Dynamics of an adaptive vibration absorber

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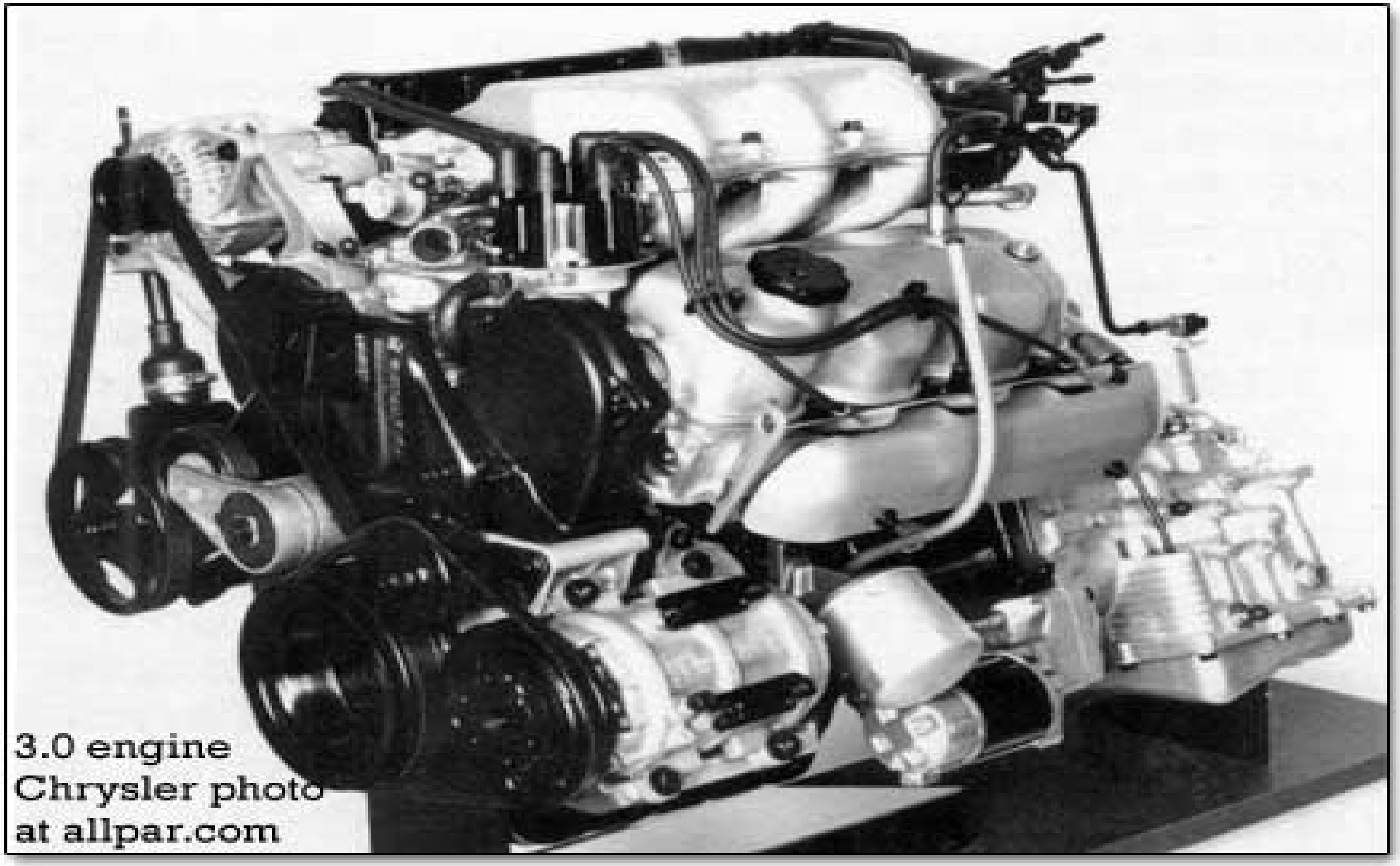
Outline

- Motivation
- Design
- Mathematical model
- Adaptive configurations
  - Stability
  - Adaptability
- Future work

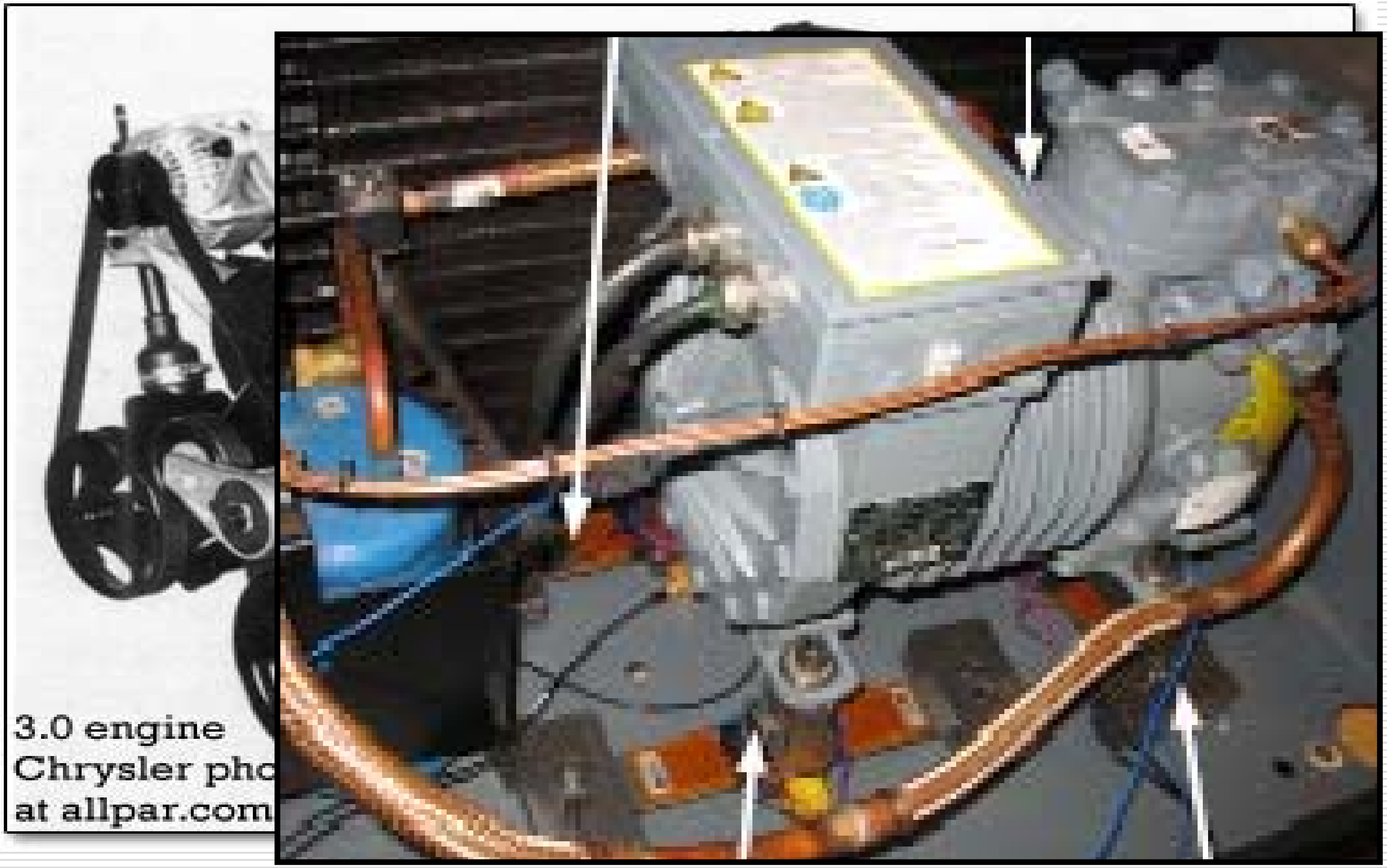
Motivation

- Vibrations generated by reciprocating machinery, such as, for example, internal combustion engines or piston type compressors.

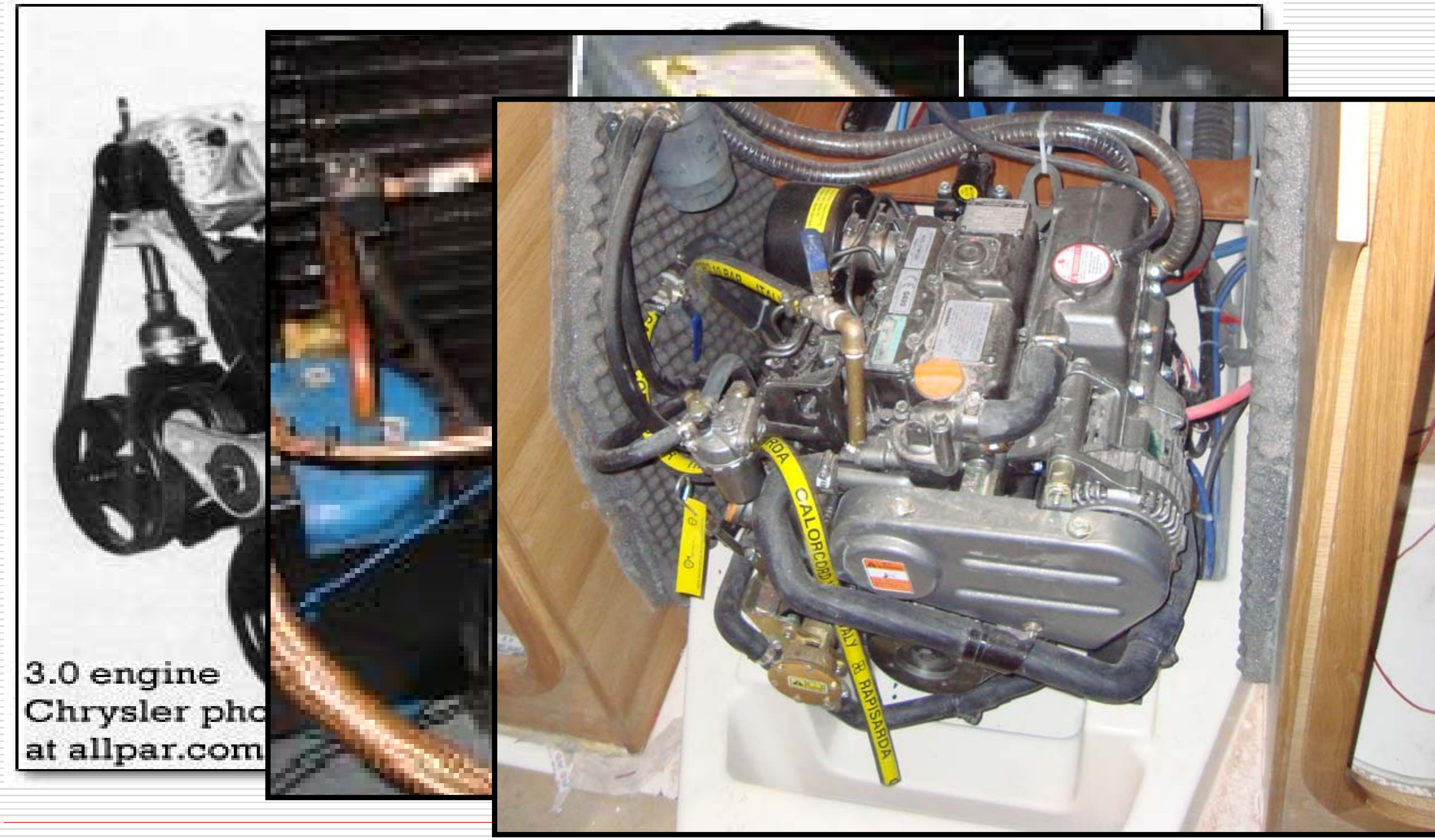
Motivation



Motivation



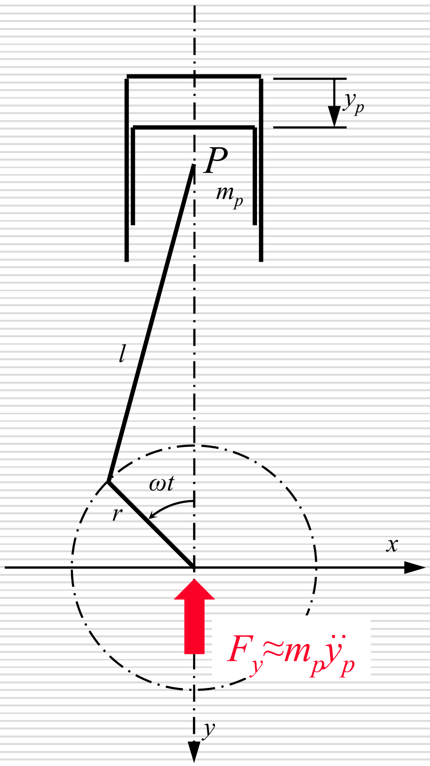
Motivation



Motivation

- Such machines generate vibrations at discrete frequencies: multiples of the machine RPM.
- These multiples are often called ENGINE ORDERS.

Motivation



$$y_p = r(1-\cos\omega t) + l\left(1-\sqrt{1-\left(\frac{r}{l}\right)^2 \sin^2\omega t}\right)$$

$\frac{l}{r} = \alpha$

$$\frac{y_p}{r} = 1-\cos\omega t + \alpha - \alpha\sqrt{1-\frac{\sin^2\omega t}{\alpha^2}}$$

Taylor's series expansion; up to the 7th order

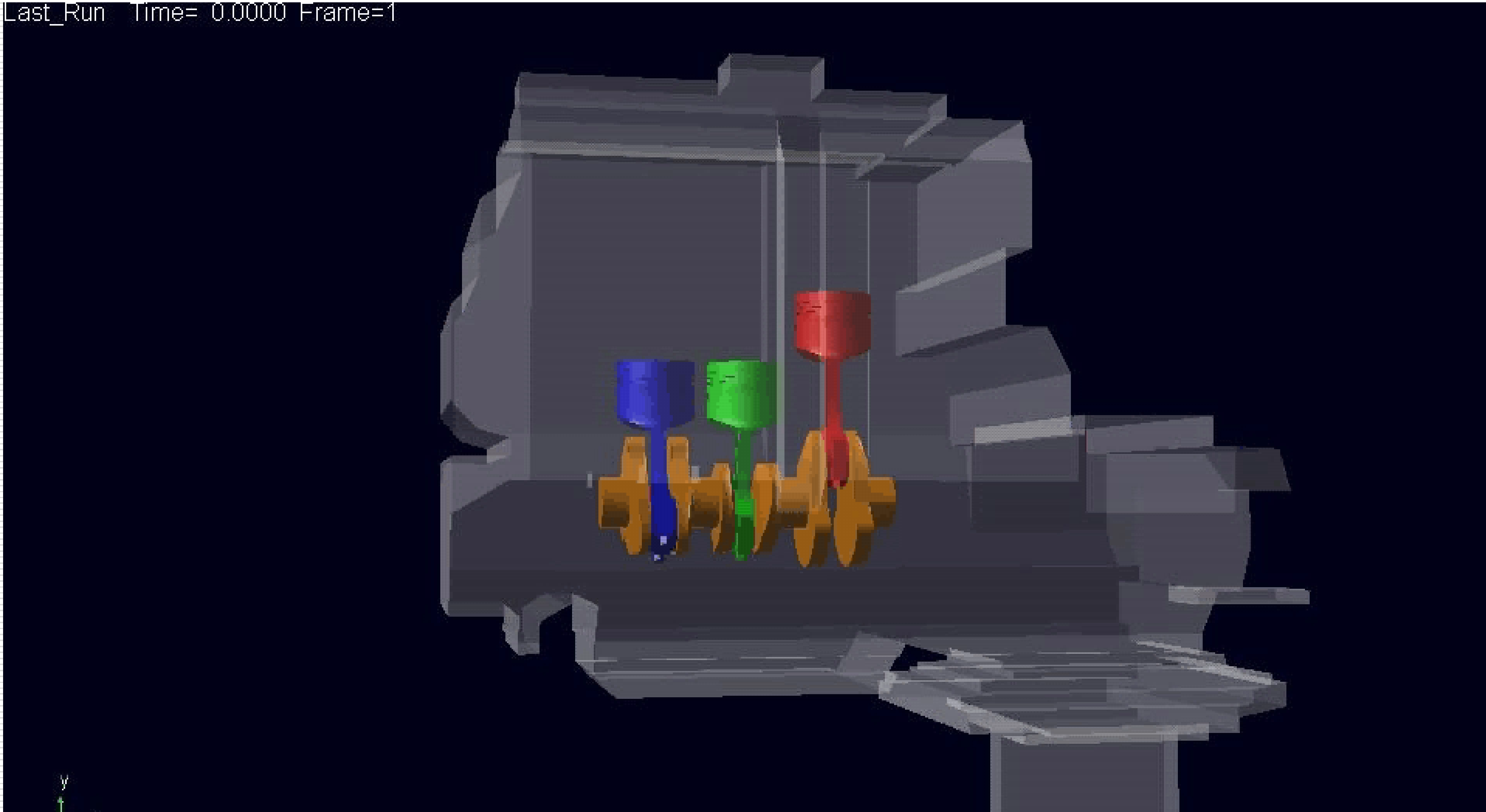
$$\frac{y_p}{r} = 1-\cos(\omega t) + \frac{1}{2\alpha}\sin^2(\omega t) + \frac{1}{8\alpha^3}\sin^4(\omega t) + \frac{1}{16\alpha^5}\sin^6(\omega t) + O\left(\frac{1}{\alpha^7}\right)$$

$\sin^2\alpha = \frac{1-\cos 2\alpha}{2}; \frac{d^2}{dt^2}$

$$\frac{\ddot{y}_p}{r} = \omega^2\left[\cos\omega t + \frac{128\alpha^4+32\alpha^2+15}{128\alpha^5}\cos(2\omega t) - \frac{8\alpha^2+6}{32\alpha^5}\cos(4\omega t) + \frac{9}{128\alpha^5}\cos(6\omega t)\right]$$

„ENGINE ORDERS"

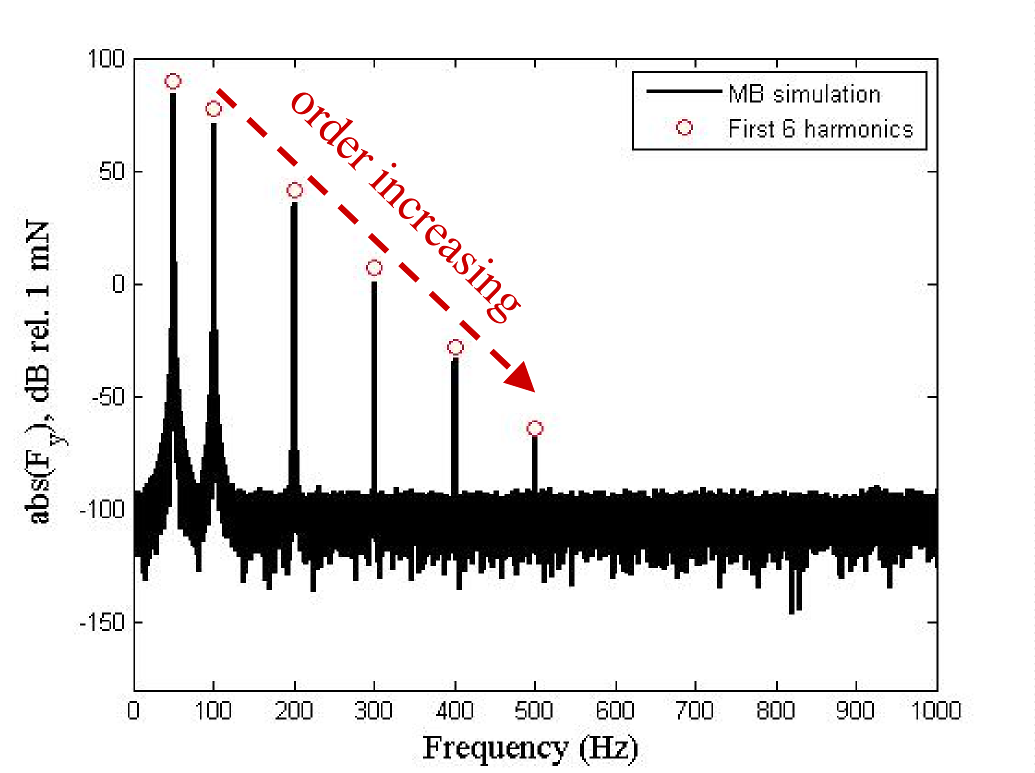
Motivation



Motivation

- Lower orders are more important because the vibration amplitude quickly decreases with increase of the engine order…

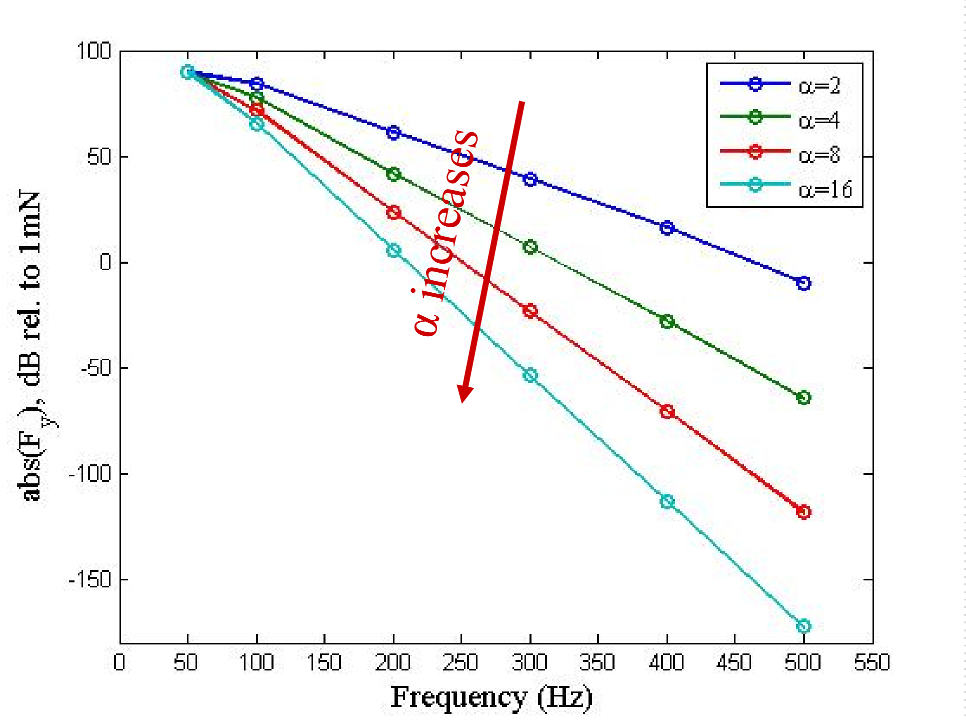
$$N = 3000\,\mathrm{min}^{-1}; \frac{l}{r} = \alpha = 4$$



Motivation

- …especially for relatively long connecting rods.

$$\alpha = \frac{l}{r}$$



Motivation

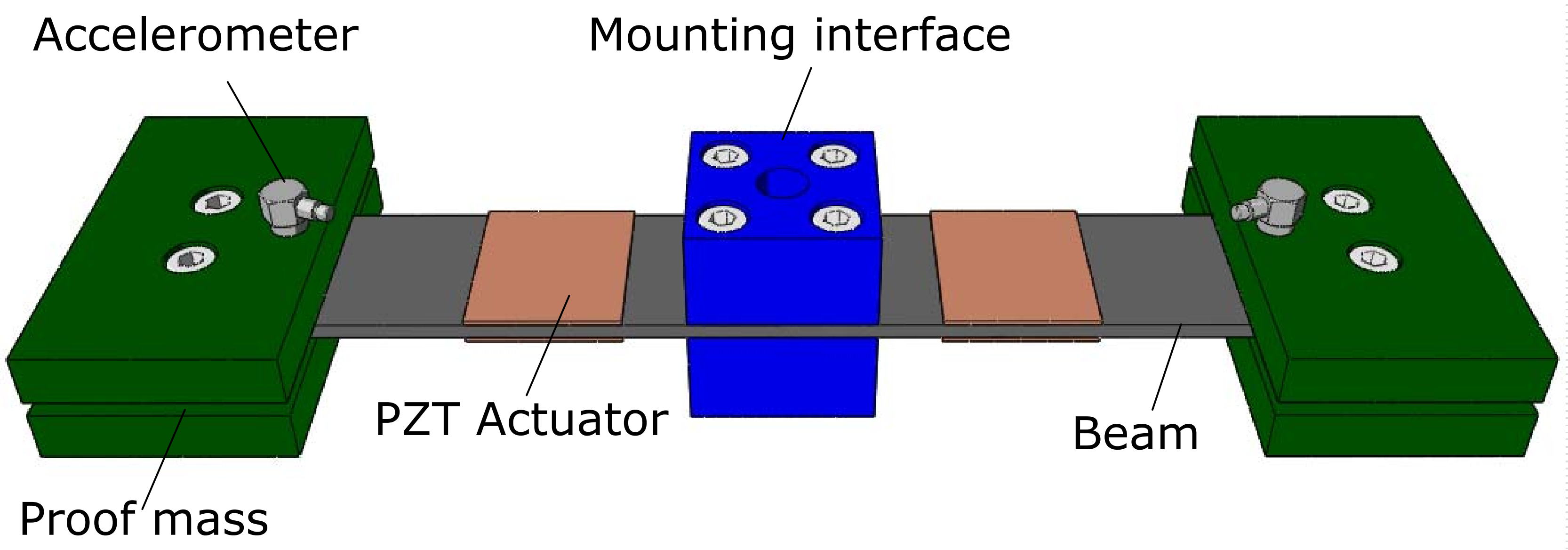
- A possible solution might be an adaptive vibration absorber.
- Such devices efficiently absorb vibrations at their resonant frequency.
- The resonant frequency should be tunable so as to follow the machine RPM.

Design

- Simple construction
- Low cost
- Little space required at the mounting point

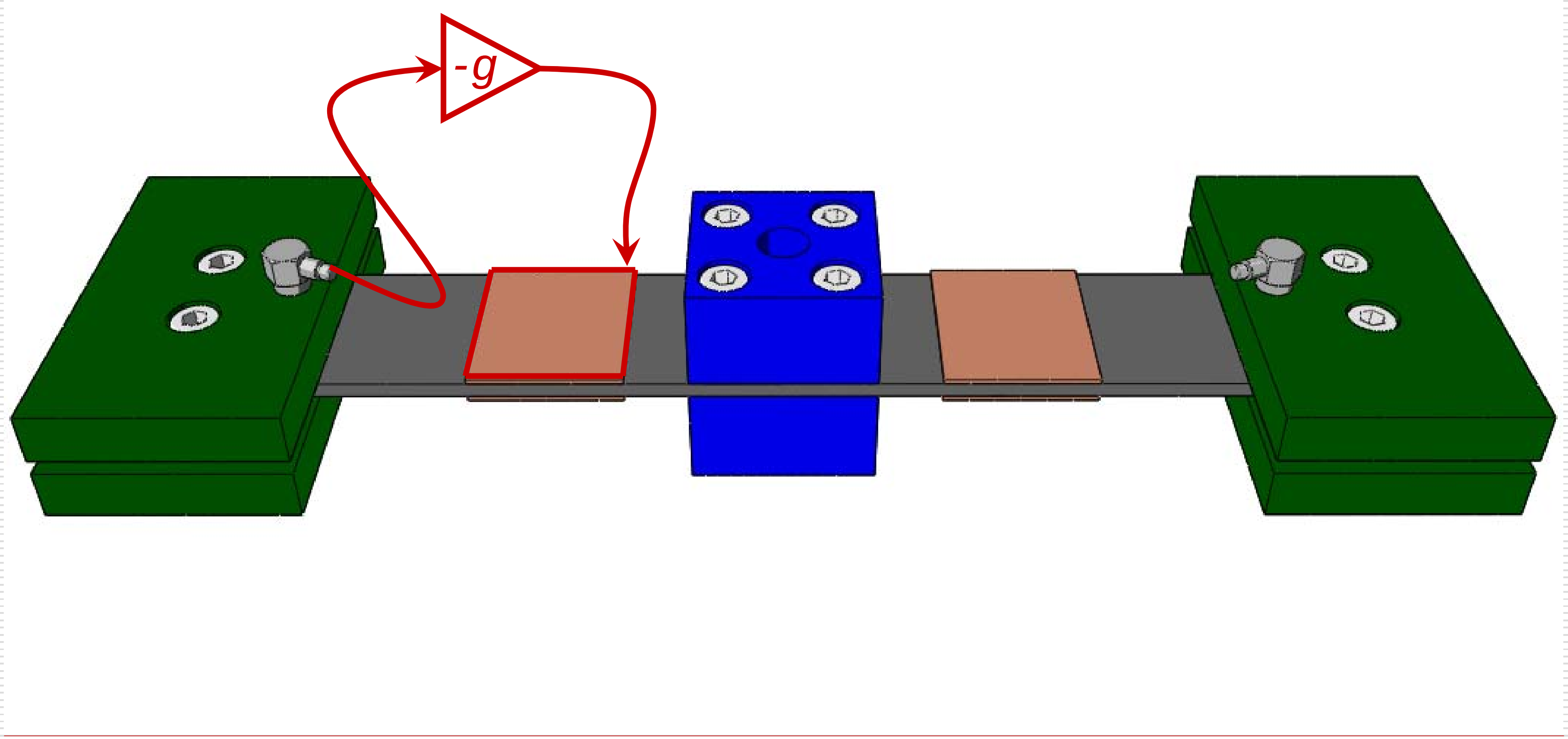
Design

- Beam type vibration absorber



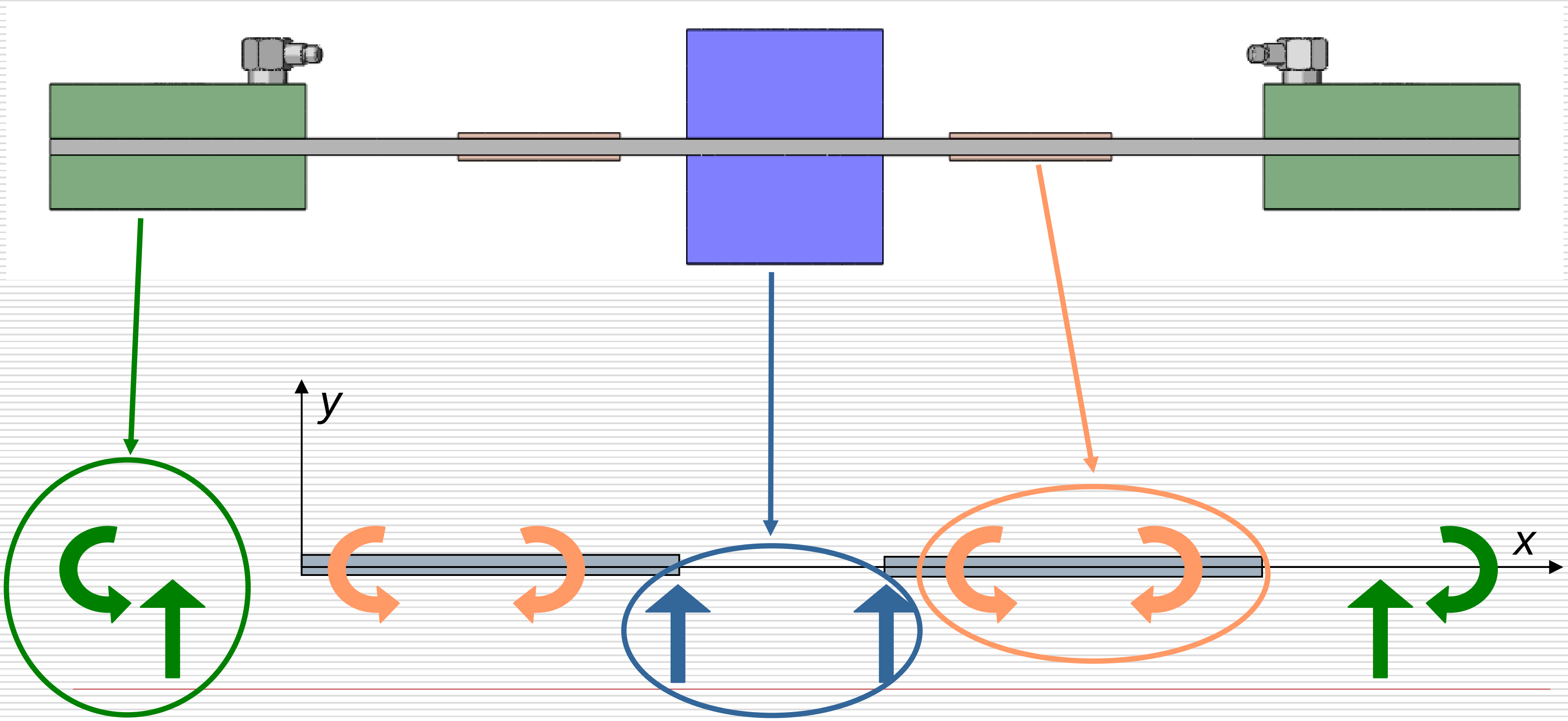
Design

- Negative acceleration feedback – active mass



Mathematical model

- Mobility – impedance approach



Mathematical model: Formulation

$$\mathbf{V} = \mathbf{YF} + \mathbf{YF'}$$

$$\mathbf{F'} = \mathbf{ZV}$$

$$(\mathbf{I} + j\omega\mathbf{QG})^{-1} = \mathbf{R}$$

$$\mathbf{F}_{\mathbf{active}} = -j\omega\mathbf{GV}_{\mathbf{active}}$$

$$\mathbf{V} = (\mathbf{I} - \mathbf{YZ})^{-1}\mathbf{YF}$$

$$\mathbf{Z} = \begin{bmatrix} \mathbf{Z}^{f,v} & \mathbf{Z}^{t,v} \\ \mathbf{Z}^{f,\alpha} & \mathbf{Z}^{t,\alpha} \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{V} \\ \boldsymbol{\alpha} \end{bmatrix} = \begin{bmatrix} \mathbf{Y}^{v,f} & \mathbf{Y}^{v,t} \\ \mathbf{Y}^{\alpha,f} & \mathbf{Y}^{\alpha,t} \end{bmatrix} \begin{bmatrix} \mathbf{f} \\ \mathbf{t} \end{bmatrix} + \begin{bmatrix} \mathbf{Y}^{v,f} & \mathbf{Y}^{v,t} \\ \mathbf{Y}^{\alpha,f} & \mathbf{Y}^{\alpha,t} \end{bmatrix} \begin{bmatrix} \mathbf{f'} \\ \mathbf{t'} \end{bmatrix}$$

$$\mathbf{V} = \mathbf{QF}$$

$$(\mathbf{I} - \mathbf{YZ})^{-1}\mathbf{Y} = \mathbf{Q}$$

$$\mathbf{V} - \mathbf{YZV} = \mathbf{YF}$$

$$\mathbf{V}_{\mathbf{active}} = \mathbf{QF} + \mathbf{Q}(-j\omega\mathbf{GV}_{\mathbf{active}})$$

$$(\mathbf{I} - \mathbf{YZ})\mathbf{V} = \mathbf{YF}$$

$$\mathbf{V}_{\mathbf{active}} + j\omega\mathbf{QGV}_{\mathbf{active}} = \mathbf{QF}$$

$$\mathbf{V} = \mathbf{YF} + \mathbf{YZV}$$

$$\mathbf{V}_{\mathbf{active}} = \mathbf{RQF}$$

$$\mathbf{RQ} = \mathbf{T}$$

$$(\mathbf{I} + j\omega\mathbf{QG})\mathbf{V}_{\mathbf{active}} = \mathbf{QF}$$

$$\mathbf{V}_{\mathbf{active}} = \mathbf{QF} + \mathbf{QF}_{\mathbf{active}}$$

$$\mathbf{V}_{\mathbf{active}} = \mathbf{TF}$$

$$\mathbf{V}_{\mathbf{active}} = (\mathbf{I} + j\omega\mathbf{QG})^{-1}\mathbf{QF}$$

Mathematical model: Mobility matrices

$$\mathbf{Y}^{v,f}=\begin{bmatrix} Y^{v,f}_{m1,m1} & Y^{v,f}_{m1,m2} & Y^{v,f}_{m1,m3} & Y^{v,f}_{m1,c1} & Y^{v,f}_{m1,c2} & \cdots & Y^{v,f}_{m1,cM} & Y^{v,f}_{m1,e1} & Y^{v,f}_{m1,e2} & \cdots & Y^{v,f}_{m1,eN} \\ Y^{v,f}_{m2,m1} & Y^{v,f}_{m2,m2} & Y^{v,f}_{m2,m3} & Y^{v,f}_{m2,c1} & Y^{v,f}_{m2,c2} & \cdots & Y^{v,f}_{m2,cM} & Y^{v,f}_{m2,e1} & Y^{v,f}_{m2,e2} & \cdots & Y^{v,f}_{m2,eN} \\ Y^{v,f}_{m3,m1} & Y^{v,f}_{m3,m2} & Y^{v,f}_{m3,m3} & Y^{v,f}_{m3,c1} & Y^{v,f}_{m3,c2} & \cdots & Y^{v,f}_{m3,cM} & Y^{v,f}_{m3,e1} & Y^{v,f}_{m3,e2} & \cdots & Y^{v,f}_{m3,eN} \\ Y^{v,f}_{c1,m1} & Y^{v,f}_{c1,m2} & Y^{v,f}_{c1,m3} & Y^{v,f}_{c1,c1} & Y^{v,f}_{c1,c2} & \cdots & Y^{v,f}_{c1,cM} & Y^{v,f}_{c1,e1} & Y^{v,f}_{c1,e2} & \cdots & Y^{v,f}_{c1,eN} \\ Y^{v,f}_{c2,m1} & Y^{v,f}_{c2,m2} & Y^{v,f}_{c2,m3} & Y^{v,f}_{c2,c1} & Y^{v,f}_{c2,c2} & \cdots & Y^{v,f}_{c2,cM} & Y^{v,f}_{c2,e1} & Y^{v,f}_{c2,e2} & \cdots & Y^{v,f}_{c2,eN} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ Y^{v,f}_{cM,m1} & Y^{v,f}_{cM,m2} & Y^{v,f}_{cM,m3} & Y^{v,f}_{cM,c1} & Y^{v,f}_{cM,c2} & \cdots & Y^{v,f}_{cM,cM} & Y^{v,f}_{cM,e1} & Y^{v,f}_{cM,e2} & \cdots & Y^{v,f}_{cM,eN} \\ Y^{v,f}_{e1,m1} & Y^{v,f}_{e1,m2} & Y^{v,f}_{e1,m3} & Y^{v,f}_{e1,c1} & Y^{v,f}_{e1,c2} & \cdots & Y^{v,f}_{e1,cM} & Y^{v,f}_{e1,e1} & Y^{v,f}_{e1,e2} & \cdots & Y^{v,f}_{e1,eN} \\ Y^{v,f}_{e2,m1} & Y^{v,f}_{e2,m2} & Y^{v,f}_{e2,m3} & Y^{v,f}_{e2,c1} & Y^{v,f}_{e2,c2} & \cdots & Y^{v,f}_{e2,cM} & Y^{v,f}_{e2,e1} & Y^{v,f}_{e2,e2} & \cdots & Y^{v,f}_{e2,eN} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ Y^{v,f}_{eN,m1} & Y^{v,f}_{eN,m2} & Y^{v,f}_{eN,m3} & Y^{v,f}_{eN,c1} & Y^{v,f}_{eN,c2} & \cdots & Y^{v,f}_{eN,cM} & Y^{v,f}_{eN,e1} & Y^{v,f}_{eN,e2} & \cdots & Y^{v,f}_{eN,eN} \end{bmatrix}$$

$$\mathbf{Y}^{v,t}=\begin{bmatrix} Y^{v,t}_{m1,m1} & Y^{v,t}_{m1,m2} & Y^{v,t}_{m1,m3} & Y^{v,t}_{m1,c1} & Y^{v,t}_{m1,c2} & \cdots & Y^{v,t}_{m1,cM} & Y^{v,t}_{m1,e1} & Y^{v,t}_{m1,e2} & \cdots & Y^{v,t}_{m1,eN} \\ Y^{v,t}_{m2,m1} & Y^{v,t}_{m2,m2} & Y^{v,t}_{m2,m3} & Y^{v,t}_{m2,c1} & Y^{v,t}_{m2,c2} & \cdots & Y^{v,t}_{m2,cM} & Y^{v,t}_{m2,e1} & Y^{v,t}_{m2,e2} & \cdots & Y^{v,t}_{m2,eN} \\ Y^{v,t}_{m3,m1} & Y^{v,t}_{m3,m2} & Y^{v,t}_{m3,m3} & Y^{v,t}_{m3,c1} & Y^{v,t}_{m3,c2} & \cdots & Y^{v,t}_{m3,cM} & Y^{v,t}_{m3,e1} & Y^{v,t}_{m3,e2} & \cdots & Y^{v,t}_{m3,eN} \\ Y^{v,t}_{c1,m1} & Y^{v,t}_{c1,m2} & Y^{v,t}_{c1,m3} & Y^{v,t}_{c1,c1} & Y^{v,t}_{c1,c2} & \cdots & Y^{v,t}_{c1,cM} & Y^{v,t}_{c1,e1} & Y^{v,t}_{c1,e2} & \cdots & Y^{v,t}_{c1,eN} \\ Y^{v,t}_{c2,m1} & Y^{v,t}_{c2,m2} & Y^{v,t}_{c2,m3} & Y^{v,t}_{c2,c1} & Y^{v,t}_{c2,c2} & \cdots & Y^{v,t}_{c2,cM} & Y^{v,t}_{c2,e1} & Y^{v,t}_{c2,e2} & \cdots & Y^{v,t}_{c2,eN} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ Y^{v,t}_{cM,m1} & Y^{v,t}_{cM,m2} & Y^{v,t}_{cM,m3} & Y^{v,t}_{cM,c1} & Y^{v,t}_{cM,c2} & \cdots & Y^{v,t}_{cM,cM} & Y^{v,t}_{cM,e1} & Y^{v,t}_{cM,e2} & \cdots & Y^{v,t}_{cM,eN} \\ Y^{v,t}_{e1,m1} & Y^{v,t}_{e1,m2} & Y^{v,t}_{e1,m3} & Y^{v,t}_{e1,c1} & Y^{v,t}_{e1,c2} & \cdots & Y^{v,t}_{e1,cM} & Y^{v,t}_{e1,e1} & Y^{v,t}_{e1,e2} & \cdots & Y^{v,t}_{e1,eN} \\ Y^{v,t}_{e2,m1} & Y^{v,t}_{e2,m2} & Y^{v,t}_{e2,m3} & Y^{v,t}_{e2,c1} & Y^{v,t}_{e2,c2} & \cdots & Y^{v,t}_{e2,cM} & Y^{v,t}_{e2,e1} & Y^{v,t}_{e2,e2} & \cdots & Y^{v,t}_{e2,eN} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ Y^{v,t}_{eN,m1} & Y^{v,t}_{eN,m2} & Y^{v,t}_{eN,m3} & Y^{v,t}_{eN,c1} & Y^{v,t}_{eN,c2} & \cdots & Y^{v,t}_{eN,cM} & Y^{v,t}_{eN,e1} & Y^{v,t}_{eN,e2} & \cdots & Y^{v,t}_{eN,eN} \end{bmatrix}$$

$$\mathbf{Y}^{\alpha,f}=\begin{bmatrix} Y^{\alpha,f}_{m1,m1} & Y^{\alpha,f}_{m1,m2} & Y^{\alpha,f}_{m1,m3} & Y^{\alpha,f}_{m1,c1} & Y^{\alpha,f}_{m1,c2} & \cdots & Y^{\alpha,f}_{m1,cM} & Y^{\alpha,f}_{m1,e1} & Y^{\alpha,f}_{m1,e2} & \cdots & Y^{\alpha,f}_{m1,eN} \\ Y^{\alpha,f}_{m2,m1} & Y^{\alpha,f}_{m2,m2} & Y^{\alpha,f}_{m2,m3} & Y^{\alpha,f}_{m2,c1} & Y^{\alpha,f}_{m2,c2} & \cdots & Y^{\alpha,f}_{m2,cM} & Y^{\alpha,f}_{m2,e1} & Y^{\alpha,f}_{m2,e2} & \cdots & Y^{\alpha,f}_{m2,eN} \\ Y^{\alpha,f}_{m3,m1} & Y^{\alpha,f}_{m3,m2} & Y^{\alpha,f}_{m3,m3} & Y^{\alpha,f}_{m3,c1} & Y^{\alpha,f}_{m3,c2} & \cdots & Y^{\alpha,f}_{m3,cM} & Y^{\alpha,f}_{m3,e1} & Y^{\alpha,f}_{m3,e2} & \cdots & Y^{\alpha,f}_{m3,eN} \\ Y^{\alpha,f}_{c1,m1} & Y^{\alpha,f}_{c1,m2} & Y^{\alpha,f}_{c1,m3} & Y^{\alpha,f}_{c1,c1} & Y^{\alpha,f}_{c1,c2} & \cdots & Y^{\alpha,f}_{c1,cM} & Y^{\alpha,f}_{c1,e1} & Y^{\alpha,f}_{c1,e2} & \cdots & Y^{\alpha,f}_{c1,eN} \\ Y^{\alpha,f}_{c2,m1} & Y^{\alpha,f}_{c2,m2} & Y^{\alpha,f}_{c2,m3} & Y^{\alpha,f}_{c2,c1} & Y^{\alpha,f}_{c2,c2} & \cdots & Y^{\alpha,f}_{c2,cM} & Y^{\alpha,f}_{c2,e1} & Y^{\alpha,f}_{c2,e2} & \cdots & Y^{\alpha,f}_{c2,eN} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ Y^{\alpha,f}_{cM,m1} & Y^{\alpha,f}_{cM,m2} & Y^{\alpha,f}_{cM,m3} & Y^{\alpha,f}_{cM,c1} & Y^{\alpha,f}_{cM,c2} & \cdots & Y^{\alpha,f}_{cM,cM} & Y^{\alpha,f}_{cM,e1} & Y^{\alpha,f}_{cM,e2} & \cdots & Y^{\alpha,f}_{cM,eN} \\ Y^{\alpha,f}_{e1,m1} & Y^{\alpha,f}_{e1,m2} & Y^{\alpha,f}_{e1,m3} & Y^{\alpha,f}_{e1,c1} & Y^{\alpha,f}_{e1,c2} & \cdots & Y^{\alpha,f}_{e1,cM} & Y^{\alpha,f}_{e1,e1} & Y^{\alpha,f}_{e1,e2} & \cdots & Y^{\alpha,f}_{e1,eN} \\ Y^{\alpha,f}_{e2,m1} & Y^{\alpha,f}_{e2,m2} & Y^{\alpha,f}_{e2,m3} & Y^{\alpha,f}_{e2,c1} & Y^{\alpha,f}_{e2,c2} & \cdots & Y^{\alpha,f}_{e2,cM} & Y^{\alpha,f}_{e2,e1} & Y^{\alpha,f}_{e2,e2} & \cdots & Y^{\alpha,f}_{e2,eN} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ Y^{\alpha,f}_{eN,m1} & Y^{\alpha,f}_{eN,m2} & Y^{\alpha,f}_{eN,m3} & Y^{\alpha,f}_{eN,c1} & Y^{\alpha,f}_{eN,c2} & \cdots & Y^{\alpha,f}_{eN,cM} & Y^{\alpha,f}_{eN,e1} & Y^{\alpha,f}_{eN,e2} & \cdots & Y^{\alpha,f}_{eN,eN} \end{bmatrix}$$

$$\mathbf{Y}^{\alpha,t}=\begin{bmatrix} Y^{\alpha,t}_{m1,m1} & Y^{\alpha,t}_{m1,m2} & Y^{\alpha,t}_{m1,m3} & Y^{\alpha,t}_{m1,c1} & Y^{\alpha,t}_{m1,c2} & \cdots & Y^{\alpha,t}_{m1,cM} & Y^{\alpha,t}_{m1,e1} & Y^{\alpha,t}_{m1,e2} & \cdots & Y^{\alpha,t}_{m1,eN} \\ Y^{\alpha,t}_{m2,m1} & Y^{\alpha,t}_{m2,m2} & Y^{\alpha,t}_{m2,m3} & Y^{\alpha,t}_{m2,c1} & Y^{\alpha,t}_{m2,c2} & \cdots & Y^{\alpha,t}_{m2,cM} & Y^{\alpha,t}_{m2,e1} & Y^{\alpha,t}_{m2,e2} & \cdots & Y^{\alpha,t}_{m2,eN} \\ Y^{\alpha,t}_{m3,m1} & Y^{\alpha,t}_{m3,m2} & Y^{\alpha,t}_{m3,m3} & Y^{\alpha,t}_{m3,c1} & Y^{\alpha,t}_{m3,c2} & \cdots & Y^{\alpha,t}_{m3,cM} & Y^{\alpha,t}_{m3,e1} & Y^{\alpha,t}_{m3,e2} & \cdots & Y^{\alpha,t}_{m3,eN} \\ Y^{\alpha,t}_{c1,m1} & Y^{\alpha,t}_{c1,m2} & Y^{\alpha,t}_{c1,m3} & Y^{\alpha,t}_{c1,c1} & Y^{\alpha,t}_{c1,c2} & \cdots & Y^{\alpha,t}_{c1,cM} & Y^{\alpha,t}_{c1,e1} & Y^{\alpha,t}_{c1,e2} & \cdots & Y^{\alpha,t}_{c1,eN} \\ Y^{\alpha,t}_{c2,m1} & Y^{\alpha,t}_{c2,m2} & Y^{\alpha,t}_{c2,m3} & Y^{\alpha,t}_{c2,c1} & Y^{\alpha,t}_{c2,c2} & \cdots & Y^{\alpha,t}_{c2,cM} & Y^{\alpha,t}_{c2,e1} & Y^{\alpha,t}_{c2,e2} & \cdots & Y^{\alpha,t}_{c2,eN} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ Y^{\alpha,t}_{cM,m1} & Y^{\alpha,t}_{cM,m2} & Y^{\alpha,t}_{cM,m3} & Y^{\alpha,t}_{cM,c1} & Y^{\alpha,t}_{cM,c2} & \cdots & Y^{\alpha,t}_{cM,cM} & Y^{\alpha,t}_{cM,e1} & Y^{\alpha,t}_{cM,e2} & \cdots & Y^{\alpha,t}_{cM,eN} \\ Y^{\alpha,t}_{e1,m1} & Y^{\alpha,t}_{e1,m2} & Y^{\alpha,t}_{e1,m3} & Y^{\alpha,t}_{e1,c1} & Y^{\alpha,t}_{e1,c2} & \cdots & Y^{\alpha,t}_{e1,cM} & Y^{\alpha,t}_{e1,e1} & Y^{\alpha,t}_{e1,e2} & \cdots & Y^{\alpha,t}_{e1,eN} \\ Y^{\alpha,t}_{e2,m1} & Y^{\alpha,t}_{e2,m2} & Y^{\alpha,t}_{e2,m3} & Y^{\alpha,t}_{e2,c1} & Y^{\alpha,t}_{e2,c2} & \cdots & Y^{\alpha,t}_{e2,cM} & Y^{\alpha,t}_{e2,e1} & Y^{\alpha,t}_{e2,e2} & \cdots & Y^{\alpha,t}_{e2,eN} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ Y^{\alpha,t}_{eN,m1} & Y^{\alpha,t}_{eN,m2} & Y^{\alpha,t}_{eN,m3} & Y^{\alpha,t}_{eN,c1} & Y^{\alpha,t}_{eN,c2} & \cdots & Y^{\alpha,t}_{eN,cM} & Y^{\alpha,t}_{eN,e1} & Y^{\alpha,t}_{eN,e2} & \cdots & Y^{\alpha,t}_{eN,eN} \end{bmatrix}$$

Mathematical model: Impedance matrices

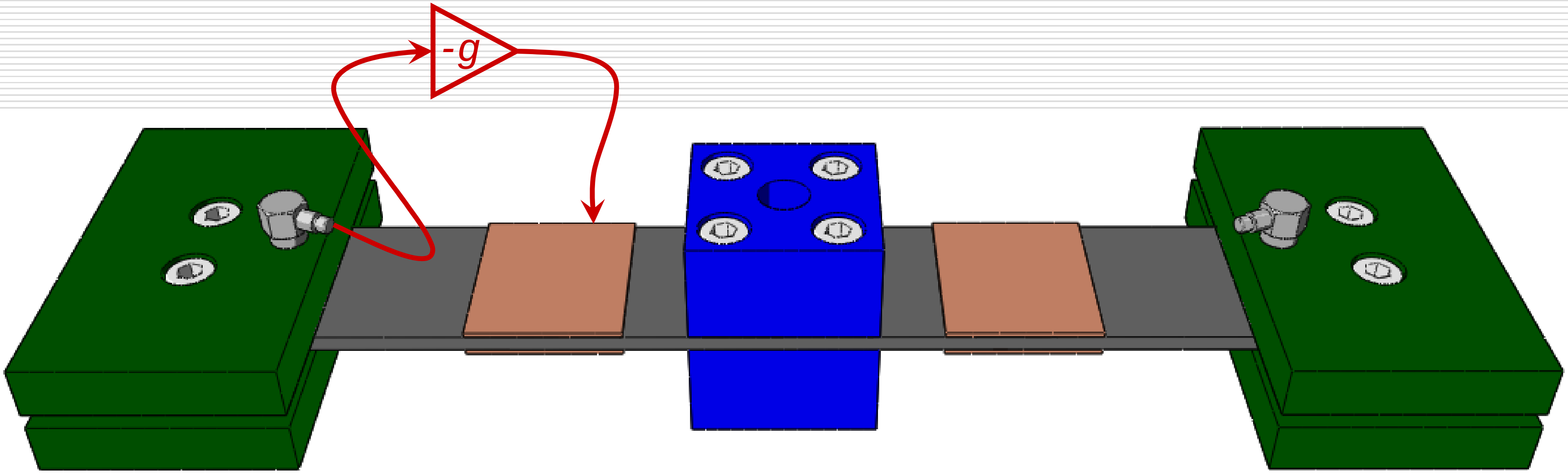
$$
\mathbf{Z}^{f,v} = \begin{bmatrix}
Z^{f,v}_{m1,m1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & Z^{f,v}_{m2,m2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & Z^{f,v}_{m3,m3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & Z^{f,v}_{c1,c1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & Z^{f,v}_{c2,c2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \ddots & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & Z^{f,v}_{cM,cM} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & Z^{f,v}_{e1,e1} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Z^{f,v}_{e2,e2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \ddots & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Z^{f,v}_{eN,eN}
\end{bmatrix}
$$

$$
\mathbf{Z}^{t,v} = \begin{bmatrix}
Z^{t,v}_{m1,m1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & Z^{t,v}_{m2,m2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & Z^{t,v}_{m3,m3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & Z^{t,v}_{c1,c1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & Z^{t,v}_{c2,c2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \ddots & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & Z^{t,v}_{cM,cM} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & Z^{t,v}_{e1,e1} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Z^{t,v}_{e2,e2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \ddots & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Z^{t,v}_{eN,eN}
\end{bmatrix}
$$

$$
\mathbf{Z}^{t,\alpha} = \begin{bmatrix}
Z^{t,\alpha}_{m1,m1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & Z^{t,\alpha}_{m2,m2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & Z^{t,\alpha}_{m3,m3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & Z^{t,\alpha}_{c1,c1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & Z^{t,\alpha}_{c2,c2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \ddots & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & Z^{t,\alpha}_{cM,cM} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & Z^{t,\alpha}_{e1,e1} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Z^{t,\alpha}_{e2,e2} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \ddots & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Z^{t,\alpha}_{eN,eN}
\end{bmatrix}
$$

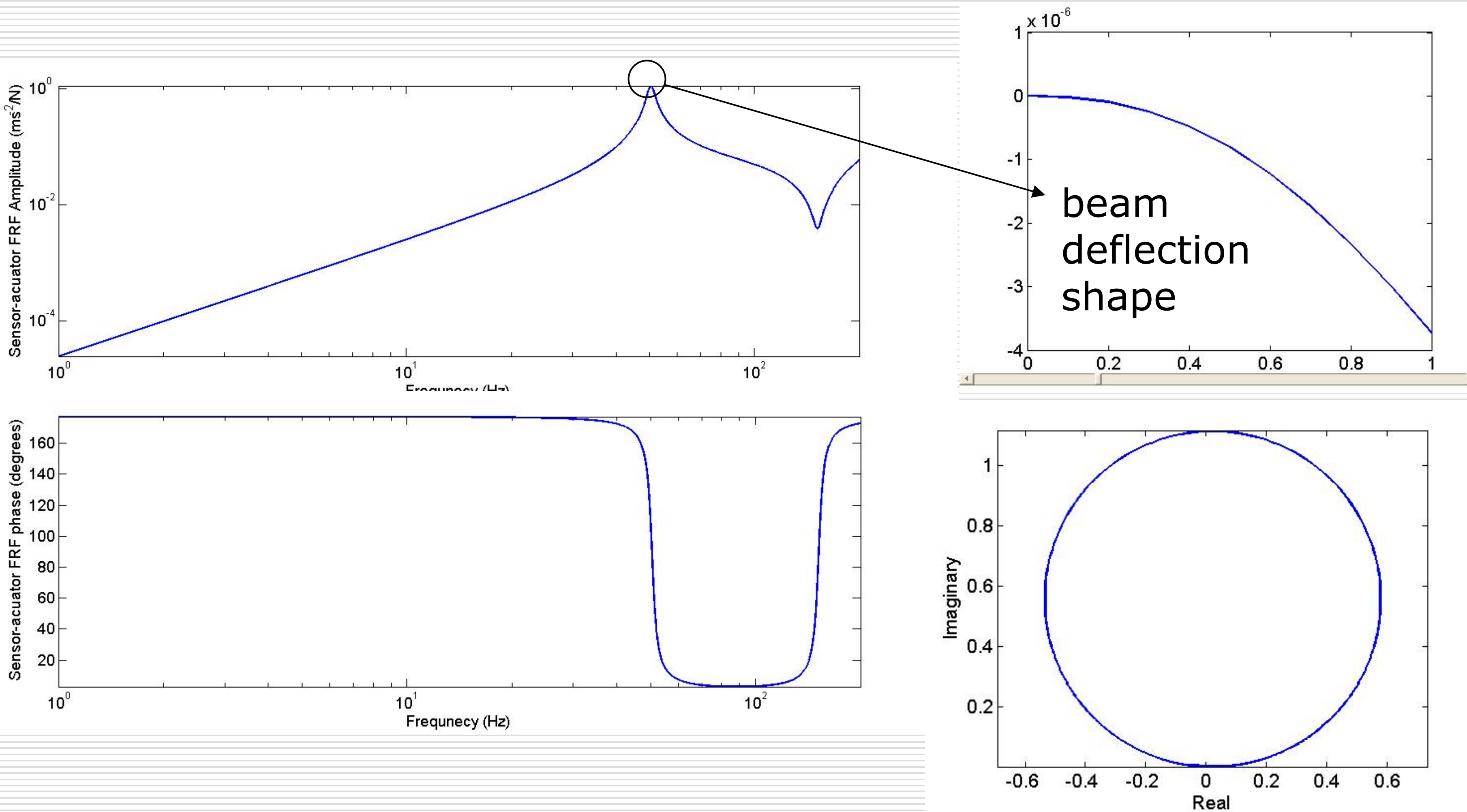
Adaptive configurations

- #1 Negative acceleration feedback with small PZT actuators and linear acceleration sensors



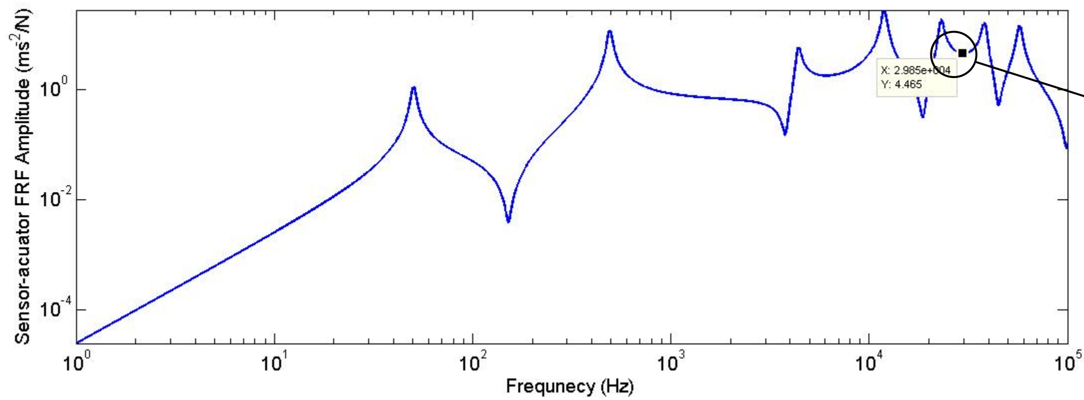
Adaptive configuration #1: Stability

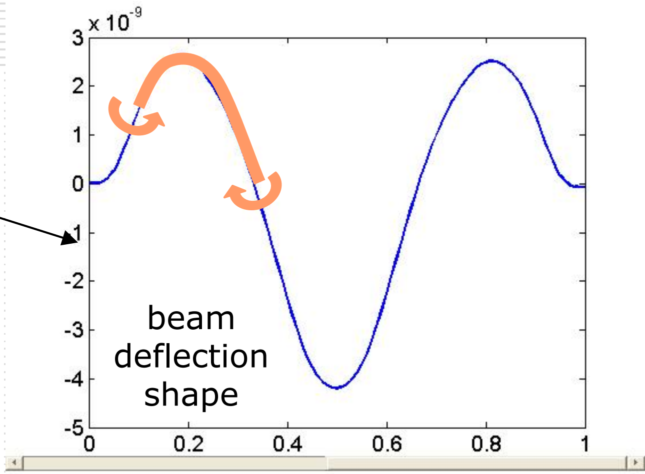
- Sensor actuator open loop frequency response function 0-200 Hz

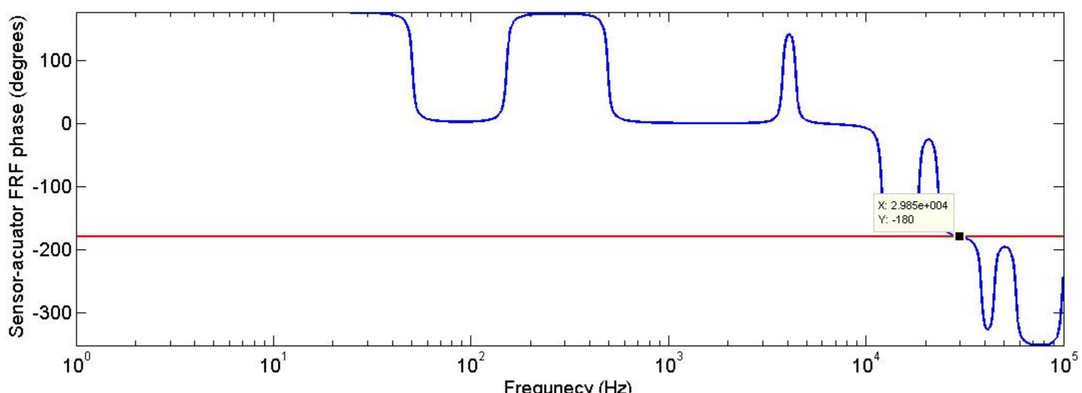


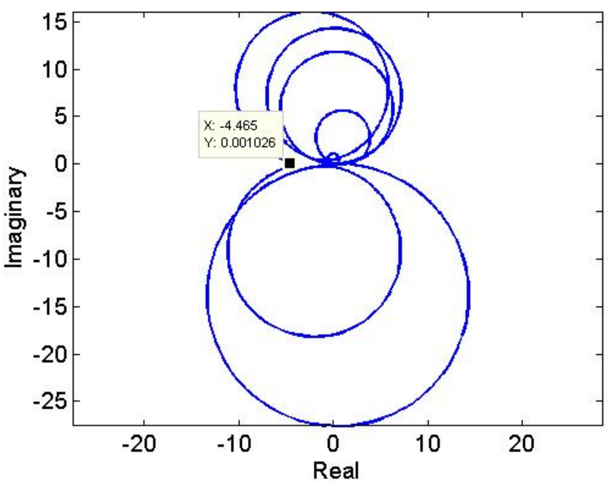
Adaptive configuration #1: Stability

- Sensor actuator open loop frequency response function 0-100 kHz





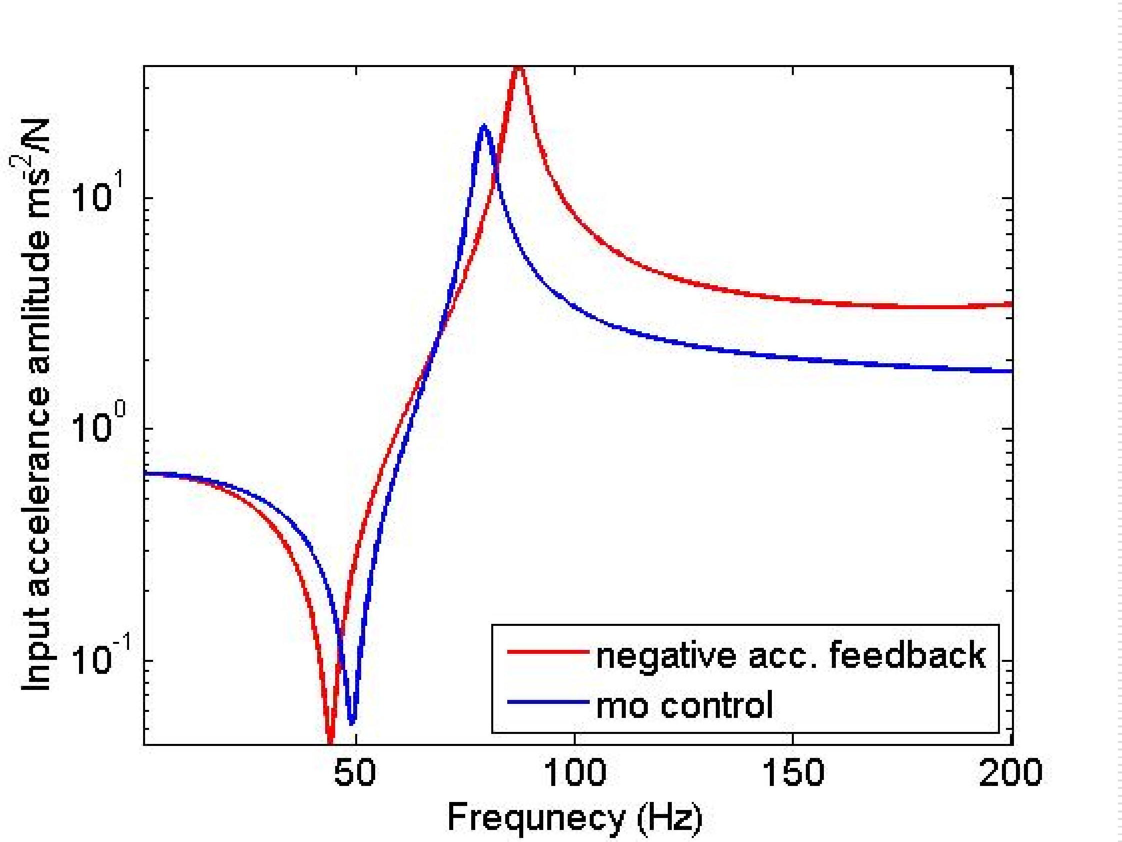




g_{max}=0.22 Ns^2

Adaptive configuration #1: Adaptability

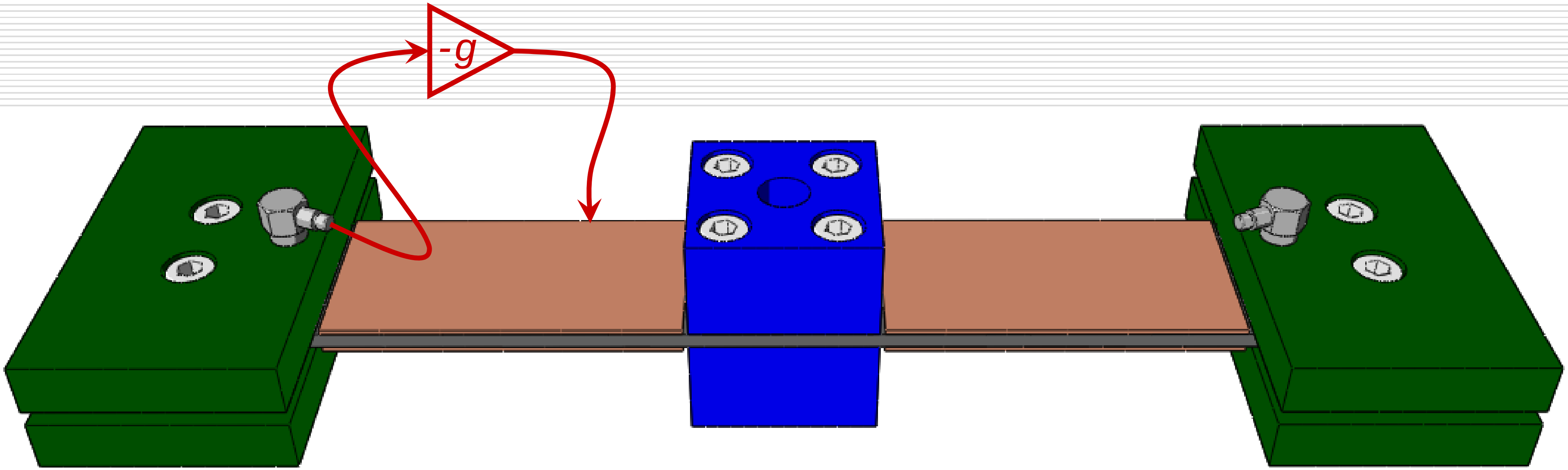
- Input accelerance 0-200 Hz



g=0.05 Ns^2

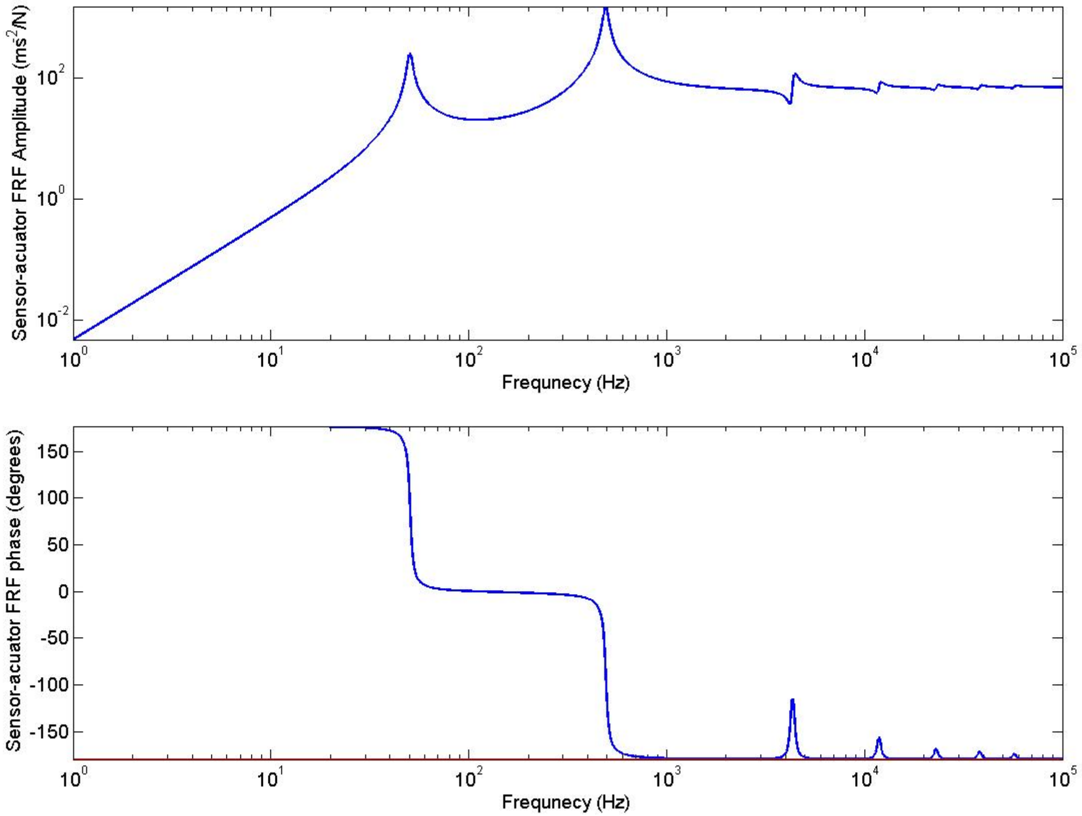
Adaptive configurations

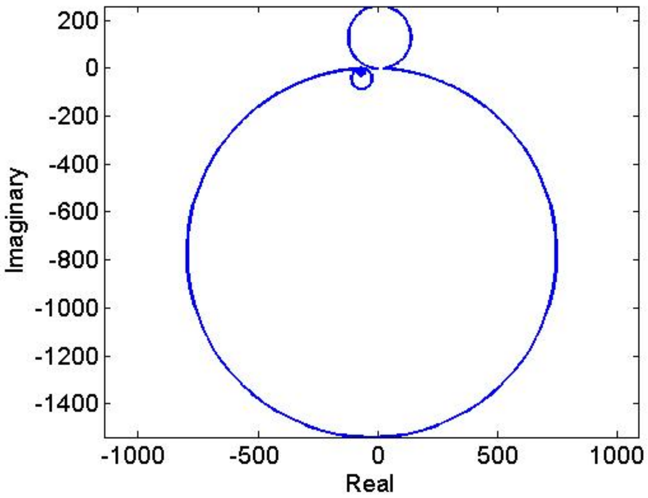
- #2 Negative acceleration feedback with large PZT actuators and linear acceleration sensors



Adaptive configuration #2: Stability

- Sensor actuator open loop frequency response function 0-100 kHz

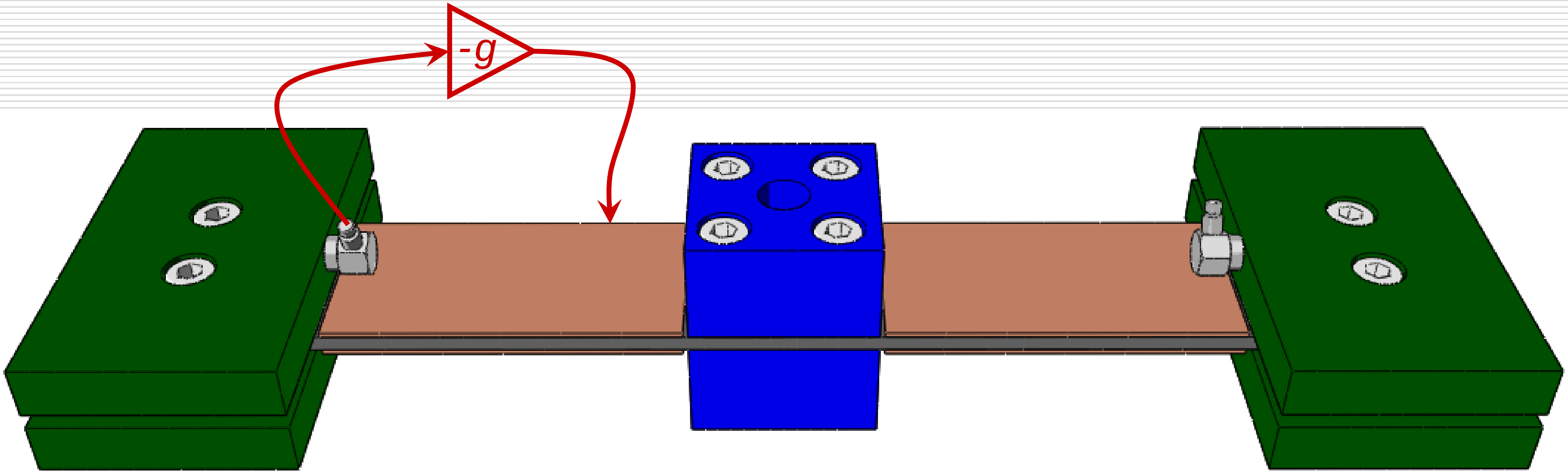




g_{max}= "∞", phase margin low

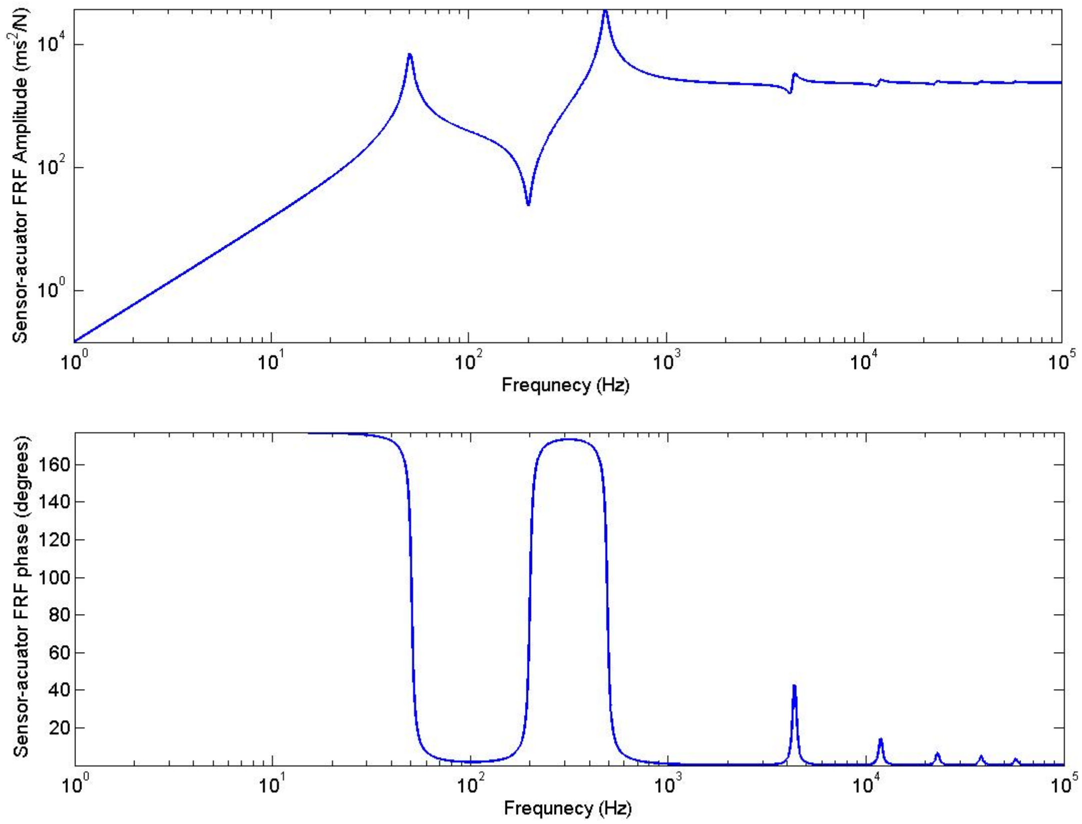
Adaptive configurations

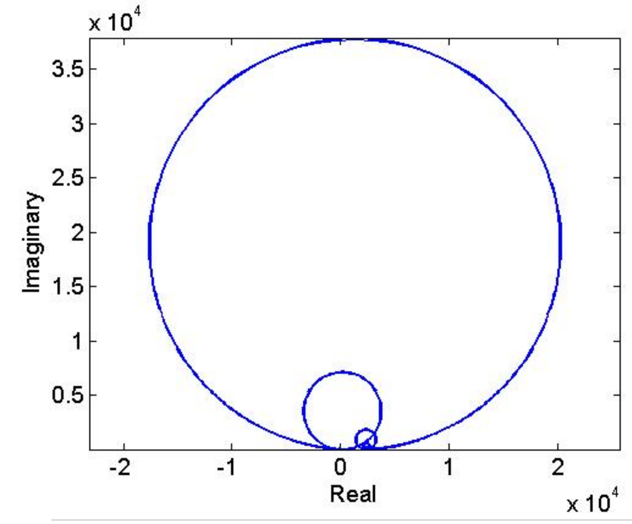
- #3 Negative acceleration feedback with large PZT actuators and angular acceleration sensors



Adaptive configuration #3: Stability

- Sensor actuator open loop frequency response function 0-100 kHz



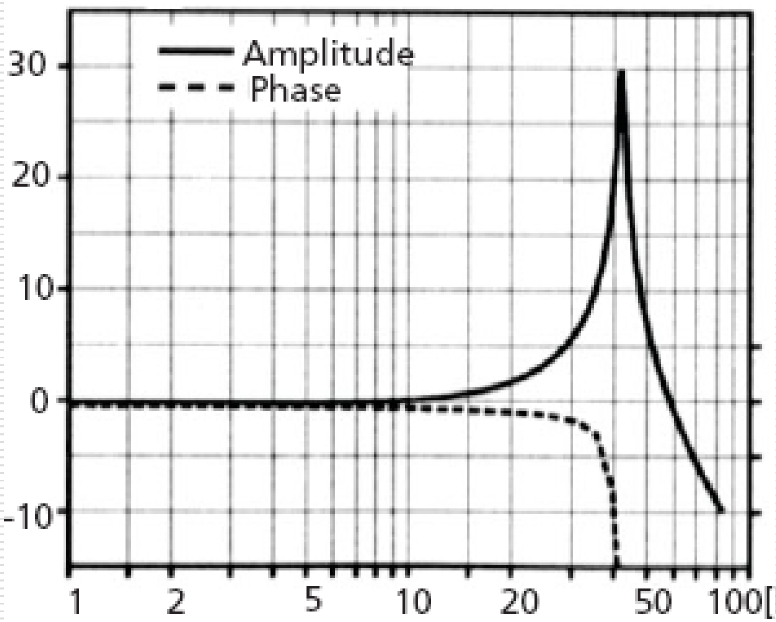


$g_{max}=\infty$, better phase margin

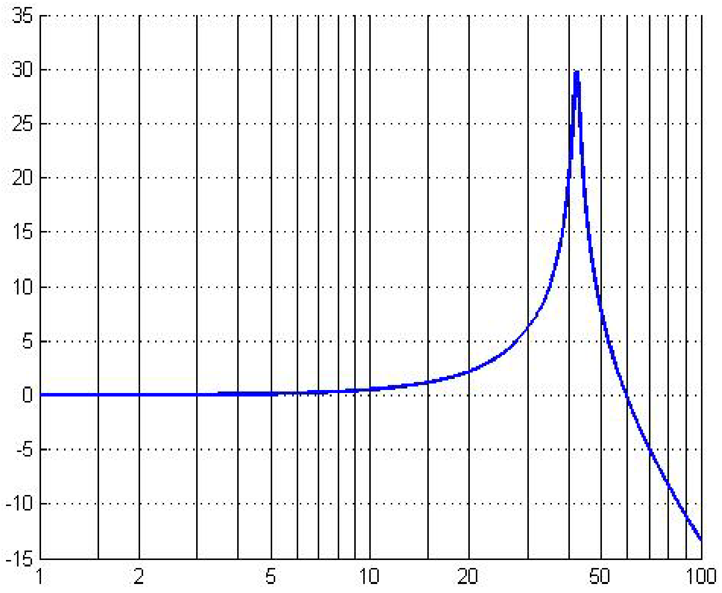
Adaptive configuration #3: Stability

- But what about the sensor dynamics?

B&K 4371calibration chart

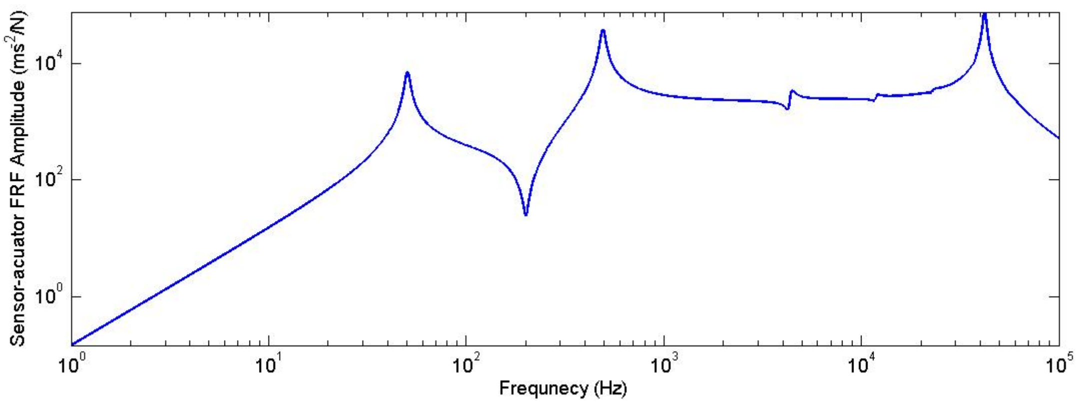


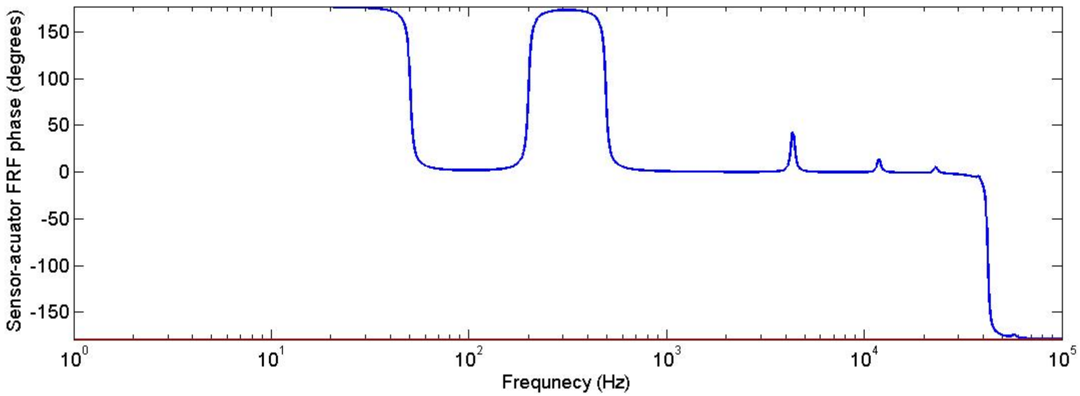
1 DOF model

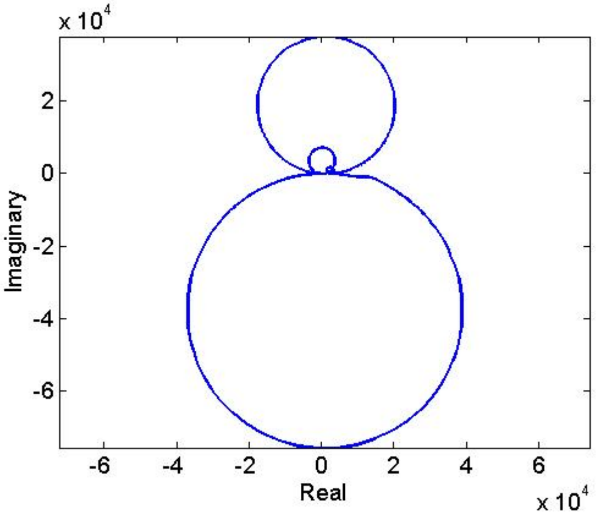


Adaptive configuration #3: Stability

- Sensor actuator open loop frequency response function 0-100 kHz including the sensor frequency response



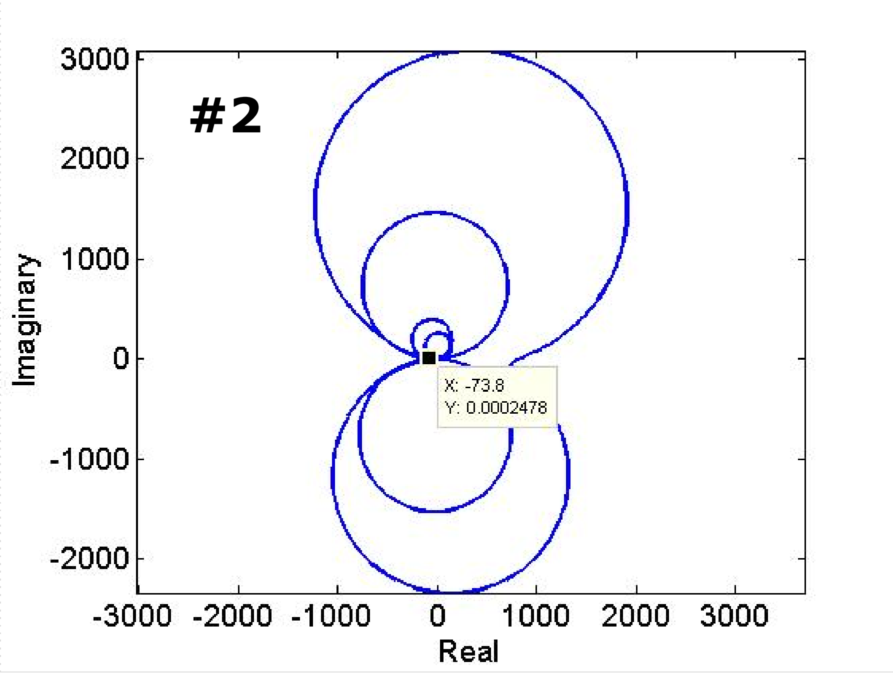


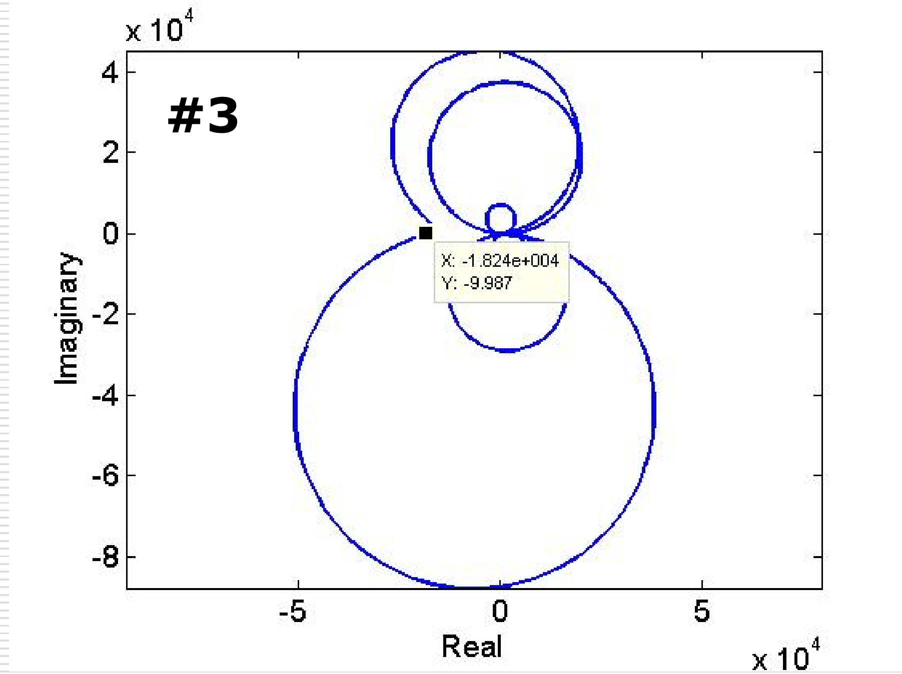


g_{max}=∞ => sensor response does not appear to disrupt the stability BUT …

Adaptive configuration #2 & #3: Stability

- …what if there is a small mismatch (3%) between the length of the piezo and the length of the beam?

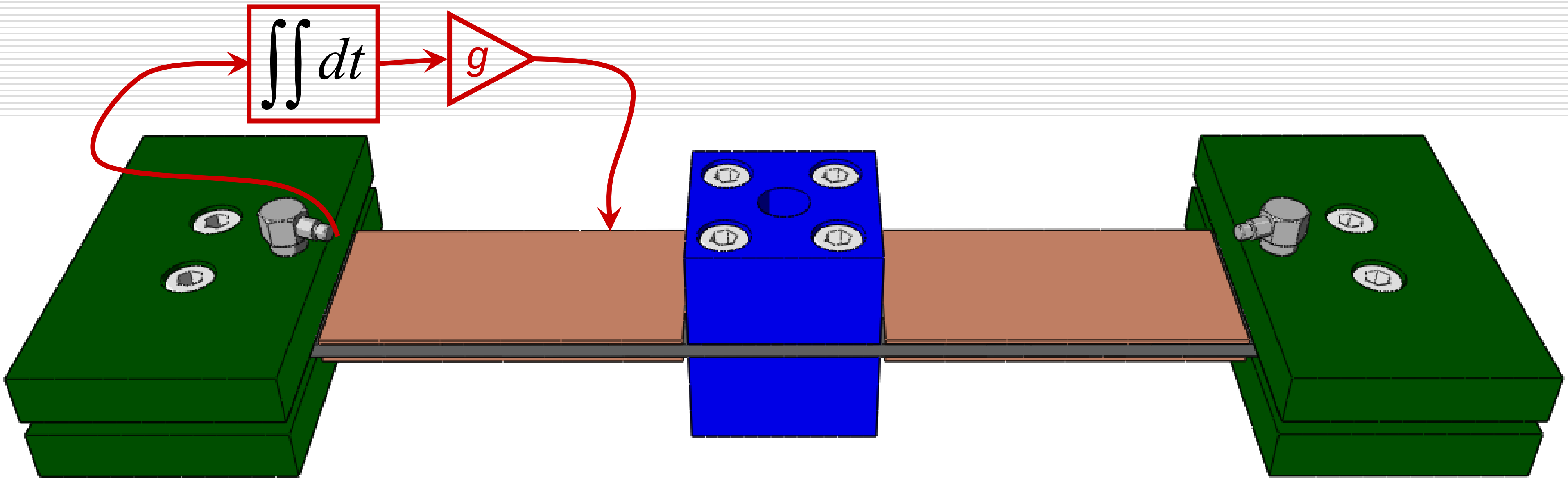




g_{max}=0.014 Ns² **g_{max}≈0**

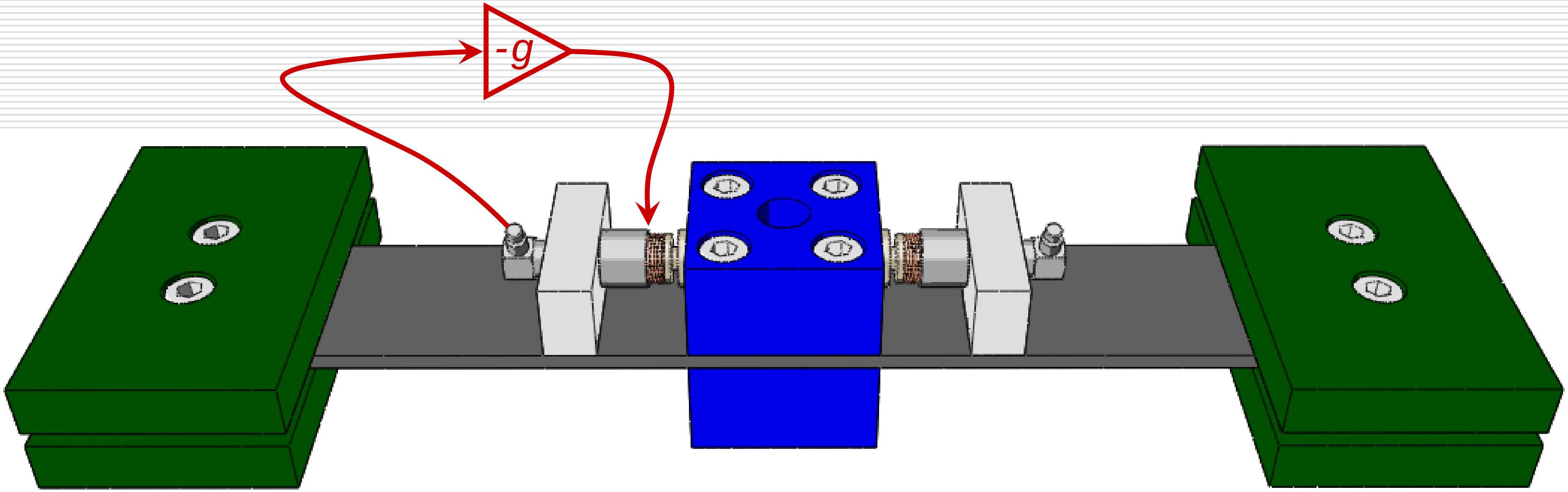
Future work: Adaptive configurations

- #4 Positive position feedback with large PZT actuators and linear displacement sensors



Future work: Adaptive configurations

- #5 negative acceleration feedback with voice coil actuators and linear acceleration sensors



Future work: Adaptive configurations

- #6 Positive position feedback with small PZT deformation angle sensors and linear force actuators