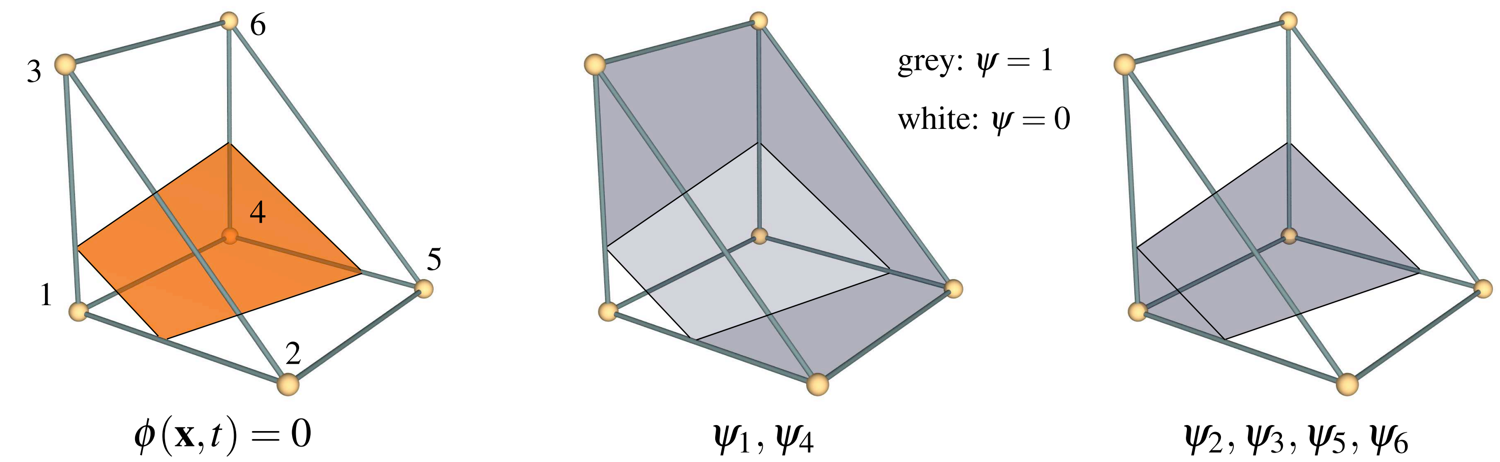
Extrinsic Enrichment of Space-Time FEM

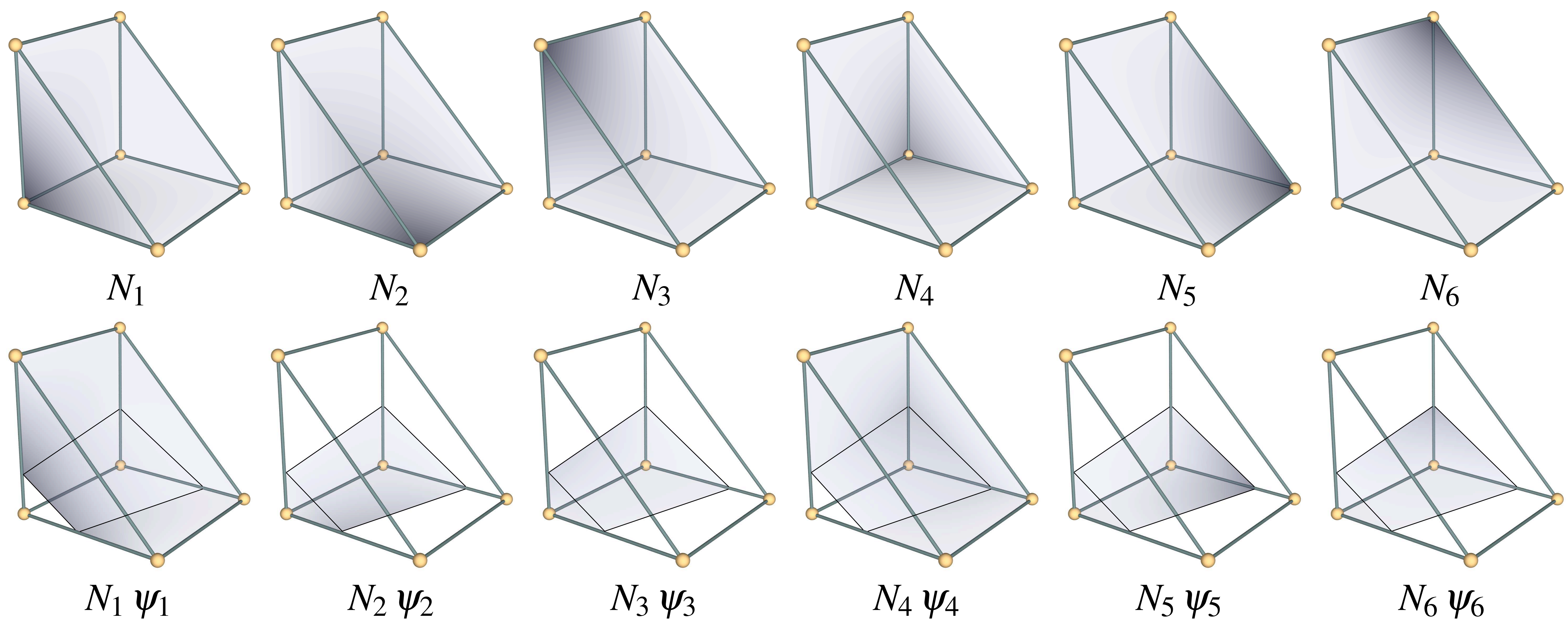
- Combination of level sets, local PUM and space-time finite elements

$$p^{\mathsf{F}}(\mathbf{x},t) = \sum_{k\in\mathcal{N}} N_k^{\mathsf{F}}(\mathbf{x},t)\; p^k + \sum_{j\in\mathcal{M}} N_j^{\mathsf{F}}(\mathbf{x},t)\,\psi_j(\mathbf{x},t)\; q^j$$

- Enrichment function (jump-type)

$$\psi_j(\mathbf{x},t) = \frac{1}{2}\left(1 - \operatorname{sign}\,\phi(\mathbf{x},t)\cdot \operatorname{sign}\,\phi(\mathbf{x}^j,t^j)\right)$$





Enriched Flow Approximation

- Enriched fluid velocity approximation

$$\mathbf{v}^{\mathsf{F}}(\mathbf{x},t) = \sum_{k\in\mathcal{N}} N_k^{\mathsf{F}}(\mathbf{x},t)\ \mathbf{v}^k + \sum_{j\in\mathcal{M}} N_j^{\mathsf{F}}(\mathbf{x},t)\psi_j(\mathbf{x},t)\ \mathbf{w}^j$$

- Enriched Fluid pressure approximation

$$p^{\mathsf{F}}(\mathbf{x},t) = \sum_{k\in\mathcal{N}} N_k^{\mathsf{F}}(\mathbf{x},t)\ p^k + \sum_{j\in\mathcal{M}} N_j^{\mathsf{F}}(\mathbf{x},t)\psi_j(\mathbf{x},t)\ q^j$$

✓ Strong and weak discontinuous flow solutions at immersed structure

➡ Enriched approximation decouples flow state at interface completely!

➡ Impose coupling conditions at fluid-structure interface!

Imposing Interface Conditions

- impose constraints at interface not explicitly discretized (meshed)
- constrained functions are touched by local enrichments (XFEM)
- varying number of involved degrees of freedom (current interface shape)

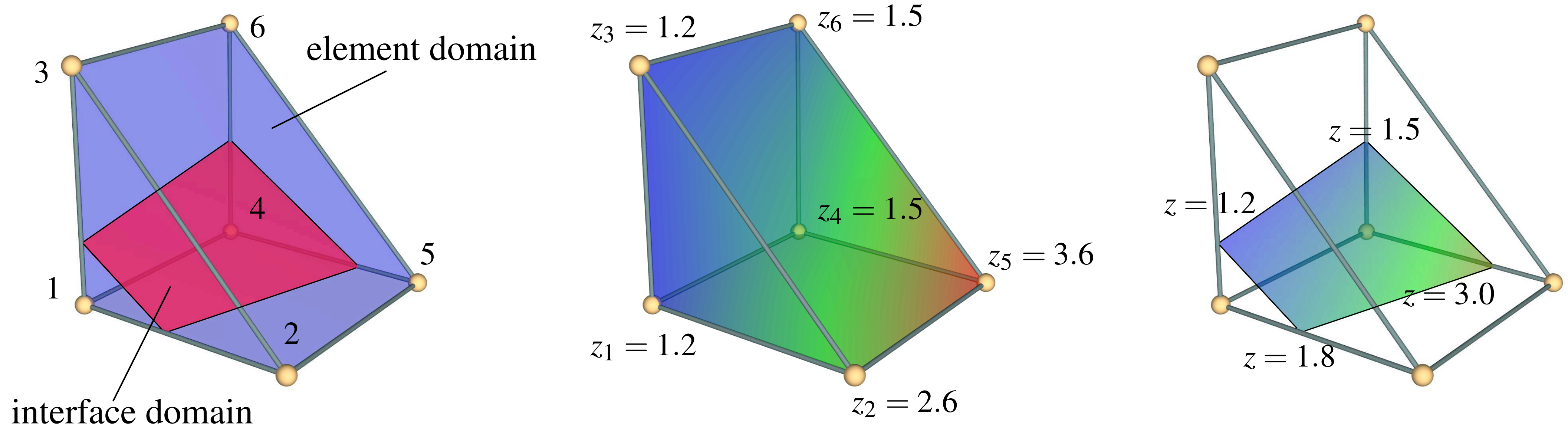
- Penalty methods?
- Lagrangian multiplier?
- Nitsche‘s method?
- etc.?

Perturbed Lagrangian Multiplier Technique

- implicit formulation of the Lagrange multiplier

$$\mathbf{t}^{\mathsf{F}} = \mathbf{z}^{\mathsf{F}}\,\delta_D(\phi)$$

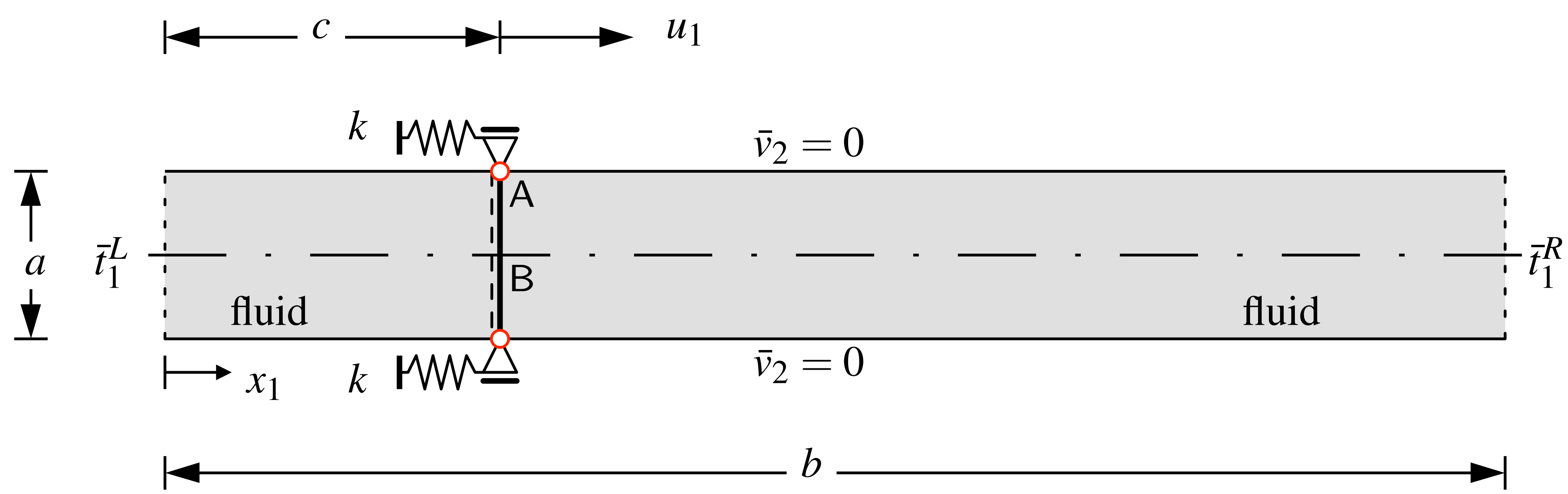
- for constraints at evolving interfaces in combination with Space-Time-FEM



- transformed weak coupling conditions (with perturbation)

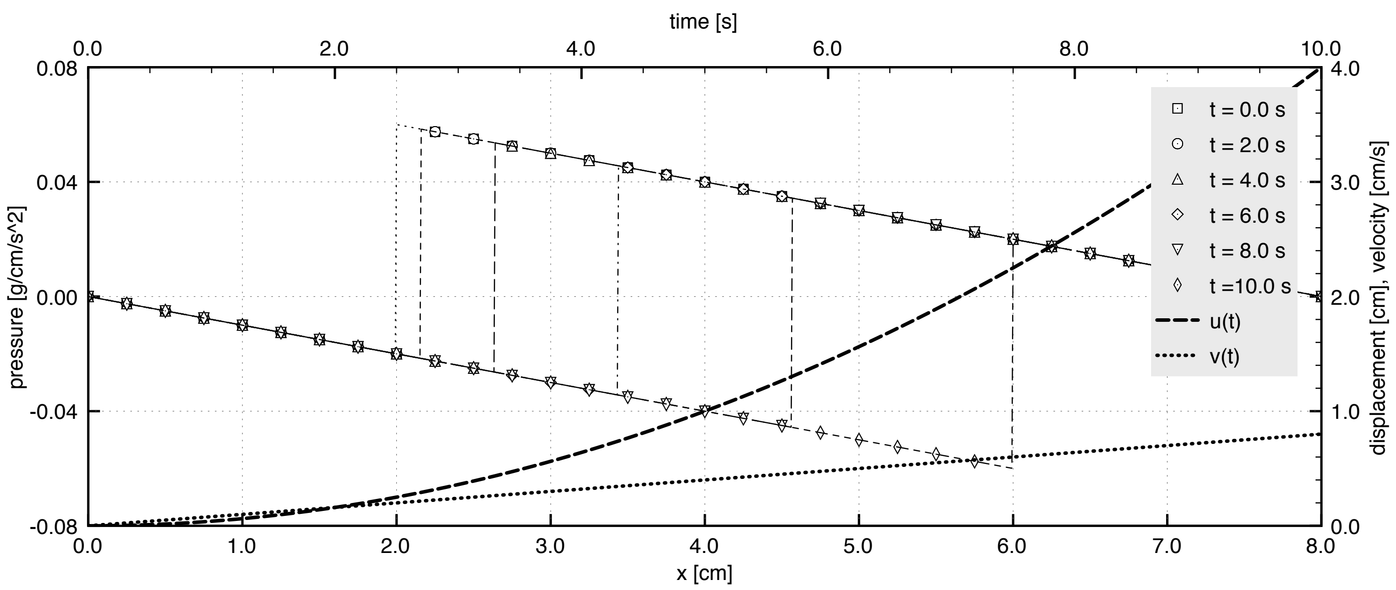
$$+\int_{R_t^{n,+}} \delta\mathbf{z}^{\mathsf{F}+}\cdot(\mathbf{v}^{\mathsf{F}+}-\mathbf{v}^{\mathsf{S}})\,\mathrm{d}R - \int_{R_t^{n,+}} \delta\mathbf{v}^{\mathsf{F}+}\cdot\mathbf{z}^{\mathsf{F}+}\,\mathrm{d}R + \int_{R_t^{n,+}} \delta\mathbf{v}^{\mathsf{S}}\cdot\mathbf{z}^{\mathsf{F}+}\,\mathrm{d}R$$
$$+\int_{R_t^{n,-}} \delta\mathbf{z}^{\mathsf{F}-}\cdot(\mathbf{v}^{\mathsf{F}-}-\mathbf{v}^{\mathsf{S}})\,\mathrm{d}R - \int_{R_t^{n,-}} \delta\mathbf{v}^{\mathsf{F}-}\cdot\mathbf{z}^{\mathsf{F}-}\,\mathrm{d}R + \int_{R_t^{n,-}} \delta\mathbf{v}^{\mathsf{S}}\cdot\mathbf{z}^{\mathsf{F}-}\,\mathrm{d}R$$
$$+\sum_{e\in\mathscr{Z}}\int_{Q_t^{n,e}} \delta\mathbf{z}^{\mathsf{F}+}\cdot\tau_z\mathbf{z}^{\mathsf{F}+} + \sum_{e\in\mathscr{Z}}\int_{Q_t^{n,e}} \delta\mathbf{z}^{\mathsf{F}-}\cdot\tau_z\mathbf{z}^{\mathsf{F}-},$$

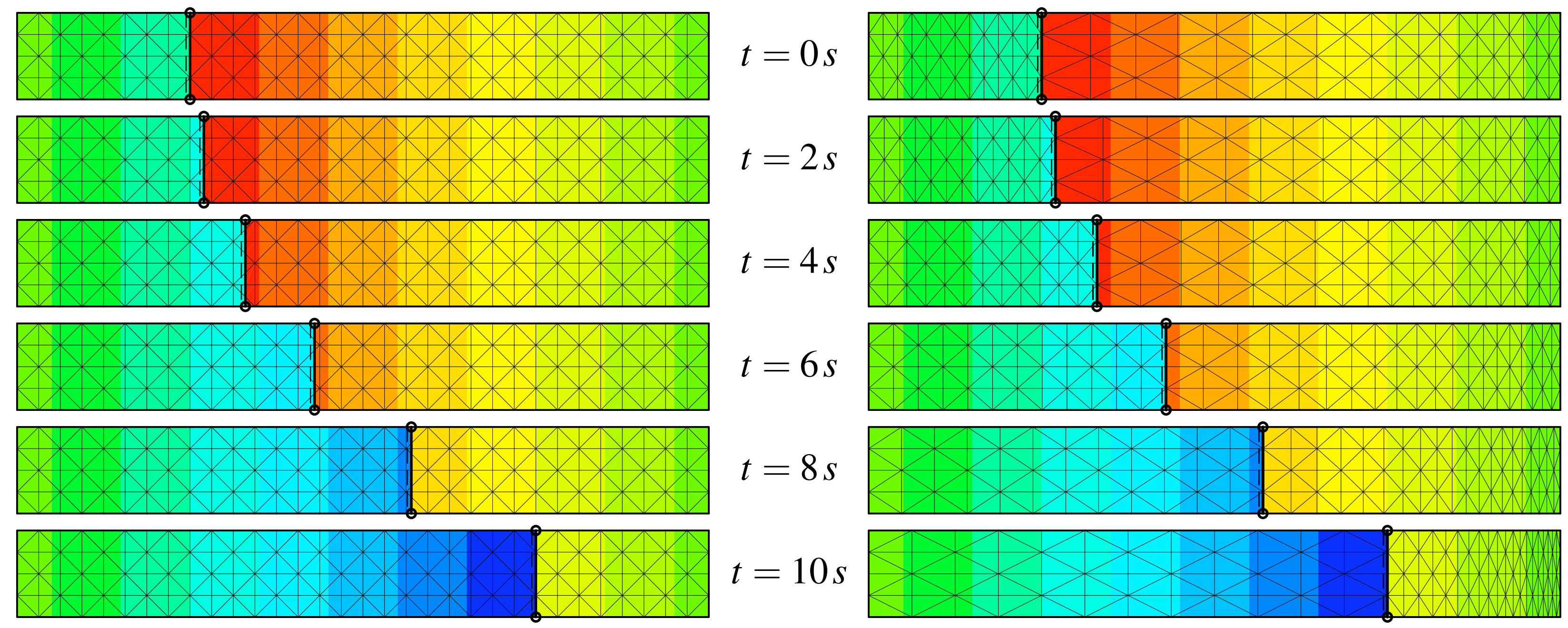
Example: Piston I - Problem Setup

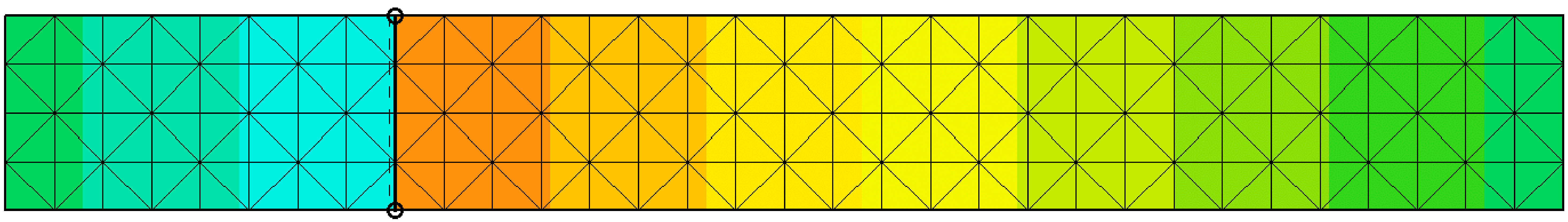


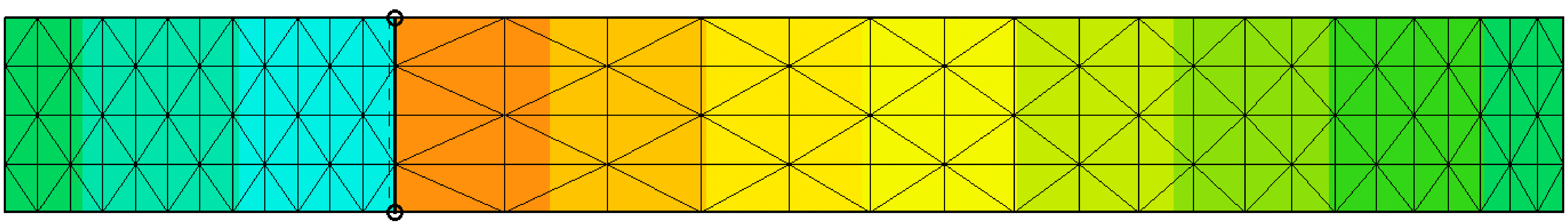
$$p(x_1,t) = \begin{cases} \bar{t}_1^L - \rho_f\, v^{\mathsf{A}}_{1,t}(t)\, x_1 & x_1 < X^{\mathsf{A}}(t) \\ \bar{t}_1^R + \rho_f\, v^{\mathsf{A}}_{1,t}(t)\,(b - x_1) & x_1 > X^{\mathsf{A}}(t) \end{cases}$$

Example: Piston II - Prescribed Motion

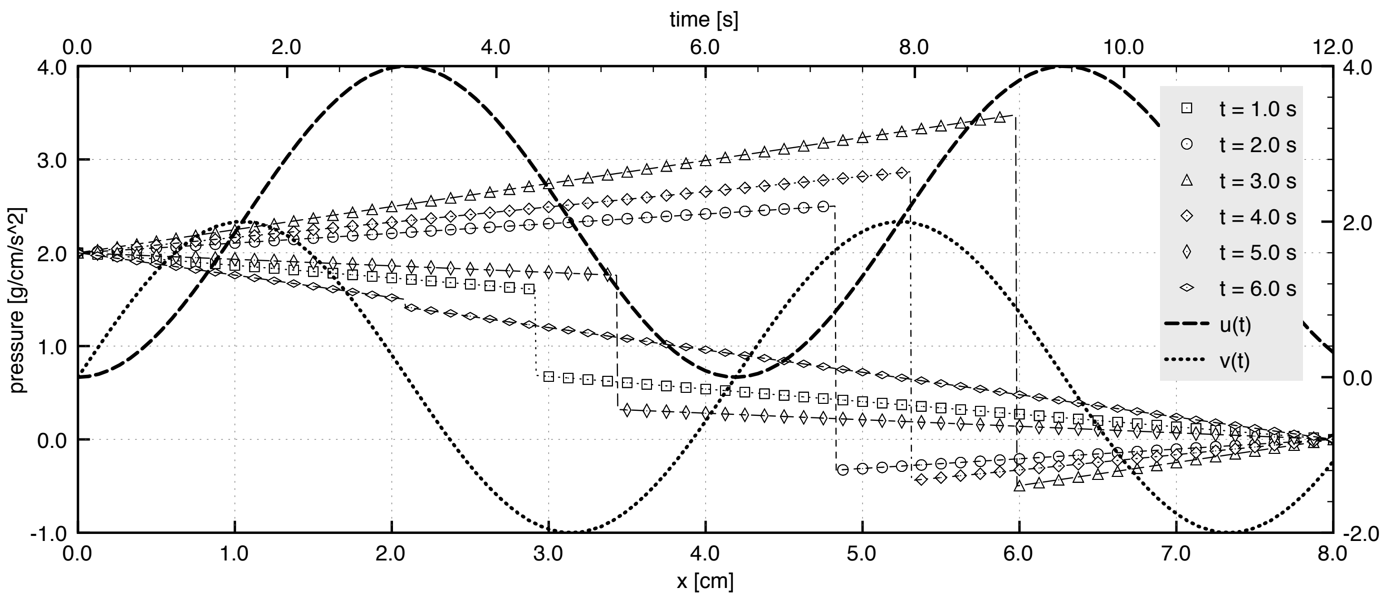


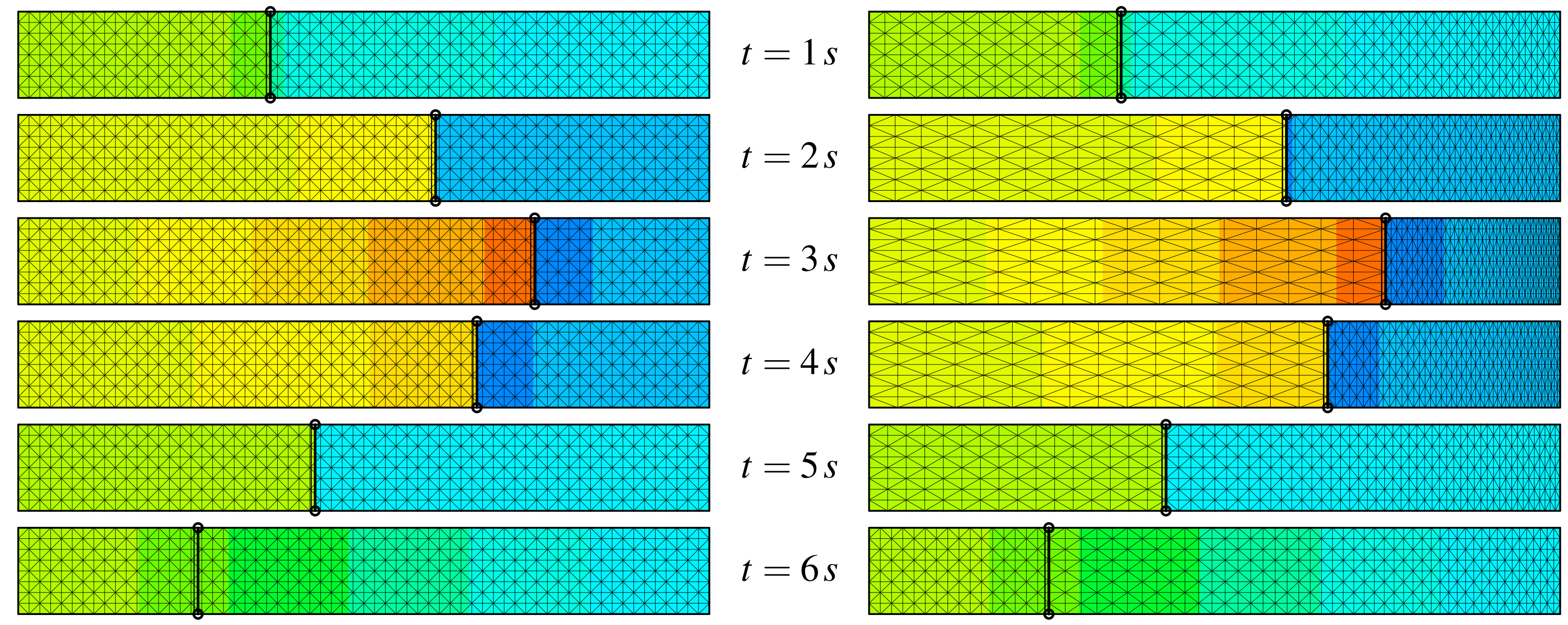


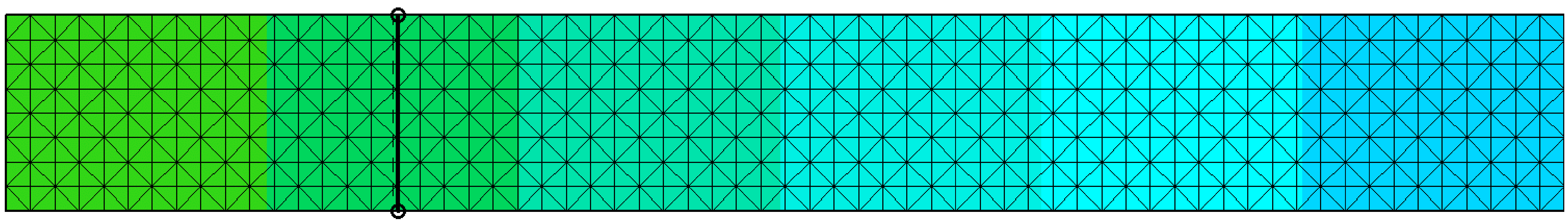


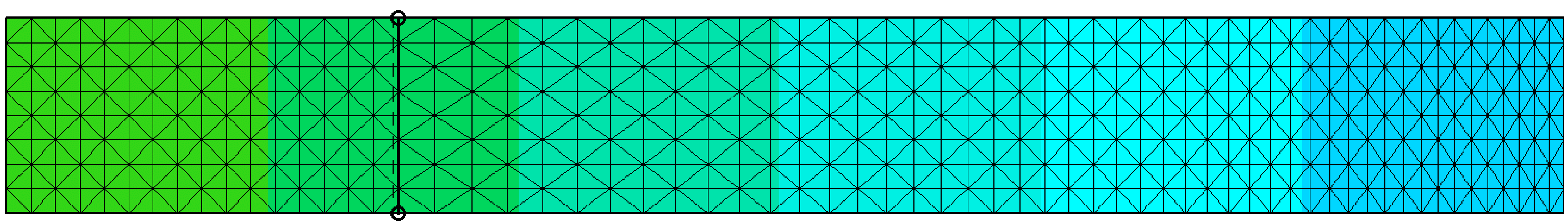


Example: Piston II - Rigid with Spring

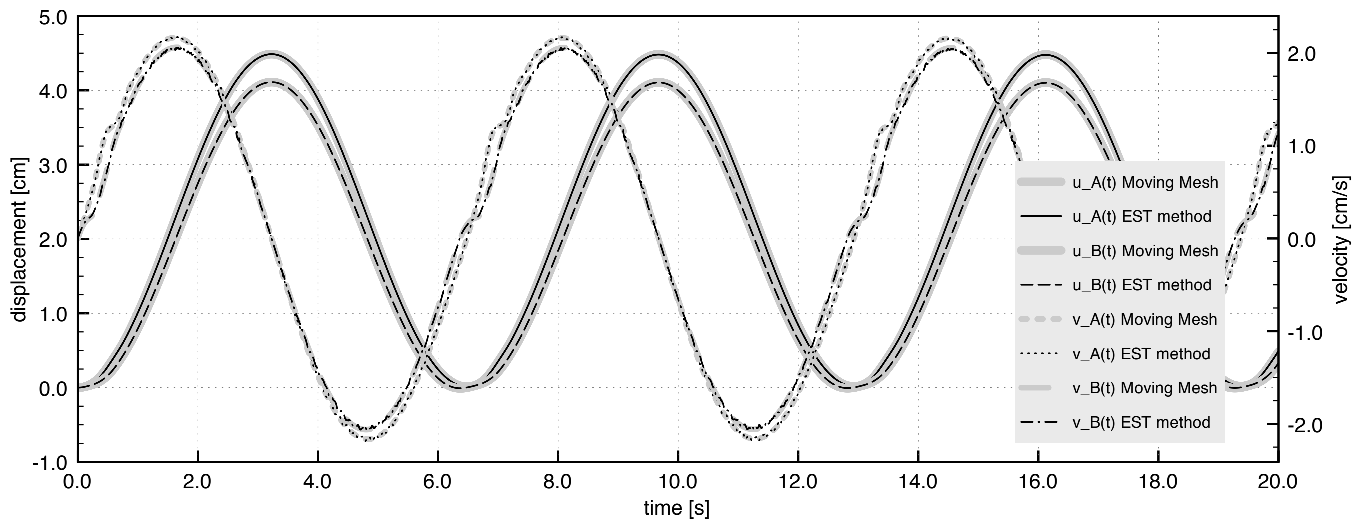


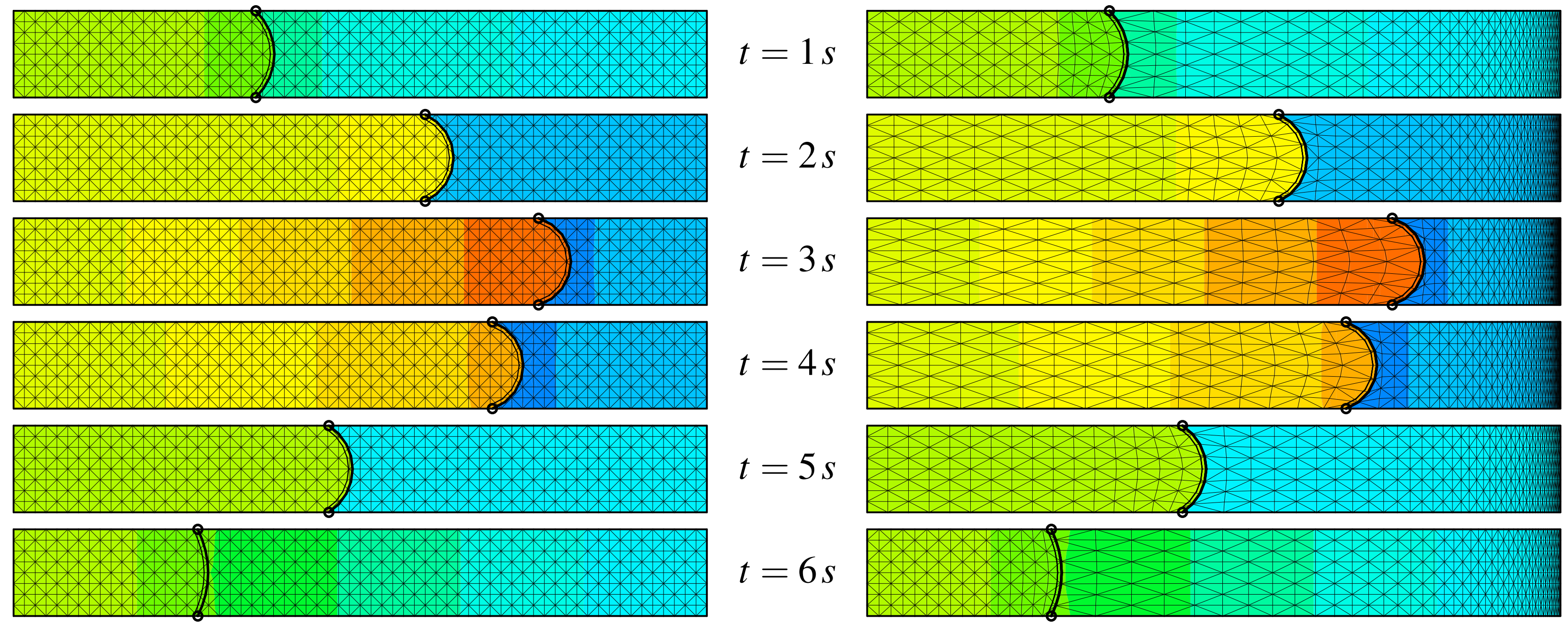


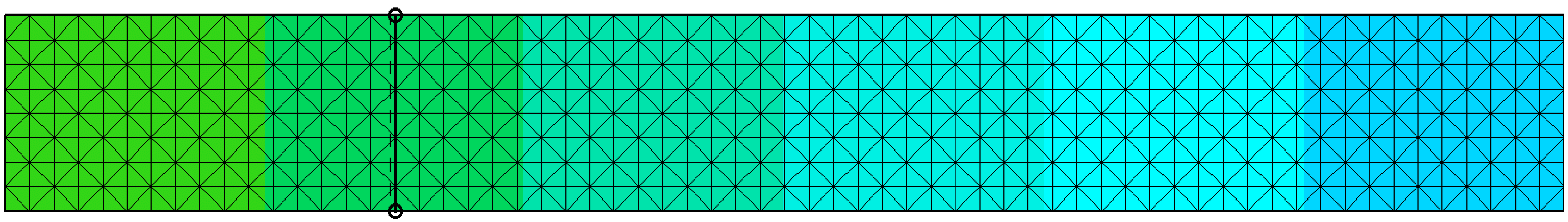


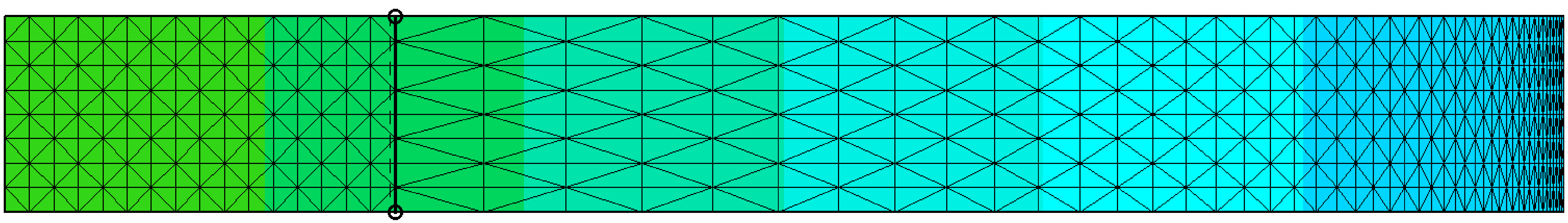


Example: Piston III - Flexible with Spring

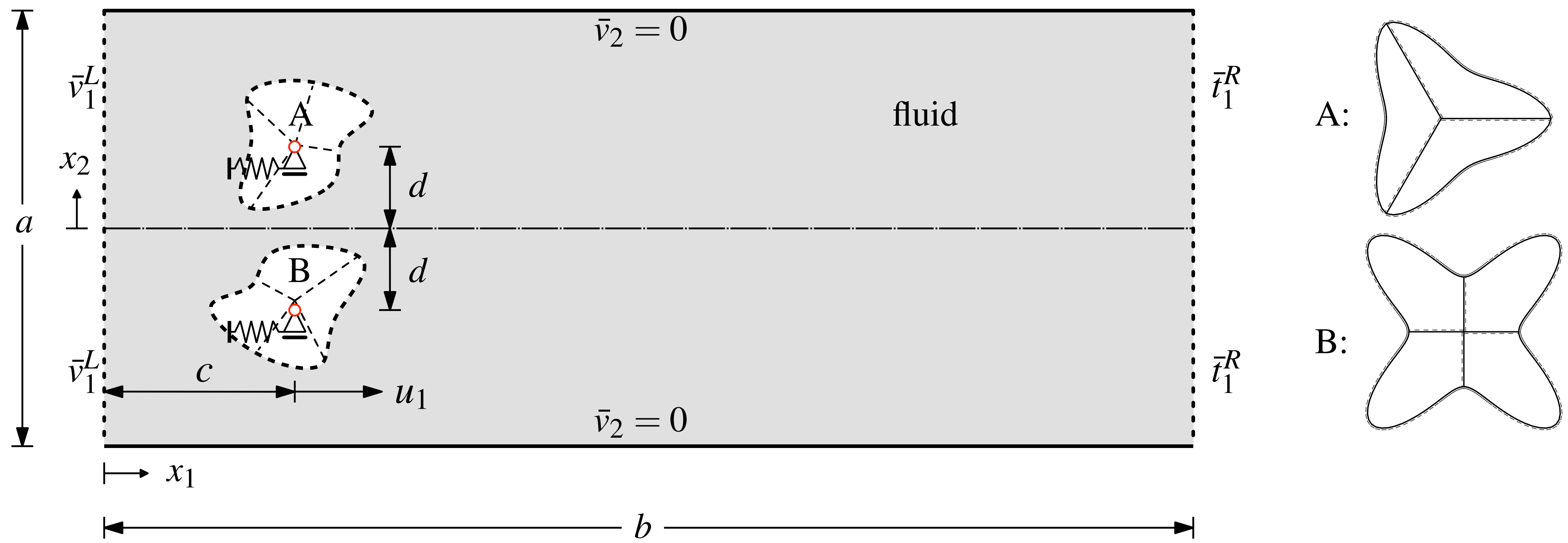


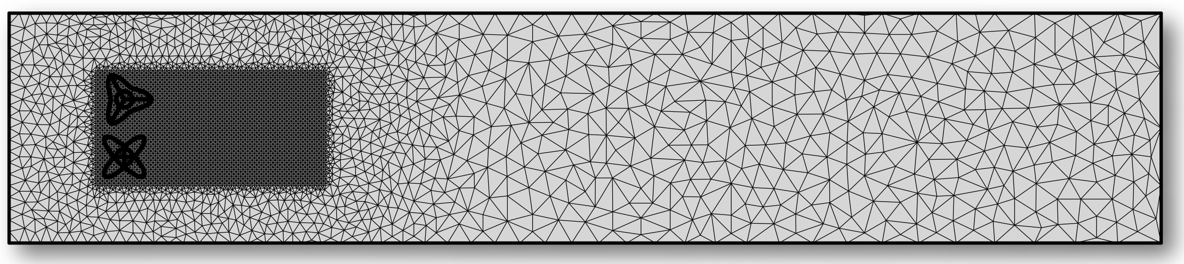


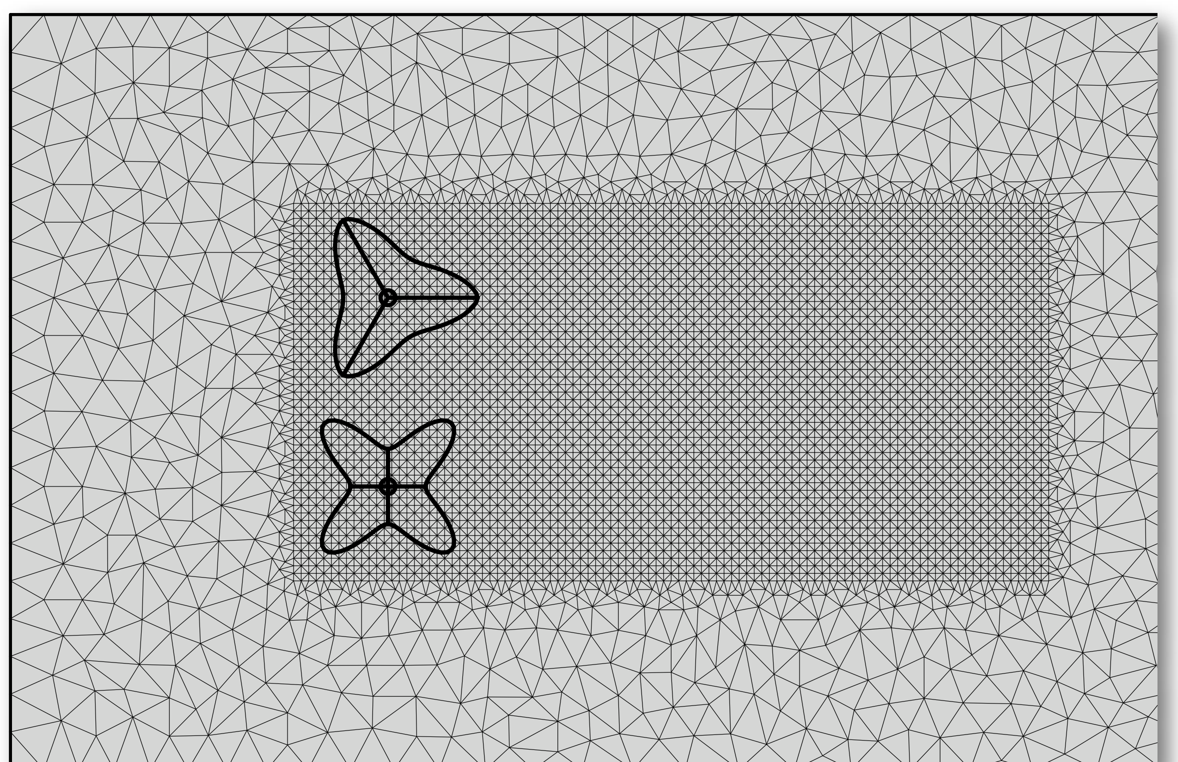


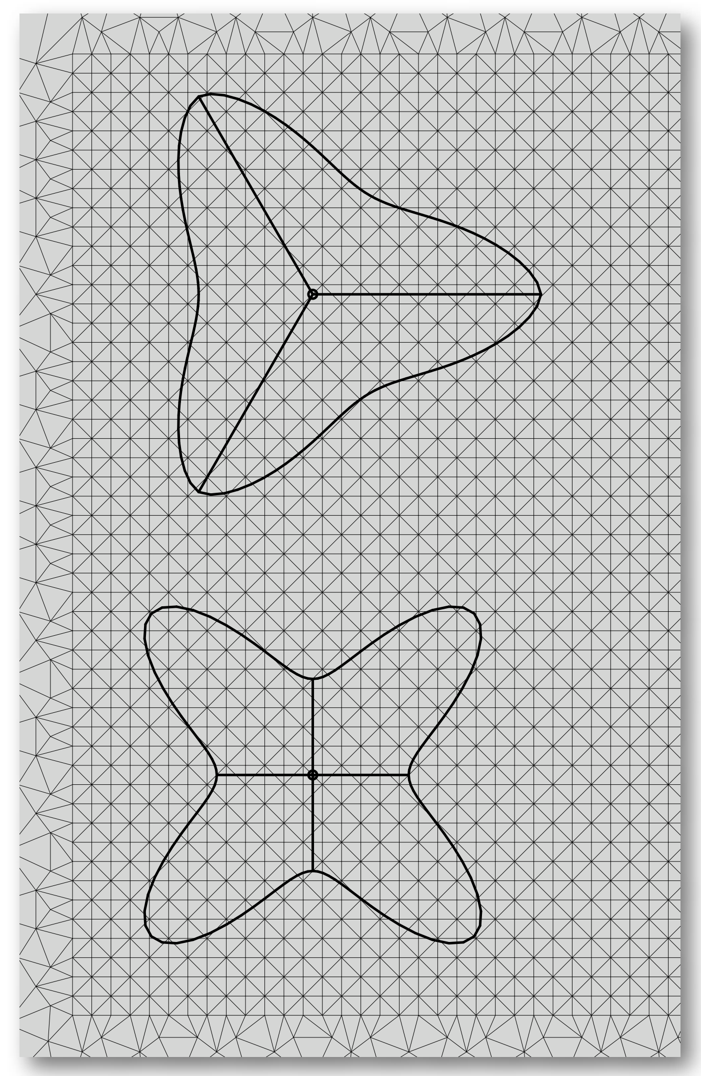


Example: Multiple Rotating Structures I

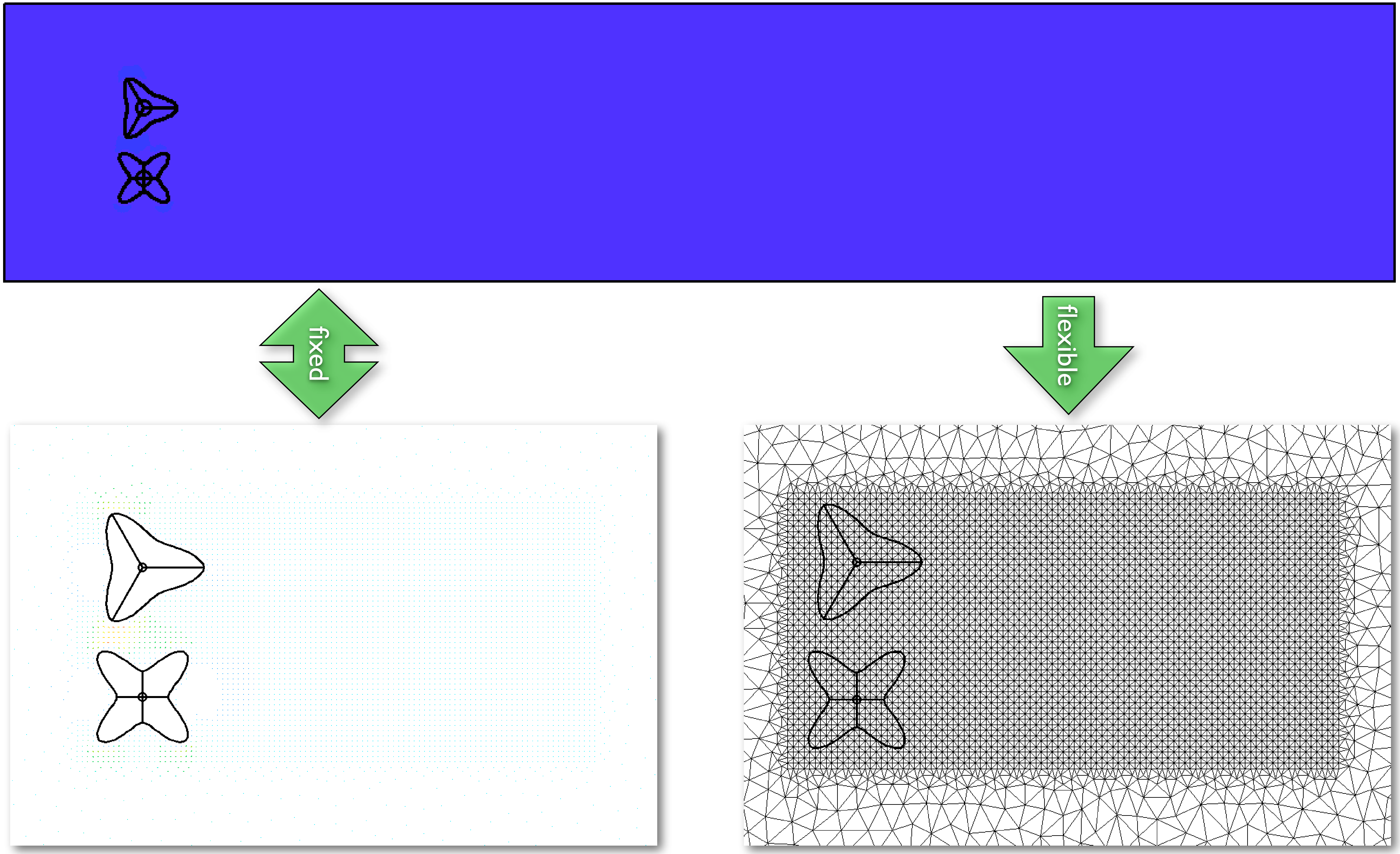








Example: Multiple Rotating Structures II



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