# Generating Parametric Models from Tabulated Data

**Sanda Lefteriu**[1,2] and Jan Mohring[1]

[1]Fraunhofer ITWM
Kaiserslautern, Germany

[2]ECE Department, Rice University
Houston, TX, USA

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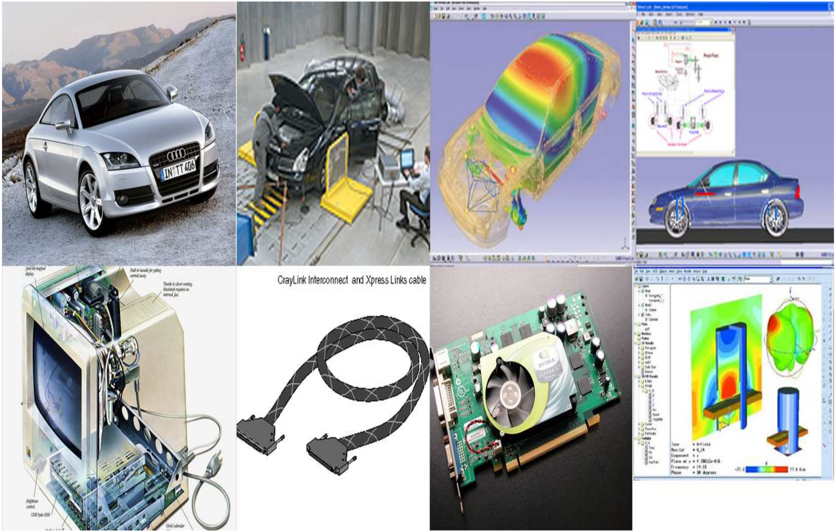
# Outline

# Motivation



# *Previous Work*

Currently available approaches:

- vector fitting (VF) used to construct models for known parameter values, followed by a parametrization of the numerator and denominator of the transfer function by linear combinations of basis functions which are piecewise linear in each parameter [9, 8].
- generalization of the Sanathanan-Koerner (SK) iteration (VF is a particular case of SK) to the parametric case [10, 7]
- multivariate formulation of the Orthonormal Vector Fitting technique [2, 1]
- recursive algorithm to compute the parametrized residues of the multivariate transfer function [3]
- generalization of multivariate Vector Fitting which includes parameter derivatives [4].

are time & memory consuming.

# Outline

# Parametric modeling from measurements

$\mathbf{\Sigma}$ in *descriptor-form*: $\mathbf{E}\dot{\mathbf{x}}(t)=\mathbf{A}\mathbf{x}(t)+\mathbf{B}\mathbf{u}(t)$, $\mathbf{y}(t)=\mathbf{C}\mathbf{x}(t)+\mathbf{D}\mathbf{u}(t)$, $\mathbf{x}(t)$: state, $\mathbf{u}(t)$: input, $\mathbf{y}(t)$: corresponding output, $\mathbf{E},\mathbf{A}\in\mathbb{R}^{n\times n}$, $\mathbf{B}\in\mathbb{R}^{n\times p}$, $\mathbf{C}\in\mathbb{R}^{p\times n}$, $\mathbf{D}\in\mathbb{R}^{p\times p}$ are constant; $[\mathbf{E},\mathbf{A},\mathbf{B},\mathbf{C},\mathbf{D}]$: a *realization* of $\mathbf{\Sigma}$.

- tabulated/measurement data (e.g. S-parameters) given wrt frequencies $f_i$, $i=1,\dots,n_f$, but also wrt one or more parameters as:

$$\left( f_i, \alpha_k, \ \mathbf{S}^{(i,k)} := \begin{bmatrix} S_{11}^{(i,k)} & \dots & S_{1p}^{(i,k)} \\ \vdots & \vdots & \vdots \\ S_{p1}^{(i,k)} & \dots & S_{pp}^{(i,k)} \end{bmatrix} \right) \tag{1}$$

- one parameter $\alpha$ taking values $\alpha_k$, $k=1,\dots,n_\alpha$

# Parametric modeling from measurements

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- one parameter $\alpha$ taking values $\alpha_k$, $k=1,\ldots,n_\alpha$
- Goal: find a parametric system $\mathbf{\Sigma}(\alpha)$ which models (1), st its transfer function computed for $\alpha_k$, evaluated at $j\cdot 2\pi f_i$, is close to $\mathbf{S}^{(i,k)}$: $\mathbf{H}^{(\alpha_k)}(2\pi j f_i)=\mathbf{C}^{(\alpha_k)}(2\pi j f_i\mathbf{I}-\mathbf{A}^{(\alpha_k)})^{-1}\mathbf{B}^{(\alpha_k)}\approx\mathbf{S}^{(i,k)}$, $i=1,\ldots,n_f$
- Moreover, $\mathbf{\Sigma}(\alpha)$ for other $\alpha$, should be close to the model one would obtain from measurements performed for these new values.

# What we propose

Modeling step: Problem: construct LTI models $\mathbf{\Sigma}(\alpha_k)$ for some fixed parameter values $\alpha_k$ (rational approximation or system identification - can be solved in many ways)

Loewner matrix framework & Tangential interpolation [6, 5]

- fast, accurate & robust
- especially designed for many ports
- models of small dimension

Generating Parametric Models: Interpolate between state-space matrices, after applying a suitable similarity transformation

- choice between different interpolation schemes
- choice between the canonical form in which systems are brought initially
- constrain the transformation to be close to unitary

# Outline

# *Unitary Constraint*

## *Motivation*

Recall: We want to solve

$$J(\mathbf{T})=\alpha\left\|\mathbf{M}^{(0)}-\mathbf{T}^{-1}\mathbf{M}^{(1)}\mathbf{T}\right\|^2+\beta\left\|\mathbf{B}^{(0)}-\mathbf{T}^{-1}\mathbf{B}^{(1)}\right\|^2+\gamma\left\|\mathbf{C}^{(0)}-\mathbf{C}^{(1)}\mathbf{T}\right\|^2 \quad (2)$$

but, instead, we solve

$$\tilde{J}(\mathbf{T})=\alpha\left\|\mathbf{T}\mathbf{M}^{(0)}-\mathbf{M}^{(1)}\mathbf{T}\right\|^2+\beta\left\|\mathbf{T}\mathbf{B}^{(0)}-\mathbf{B}^{(1)}\right\|^2+\gamma\left\|\mathbf{C}^{(0)}-\mathbf{C}^{(1)}\mathbf{T}\right\|^2 \quad (3)$$

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They are different, but by introducing an additional constraint in (3) to enforce $\mathbf{T}$ to be close to unitary ($\mathbf{T}$ is unitary if $\mathbf{T}^*\mathbf{T} = \mathbf{T}\mathbf{T}^* = \mathbf{I}$), the approximation of (2) by (3) makes sense.

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Why?

$$\left\| \mathbf{T}\mathbf{M}^{(0)} - \mathbf{M}^{(1)}\mathbf{T} \right\|^2 = \left\| \mathbf{T}^{-1}\left( \mathbf{T}\mathbf{M}^{(0)} - \mathbf{M}^{(1)}\mathbf{T} \right) \right\|^2 = \left\| \mathbf{M}^{(0)} - \mathbf{T}^{-1}\mathbf{M}^{(1)}\mathbf{T} \right\|^2$$

because the 2-norm is invariant under a unitary change of basis.

# *Unitary Constraint*

We add an additional term to (3)

$$\hat{\mathcal{J}}(\mathbf{T}) = \alpha \left\| \mathbf{T}\mathbf{M}^{(0)} - \mathbf{M}^{(1)}\mathbf{T} \right\|^2 + \beta \left\| \mathbf{T}\mathbf{B}^{(0)} - \mathbf{B}^{(1)} \right\|^2 + \gamma \left\| \mathbf{C}^{(0)} - \mathbf{C}^{(1)}\mathbf{T} \right\|^2 + \delta \left\| \mathbf{T}^*\mathbf{T} - \mathbf{I} \right\|^2,$$

$\delta$ appropriate scaling factor. Forming the Fréchet derivative, we have

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$$\left(\alpha \mathbf{M}^{(1)*}\mathbf{M}^{(1)} + \gamma \mathbf{C}^{(1)*}\mathbf{C}^{(1)}\right)\mathbf{T} + \mathbf{T}\left(\alpha \mathbf{M}^{(0)}\mathbf{M}^{(0)*} + \beta \mathbf{B}^{(0)}\mathbf{B}^{(0)*}\right) - \alpha\left(\mathbf{M}^{(1)}\mathbf{T}\mathbf{M}^{(0)*} + \mathbf{M}^{(1)*}\mathbf{T}\mathbf{M}^{(0)}\right)$$
$$+ \delta\, 4\mathbf{T}(\mathbf{T}^*\mathbf{T} - \mathbf{I}) = \beta \mathbf{B}^{(1)}\mathbf{B}^{(0)*} + \gamma \mathbf{C}^{(1)*}\mathbf{C}^{(0)} \qquad (4)$$

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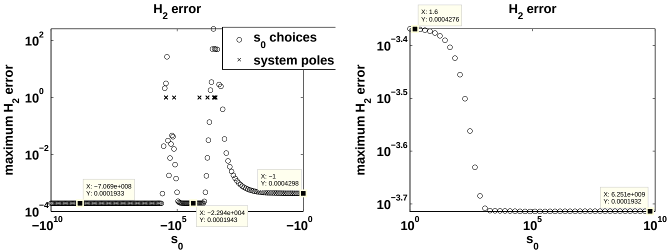
- $\mathbf{T}(\mathbf{T}^*\mathbf{T} - \mathbf{I})$ makes (4) nonlinear in $\mathbf{T}$, so it cannot be solved directly
- we use a Newton-like procedure on the linearized (4), obtained by writing $\mathbf{T}$ as $\mathbf{T}_0 + \Delta\mathbf{T}$ and disregarding higher order terms in $\Delta\mathbf{T}$
- we start with the initial guess $\mathbf{T}_0$ and add, at each step, a correction $\Delta\mathbf{T}$ which will make the solution $\mathbf{T}$ close to being unitary.

# Outline

# Where to choose the test frequency?

Recall: $\Delta^{(\alpha)}(s_0) = (1-\alpha)\mathbf{H}^{(0)}(s_0) + \alpha\mathbf{H}^{(1)}(s_0) - \mathbf{H}^{(\alpha)}(s_0)$.

- chosen away from the system poles
- $s_0 \in \mathbb{R}$




(a) Negative $s_0$ (b) Positive $s_0$

*Figure:* The influence of $s_0$ on the errors

# Outline

# Stability

Fact: Systems built for known parameter configurations are stable.
Question: Will the system obtained with our procedure also be stable for any new parameter value?
Answer: Lyapunov stability theory [11].

## Definition

Let $L$ be a continous map from $\mathbb{R}^n$ to $\mathbb{R}$. It is called a Lyapunov function for system $\dot{\mathbf{x}}(t)=f(\mathbf{x}(t))$ if:

- $L$ is locally positive definite ($L(\mathbf{x})>0$, $0<\|\mathbf{x}\|<r_1$, for some $r_1$) &
- $\dot{L}$ is locally negative semidefinite ($\dot{L}(\mathbf{x})\leq 0$, $0<\|\mathbf{x}\|<r_2$).

## Theorem

- *If $\exists$ $L(\mathbf{x})$ for system $\dot{\mathbf{x}}(t)=f(\mathbf{x}(t))$, then $\mathbf{x}=\mathbf{0}$ is a stable equilibrium point in the sense of Lyapunov.*
- *If $\dot{L}(\mathbf{x})<0$, $0<\|\mathbf{x}\|<r_2$, for some $r_2$, then $\mathbf{x}=\mathbf{0}$ is an asymptotically stable equilibrium point.*

# Stability

Define a Lyapunov candidate function $L(\mathbf{x}) = \mathbf{x}^*\mathbf{P}\mathbf{x}$, where $\mathbf{P}$ is symmetric positive definite.

$$\dot{L}(\mathbf{x}) = \dot{\mathbf{x}}^*\mathbf{P}\mathbf{x} + \mathbf{x}^*\mathbf{P}\dot{\mathbf{x}} = \mathbf{x}^*\mathbf{A}^*\mathbf{P}\mathbf{x} + \mathbf{x}^*\mathbf{P}\mathbf{A}\mathbf{x} = \mathbf{x}^*\underbrace{(\mathbf{A}^*\mathbf{P} + \mathbf{P}\mathbf{A})}_{-\mathbf{Q}}\mathbf{x} < 0 \quad (5)$$

- if $\mathbf{Q}\geq\mathbf{0}$, $\mathbf{x}=\mathbf{0}$ is a stable equilibrium point.
- if $\mathbf{Q}>\mathbf{0}$, $\mathbf{x}=\mathbf{0}$ is globally asymptotically stable & the system is stable.
- this can be expressed as a *linear matrix inequality* (LMI): $\mathbf{P}\mathbf{A}+\mathbf{A}^*\mathbf{P}<\mathbf{0}$ which always has a solution for $\mathbf{A}$ stable.

# Stability

$\mathbf{\Sigma}(\alpha_k)$ are stable, for $k{=}1,\ldots,n_\alpha$, and for a new $\alpha$,

$$\mathbf{A}^{(\alpha)} = \sum_{k=1}^{n_{\alpha_k}} w_k \mathbf{A}^{(\alpha_k)}, \;\; \mathbf{B}^{(\alpha)} = \sum_{k=1}^{n_{\alpha_k}} w_k \mathbf{B}^{(\alpha_k)} \tag{6}$$

and similarly for $\mathbf{C}^{(\alpha)}$, $\mathbf{D}^{(\alpha)}$, with $\sum_{k=1}^{n_{\alpha_k}} w_k{=}1$.

Goal: Find a common solution $\mathbf{P}$, $\forall k{=}1,\ldots,n_\alpha$, to the inequalities

$$\mathbf{A}^{(\alpha_k)*}\mathbf{P} + \mathbf{P}\mathbf{A}^{(\alpha_k)} < \mathbf{0} \Leftrightarrow \mathbf{A}^{*}\text{diag}(\mathbf{P},\ldots,\mathbf{P}) + \text{diag}(\mathbf{P},\ldots,\mathbf{P})\mathbf{A} < \mathbf{0} \tag{7}$$

$$w_k\mathbf{A}^{(\alpha_k)*}\mathbf{P} + \mathbf{P}w_k\mathbf{A}^{(\alpha_k)} < \mathbf{0} \Rightarrow \underbrace{\sum_{k=1}^{n_\alpha} w_k \mathbf{A}^{(\alpha_k)*}}_{\mathbf{A}^{(\alpha)}}\mathbf{P} + \mathbf{P}\sum_{k=1}^{n_\alpha} w_k \mathbf{A}^{(\alpha_k)} < \mathbf{0}$$

where $\mathbf{A}{=}\text{diag}(\mathbf{A}^{(\alpha_1)},\ldots,\mathbf{A}^{(\alpha_{n_\alpha})})$.

Solved with Matlab's LMI toolbox.

# Outline

# Error Measures

- normalized $\mathcal{H}_\infty$-norm of the error system

$$\mathcal{H}_\infty \text{ error } = \frac{\max_{i=1\ldots k} \sigma_1 \left(\mathbf{H}(j\omega_i) - \mathbf{S}^{(i)}\right)}{\max_{i=1\ldots k} \sigma_1 \left(\mathbf{S}^{(i)}\right)},$$

- normalized $\mathcal{H}_2$-norm of the error system

$$\mathcal{H}_2 \text{ error } = \frac{\sum_{i=1}^{k} \left\|\mathbf{H}(j\omega_i) - \mathbf{S}^{(i)}\right\|_F^2}{\sum_{i=1}^{k} \left\|\mathbf{S}^{(i)}\right\|_F^2}.$$

where

$$\left\|\mathbf{H}(j\omega_i) - \mathbf{S}^{(i)}\right\|_F^2 = \sum_{k_1=1}^{p} \sum_{k_2=1}^{p} \left|\mathbf{H}_{k_1,k_2}(j\omega_i) - \mathbf{S}^{(i)}_{k_1,k_2}\right|^2.$$

# Microstrip lines and an RC pair

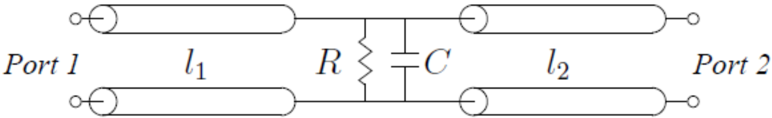


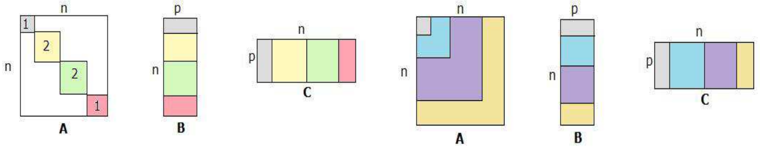

*Figure:* Microstrip lines and an RC pair

Nominal values: $R = 4\text{k}\Omega$, $C = 0.2\text{pF}$, $w = 80\mu\text{m}$, $l_1 = 3\text{cm}$, $l_2 = 2\text{cm}$, for width & lengths of the microstrips, and $h = 0.3\text{mm}$, $\epsilon_r = 4$, for the dielectric height & permittivity.
This reproduces an interconnect link loaded by a device.

# Design parameter: width $w$

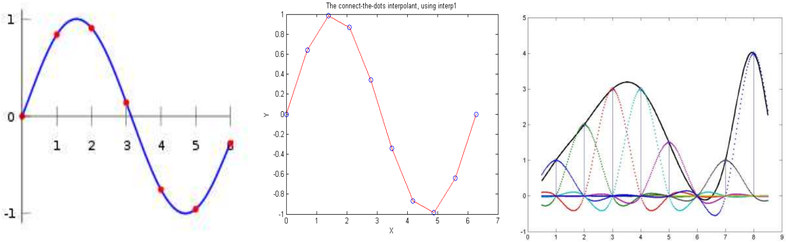
- S-parameters of this 2-port system computed (via a full wave simulation) for 100 frequencies between 10MHz and 10GHz
- 15 values of $w$ between 60 and 130$\mu$m in steps of 5$\mu$m
- for better conditioning, frequencies were scaled by $10^{-6}$
- we use 8 responses for $w$=$\{60, 70, \ldots, 120, 130\}$ $\mu$m for modeling
- we identified systems of order 20 with $\mathbf{D} = \mathbf{0}$
- CPU time for one model was 0.03s on average, and all 8 took 0.25s
- LMI (7) was solved for a common $\mathbf{P}$ $\Rightarrow$ all our parametric models will be stable (no matter how the weights are chosen)
- $w = 90\mu$m was chosen as the reference system
- for all remaining systems, $\mathbf{T}$ in (3) was applied as a similarity transformation; this took 0.51s

# Different canonical forms and interpolation schemes




(a) Modal (b) Balanced

*Figure:* Different canonical forms




(a) Polynomial (b) Piecewise (c) Spline

*Figure:* Plots for different interpolation schemes

# Results for design parameter: width $w$

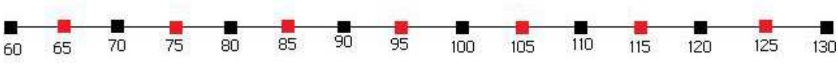



*Figure:* Values for $w$

| | $\mathcal{H}_2$ max | $\mathcal{H}_2$ min | $\mathcal{H}_\infty$ max | $\mathcal{H}_\infty$ min |
|---|---|---|---|---|
| modeling | 1.3309e-4 | 1.1719e-4 | 1.2084e-3 | 1.1030e-3 |
| polynomial with balanced | 1.9319e-4 | 1.1939e-4 | 1.2491e-3 | 1.0970e-3 |
| polynomial with modal | 7.3754e-3 | 5.3365e-4 | 2.2093e-2 | 1.4944e-3 |
| piecewise with balanced | 5.8575e-4 | 2.4685e-4 | 1.3116e-3 | 1.1477e-3 |
| piecewise with modal | 5.4896e-3 | 8.2878e-4 | 1.3753e-2 | 2.2075e-3 |
| spline with balanced | 1.4855e-4 | 1.1979e-4 | 1.2202e-3 | 1.1022e-3 |
| spline with modal | 5.2803e-3 | 1.5953e-4 | 1.2509e-2 | 1.1517e-3 |
| *validation* | *1.3213e-4* | *1.1860e-4* | *1.2032e-3* | *1.1140e-3* |

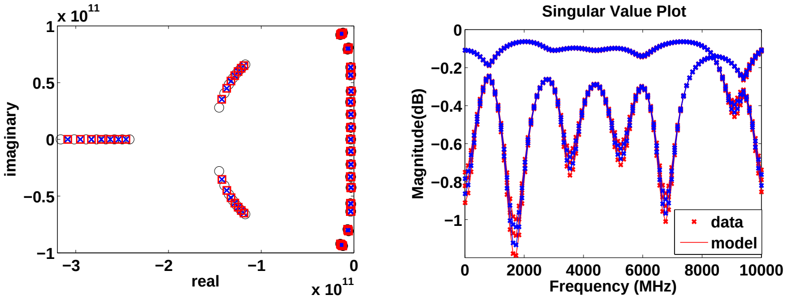
*Table:* Errors

# Results for unitary constraint for design parameter: width $w$

| | $\mathcal{H}_2$ max | $\mathcal{H}_2$ min | $\mathcal{H}_\infty$ max | $\mathcal{H}_\infty$ min |
|---|---|---|---|---|
| polynomial with balanced | 1.9236e-4 | 1.1938e-4 | 1.2483e-3 | 1.0962e-3 |
| polynomial with modal | 1.3852 | 1.6774e-1 | 4.6274 | 3.6683e-1 |
| piecewise with balanced | 6.4347e-4 | 3.0443e-4 | 1.2204e-3 | 1.0917e-3 |
| piecewise with modal | 7.8957e-1 | 3.7693e-3 | 2.4783 | 1.4491e-2 |
| spline with balanced | 1.4854e-4 | 1.1956e-4 | 1.2202e-3 | 1.1022e-3 |
| spline with modal | 9.0187e-1 | 6.1852e-2 | 2.7540 | 1.1561e-1 |

*Table:* Errors when adding the unitary constraint

# *Plots for design parameter: width* $w$




(a) Evolution of poles wrt parameters (black circles: poles of the systems used for modeling, blue crosses: poles of the parametric systems, red squares: true poles of the validation systems)

(b) Singular values of the S-parameter matrices wrt to frequency and parameters (red: systems used for modeling, blue: parametric systems)

*Figure:* Plots for design parameter: width $w$

# Results for design parameters: resistor $R$ & capacitor $C$

| | $\mathcal{H}_2$ max | $\mathcal{H}_2$ min | $\mathcal{H}_\infty$ max | $\mathcal{H}_\infty$ min |
|---|---|---|---|---|
| modeling | 1.9043e-4 | 1.7928e-4 | 1.8470e-3 | 1.7432e-3 |
| polynomial with balanced | 3.0840e-4 | 1.7929e-4 | 1.9073e-3 | 1.7233e-3 |
| polynomial with modal | 1.6616e-2 | 1.7894e-4 | 6.1901e-2 | 1.7532e-3 |
| piecewise with balanced | 1.5603e-3 | 1.7927e-4 | 5.5619e-3 | 1.7090e-3 |
| piecewise with modal | 1.6616e-2 | 1.7927e-4 | 6.1901e-2 | 1.7540e-3 |
| spline with balanced | 3.0840e-4 | 1.7929e-4 | 1.9073e-3 | 1.7233e-3 |
| spline with modal | 1.6616e-2 | 1.7894e-4 | 6.1901e-2 | 1.7532e-3 |
| *validation* | *1.8938e-4* | *1.7941e-4* | *1.8481e-3* | *1.7531e-3* |

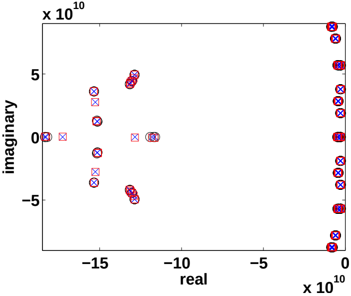
*Table:* Errors

# *Results for unitary constraint for design parameters: resistor $R$ & capacitor $C$*

| | $\mathcal{H}_2$ max | $\mathcal{H}_2$ min | $\mathcal{H}_\infty$ max | $\mathcal{H}_\infty$ min |
|---|---|---|---|---|
| polynomial with balanced | 1.4544e-3 | 1.7929e-4 | 6.8113e-3 | 1.7171e-3 |
| polynomial with modal | 4.0447 | 2.9460e-4 | 2.9092e+1 | 1.8190e-3 |
| piecewise with balanced | 7.3779e-3 | 1.7926e-4 | 3.8669e-2 | 1.6939e-3 |
| piecewise with modal | 3.7449 | 2.4148e-4 | 8.1928 | 1.8347e-3 |
| spline with balanced | 1.4544e-3 | 1.7929e-4 | 6.8113e-3 | 1.7171e-3 |
| spline with modal | 4.0447 | 2.9460e-4 | 2.9092e+1 | 1.8190e-3 |

*Table:* Errors when adding the unitary constraint

# Plots for design parameters: resistor $R$ & capacitor $C$

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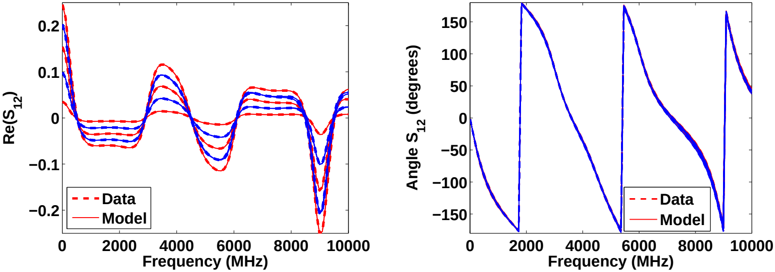


(a) Re($S_{12}$) (b) Angle $S_{21}$

*Figure:* Entries for some parameter values (red: systems used for modeling, blue: parametric systems)

# Outline

# Conclusion

Results depend on

- initial canonical form:
    - modal form suitable for purely mechanical systems
    - balanced form suitable for systems with high damping
- interpolation scheme
    - piecewise is cheap, with not so good results
    - polynomial may loose accuracy when too many points are used
    - spline is accurate, but expensive (with Matlab's spline toolbox)

# Thank you!

# Questions?

# References


D. Deschrijver and T. Dhaene.
Stability and passivity enforcement of parametric macromodels in time and frequency domain.
*IEEE Trans. Microw. Theory Tech.*, 56(11):2435–2441, Nov. 2008.

D. Deschrijver, T. Dhaene, and D. De Zutter.
Robust parametric macromodeling using multivariate orthonormal vector fitting.
*IEEE Trans. Microw. Theory Tech.*, 56(7):1661–1667, July 2008.

T. Dhaene and D. Deschrijver.
Stable parametric macromodeling using a recursive implementation of the vector fitting algorithm.
*IEEE Microw. Wireless Compon. Lett.*, 19(2):59–61, Feb. 2009.

F. Ferranti, D. Deschrijver, L. Knockaert, and T. Dhaene.
Fast parametric macromodeling of frequency responses using parameter derivatives.
*IEEE Microw. Wireless Compon. Lett.*, 18(12):761–763, Dec. 2008.

S. Lefteriu and A. C. Antoulas.
A new approach to modeling multi-port systems from frequency domain data.
*IEEE Trans. Comput.-Aided Design Integr. Circuits Syst.*, 29, Jan. 2010.

A. J. Mayo and A. C. Antoulas.
A framework for the solution of the generalized realization problem.
*Linear Algebra and Its Applications*, 405:634–662, 2007.

P. Triverio, S. Grivet-Talocia, and M. Nakhla.
An improved fitting algorithm for parametric macromodeling from tabulated data.
In *IEEE Workshop on Signal Propagation on Interconnects*, pages 1–4, May 2008.

P. Triverio, S. Grivet-Talocia, and M. Nakhla.
On the construction of uniformly stable multivariate interconnect macromodels.
In *IEEE Workshop on Signal Propagation on Interconnects*, pages 1–4, May 2009.

P. Triverio, S. Grivet-Talocia, and M. Nakhla.