Progress in Parametric Model Reduction

Fraunhofer Institut Techno- und Wirtschaftsmathematik

J. Mohring, Paris, 15.12.2009

Overview

- Need for parametric model reduction
- Optimal H_2 controller for LMS concrete car benchmark
- The LBF acoustic aquarium
- Numerical improvements in Fraunhofer model reduction toolbox
- New theoretical results

Need for parametric model reduction

Need for parametric model reduction

- **Fitting** FE-models to measurements
- Treatment of **non-linear** boundary conditions
- **Sensitivity analyses**
- Model based **controller design**
- Simulation based **design optimization**

Whenever evaluating **large FE-models**

- must be **repeated** many times (e.g. optimization)
- is very **expensive** (e.g. frequency response)

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Optimal H_2 controller for

LMS concrete Car benchmark

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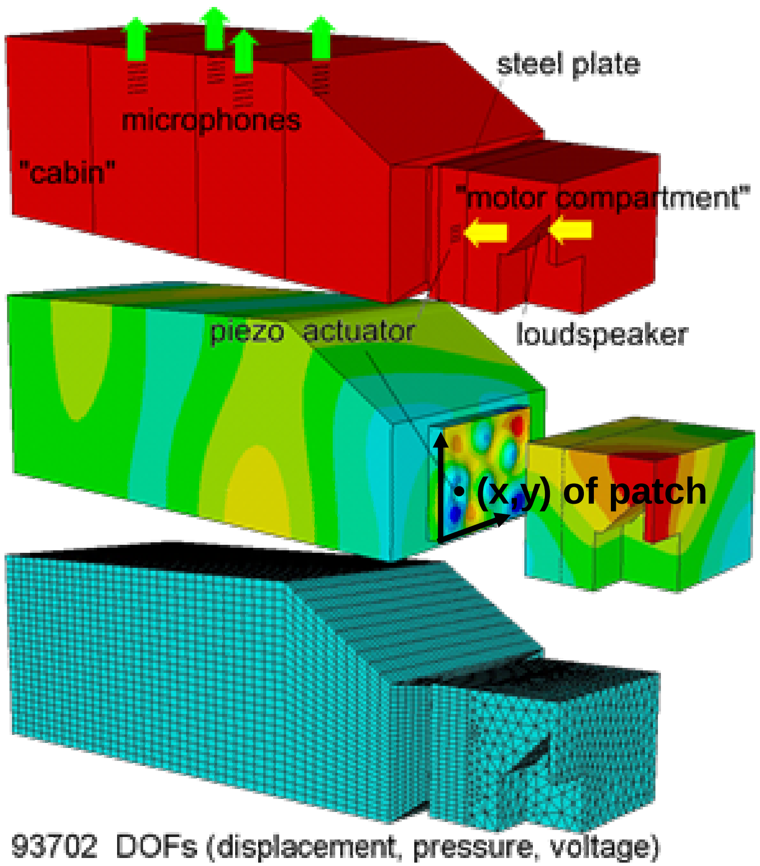
Optimal model based H_2 controller of LMS concrete car



Inputs:
piezo voltage & volume velocity of loudspeaker

Outputs:
SPL at passengers' heads

Optimization parameters:
Controller gains + Patch coordinates

FE Model:
multi-field, ~100.000 DOFs

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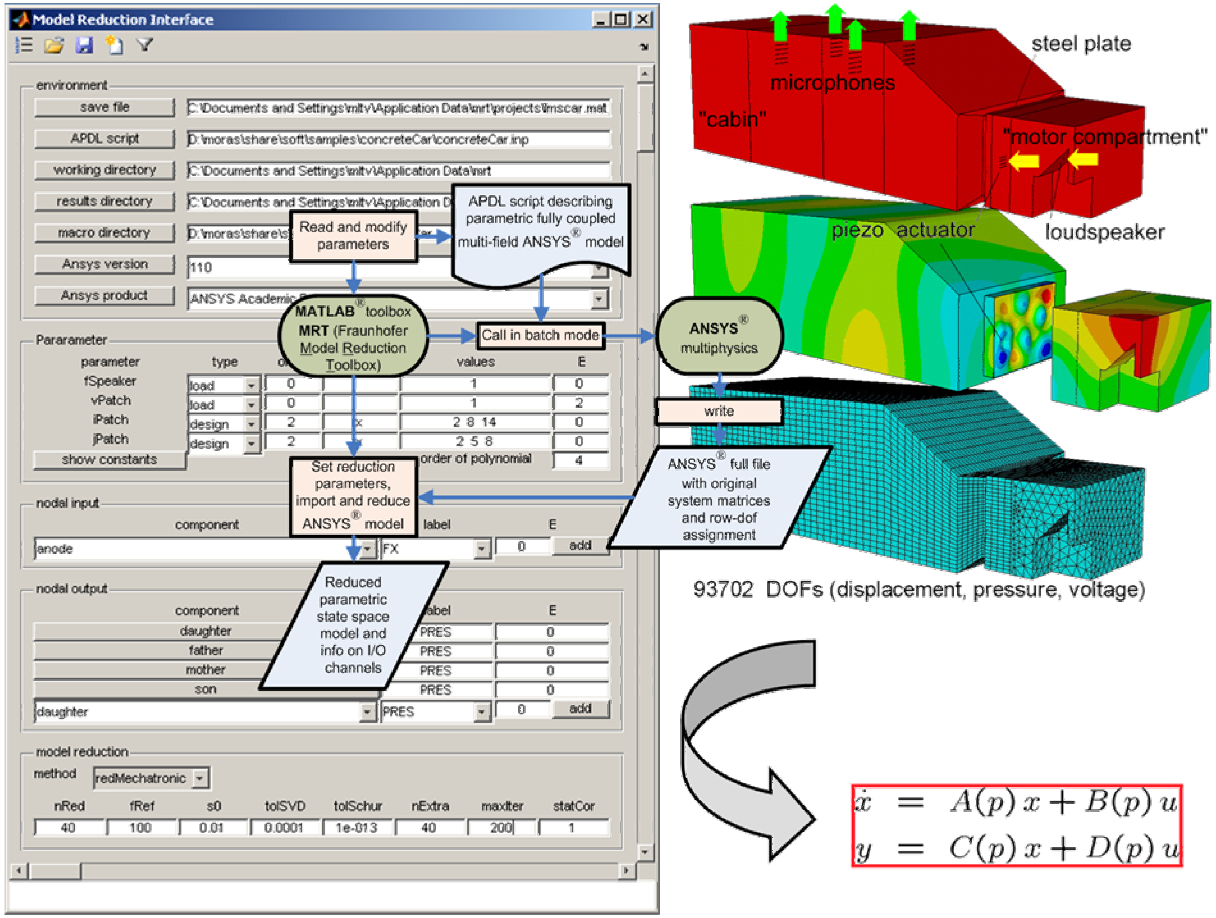
Step 1: create reduced parametric model using Fraunhofer model reduction toolbox



$$\dot{x} = A(p)\,x + B(p)\,u$$
$$y = C(p)\,x + D(p)\,u$$

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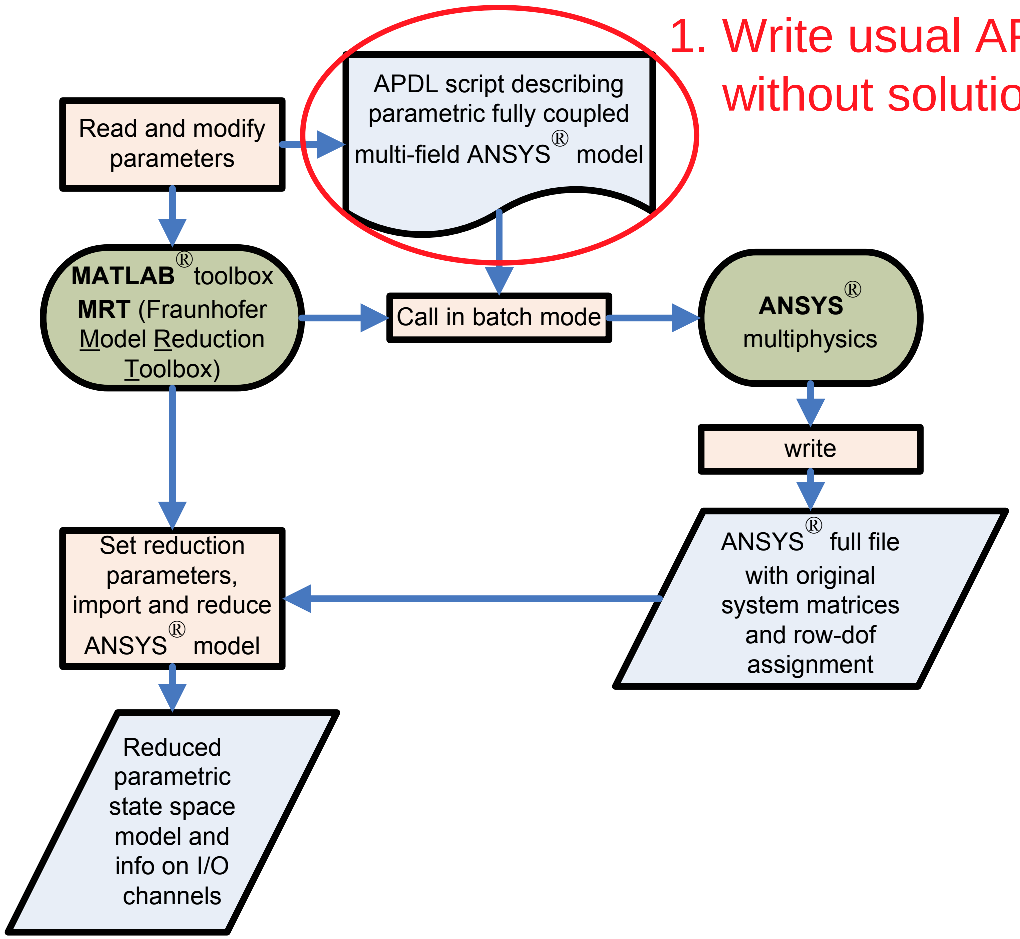
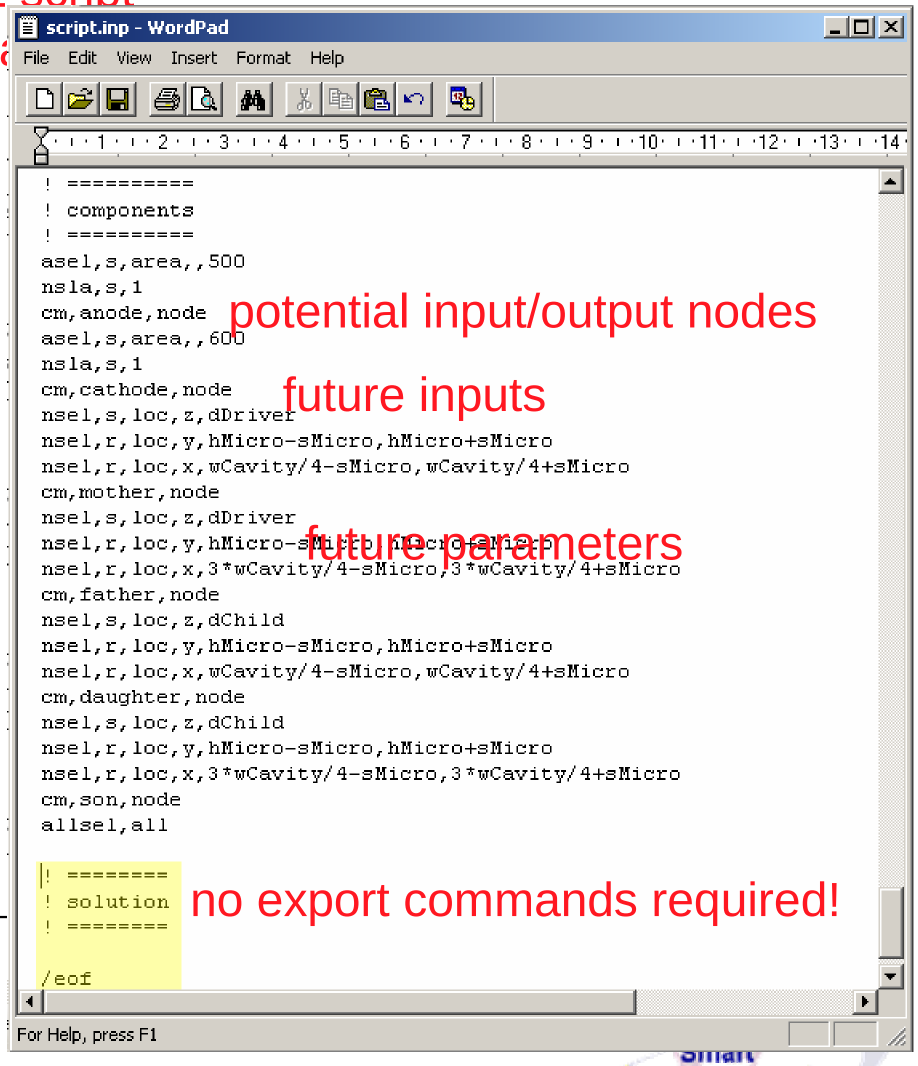
How to generate a parametric reduced model with MRT

1. Write usual APDL script without solution pa

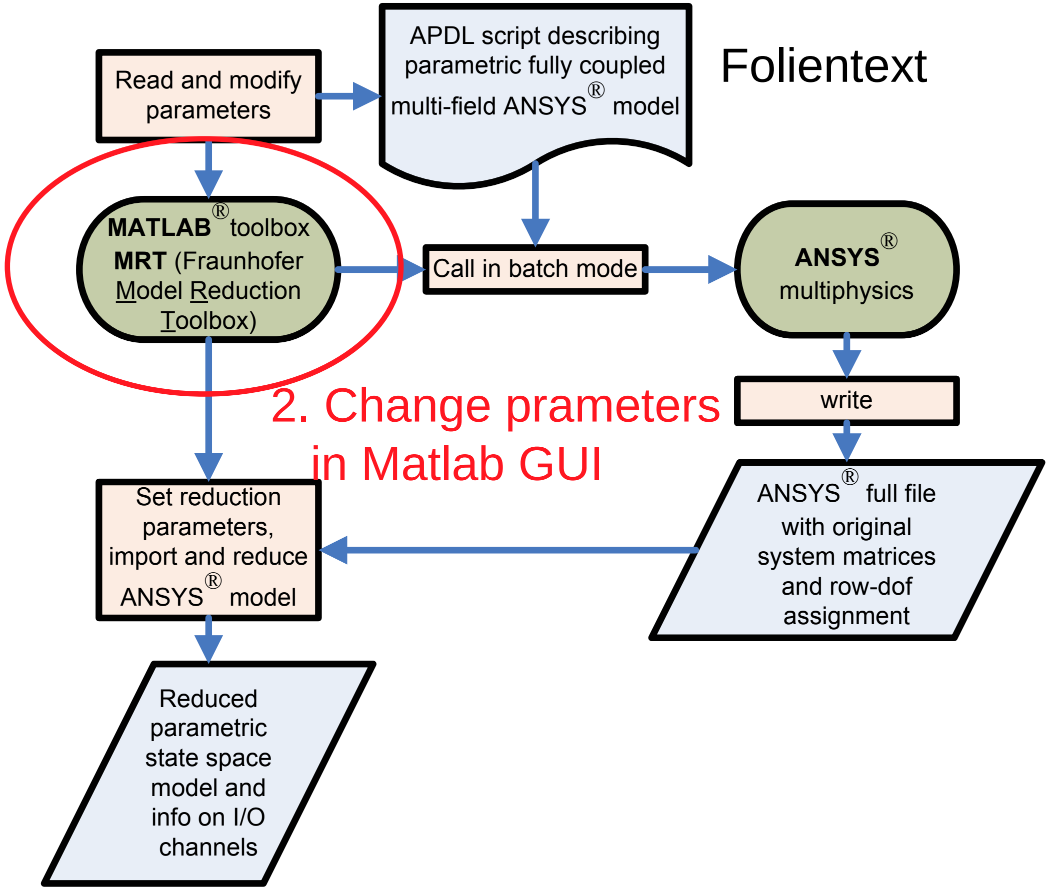


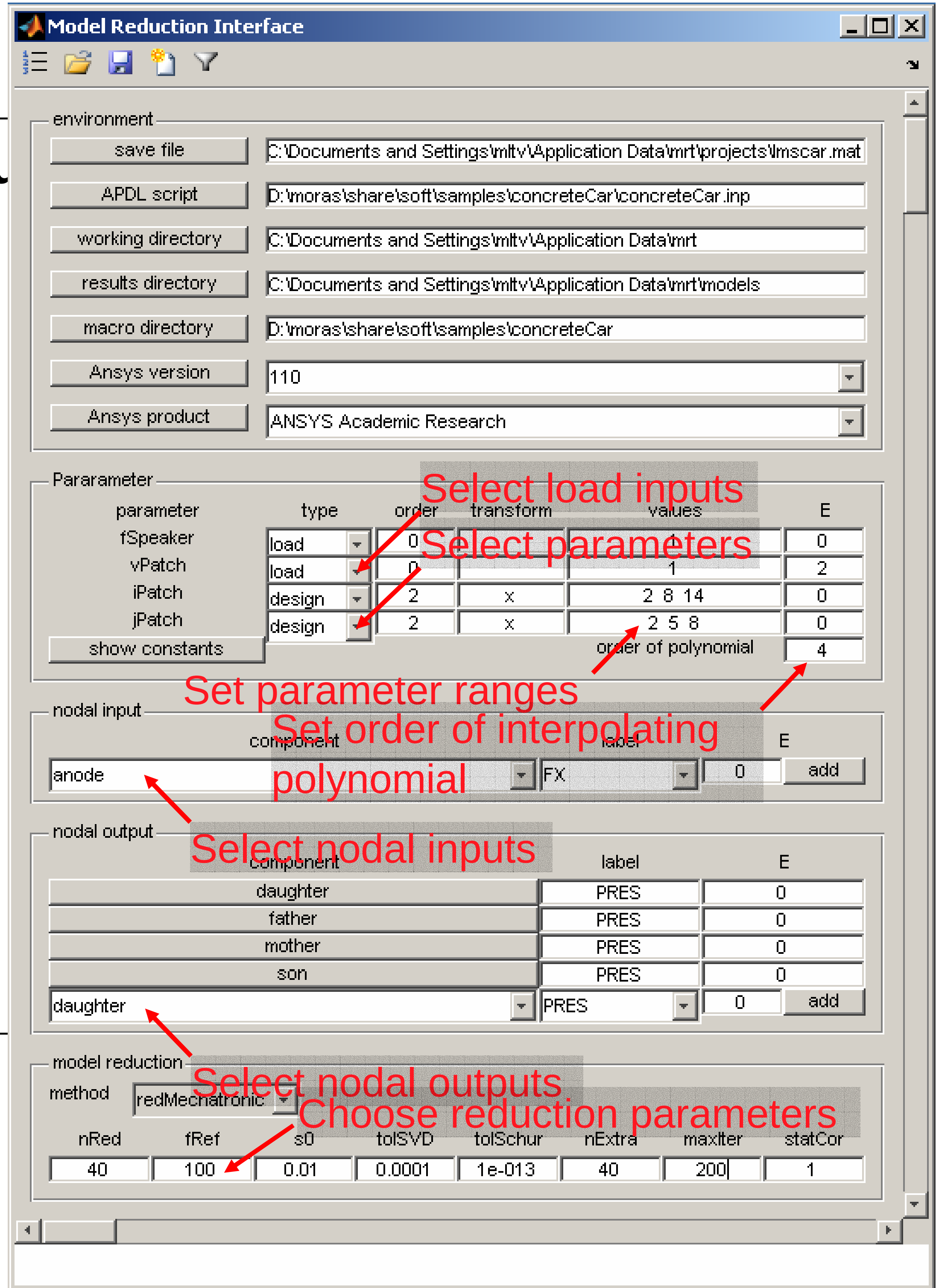



```
! ==========
! components
! ==========
asel,s,area,,500
nsla,s,1
cm,anode,node
asel,s,area,,600
nsla,s,1
cm,cathode,node
nsel,s,loc,z,dDriver
nsel,r,loc,y,hMicro-sMicro,hMicro+sMicro
nsel,r,loc,x,wCavity/4-sMicro,wCavity/4+sMicro
cm,mother,node
nsel,s,loc,z,dDriver
nsel,r,loc,y,hMicro-sMicro,hMicro+sMicro
nsel,r,loc,x,3*wCavity/4-sMicro,3*wCavity/4+sMicro
cm,father,node
nsel,s,loc,z,dChild
nsel,r,loc,y,hMicro-sMicro,hMicro+sMicro
nsel,r,loc,x,wCavity/4-sMicro,wCavity/4+sMicro
cm,daughter,node
nsel,s,loc,z,dChild
nsel,r,loc,y,hMicro-sMicro,hMicro+sMicro
nsel,r,loc,x,3*wCavity/4-sMicro,3*wCavity/4+sMicro
cm,son,node
allsel,all

! ========
! solution
! ========

/eof
```

How to generate a parametric redu

Folientext





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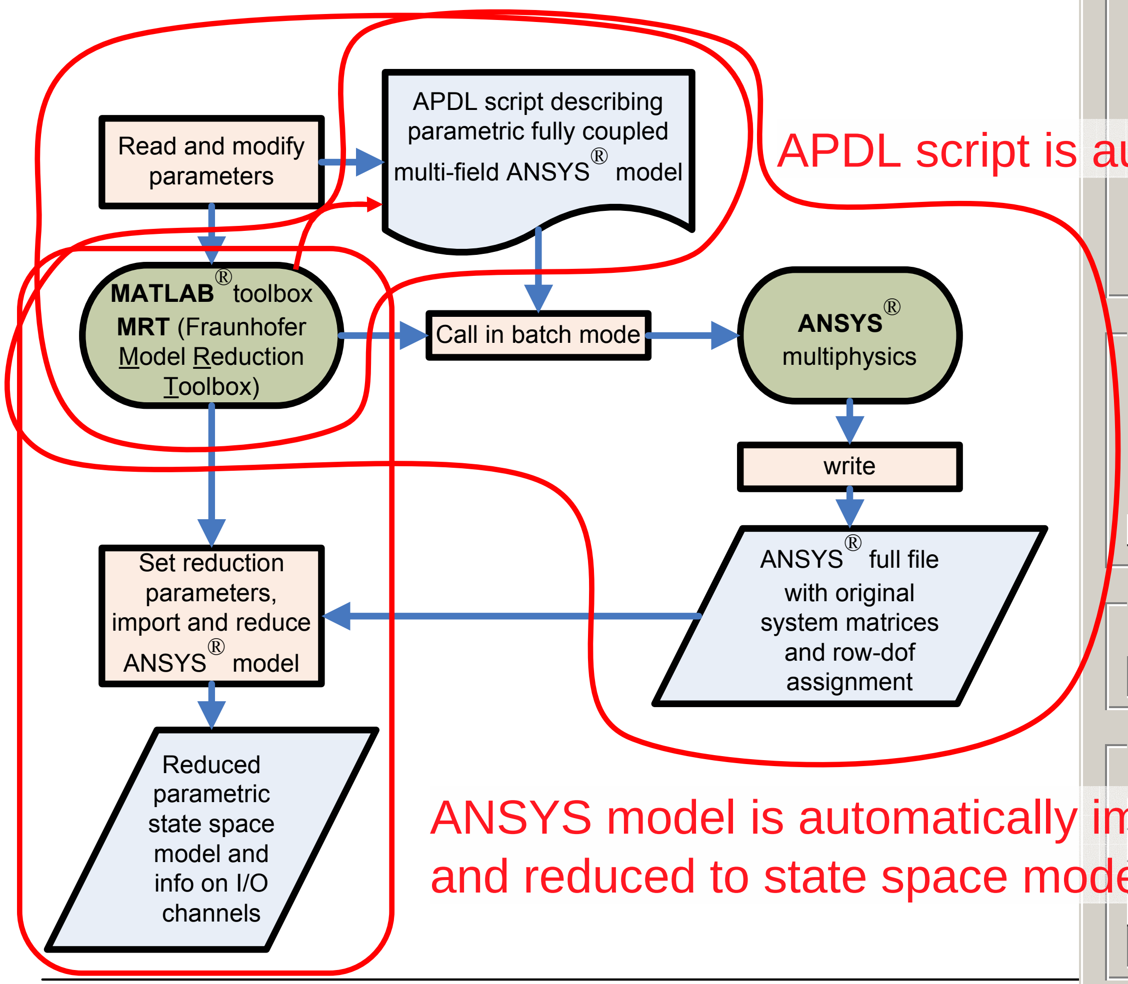
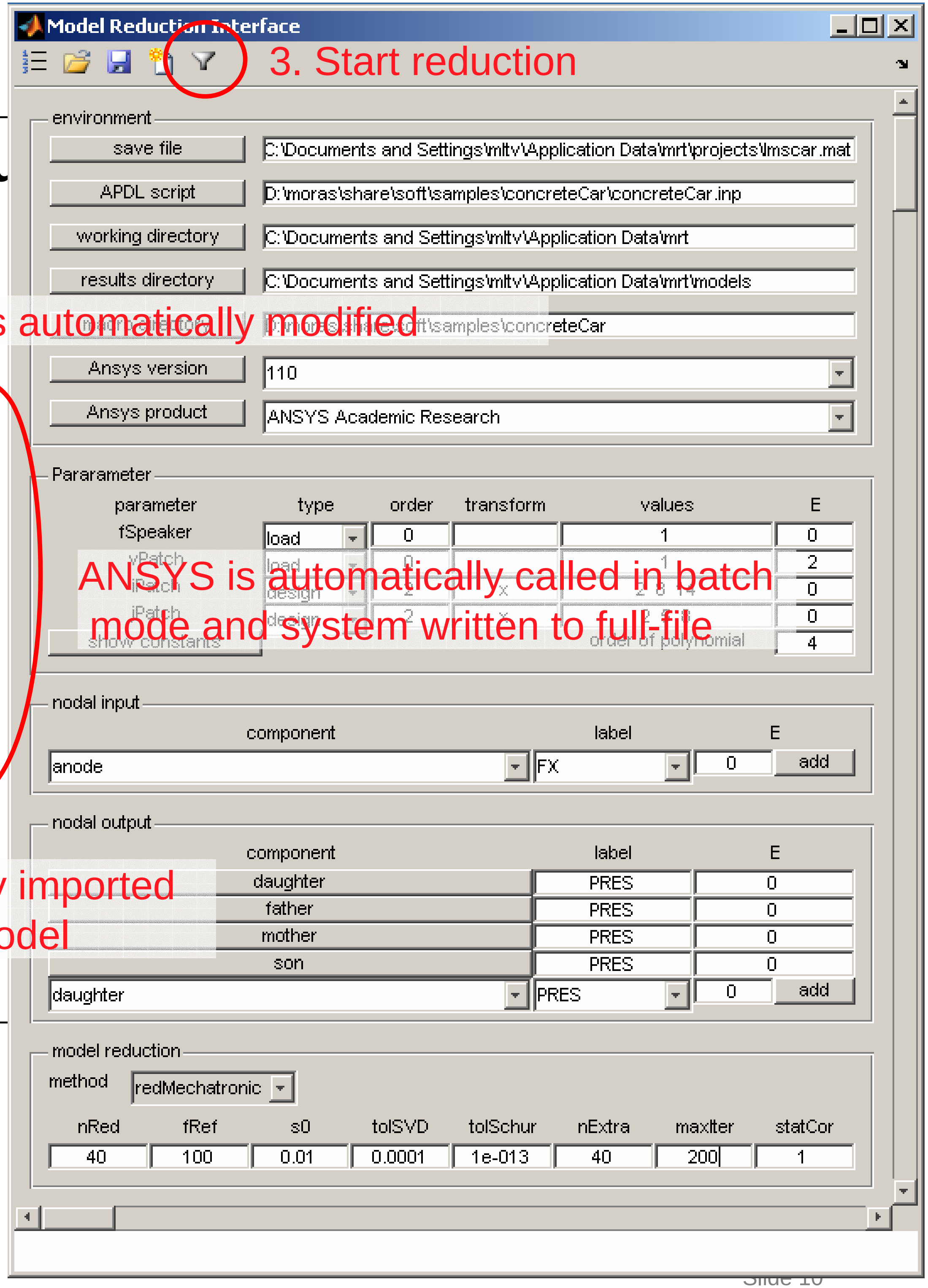
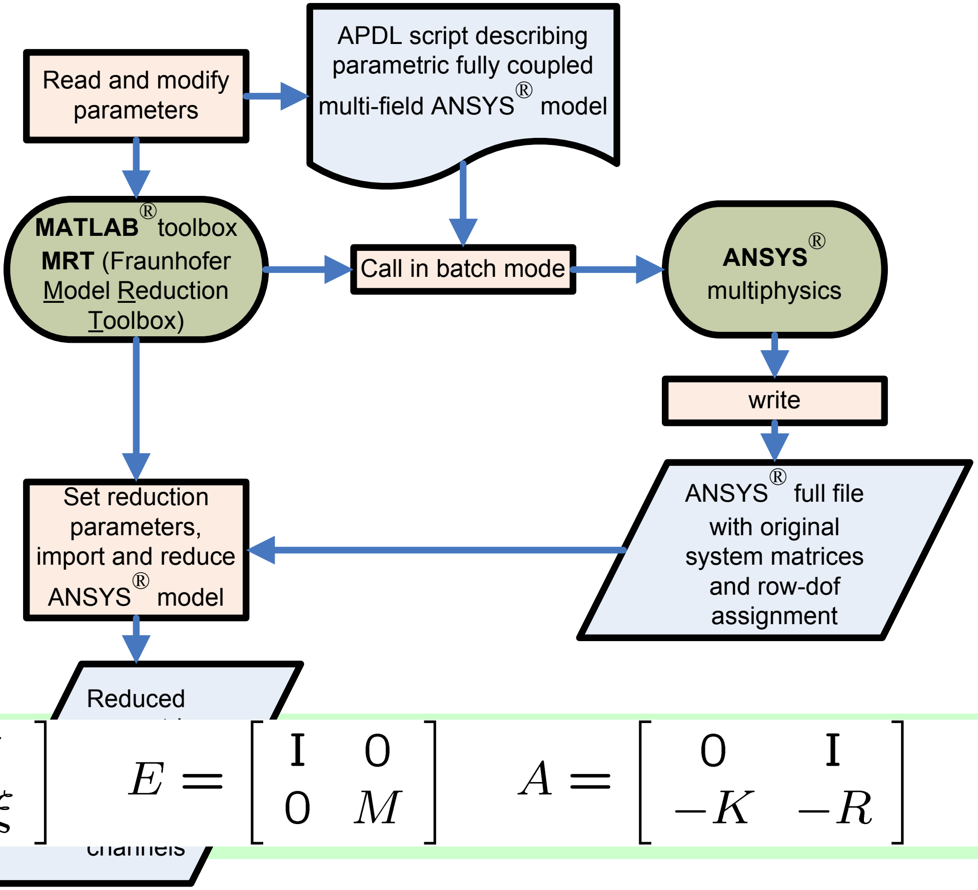
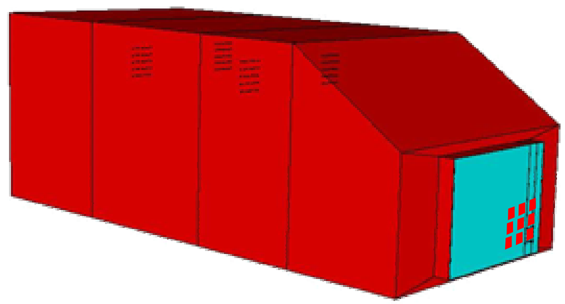
How to generate a parametric redu





How to generate a parametric reduced model with MRT



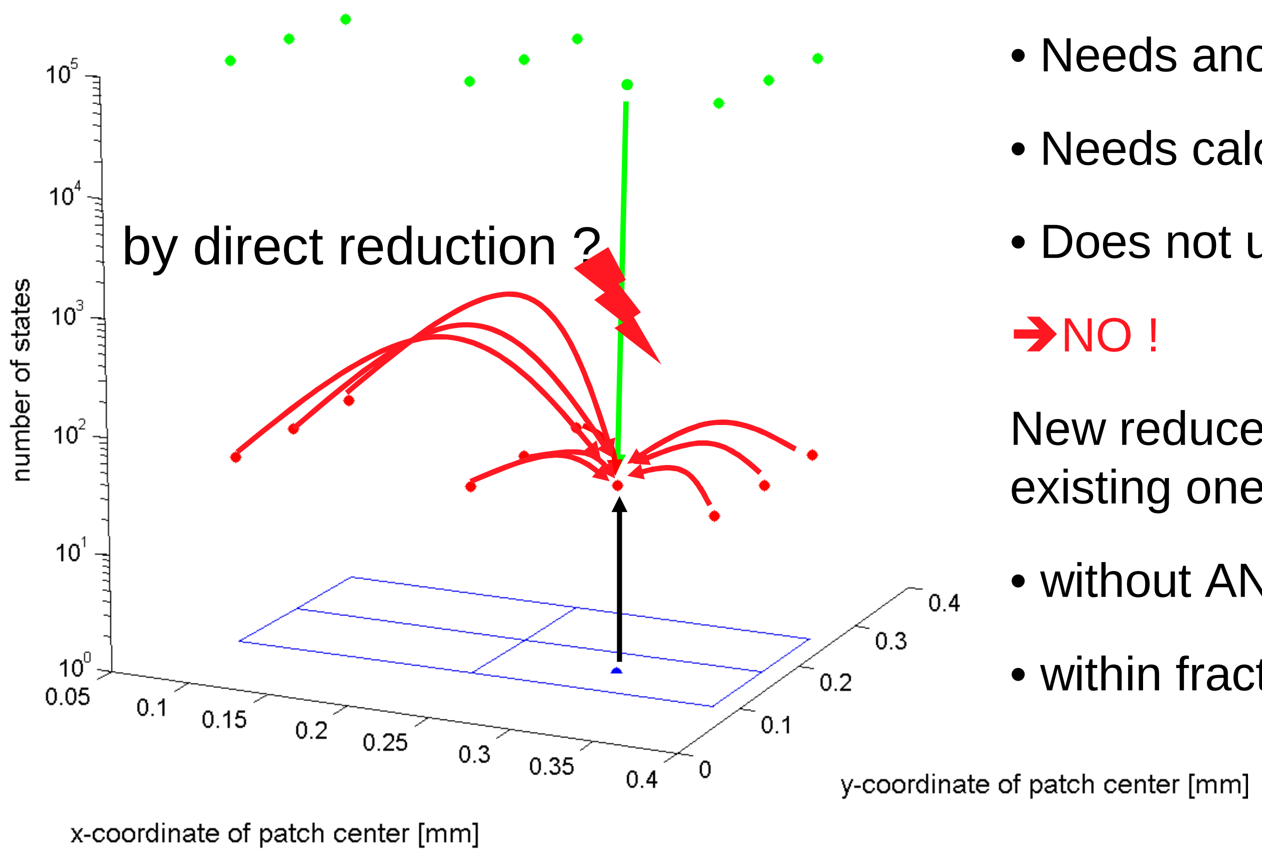


This is repeated for different automatically chosen parameters covering design space

$$= \begin{bmatrix} \xi \\ s\xi \end{bmatrix} \qquad E = \begin{bmatrix} \mathrm{I} & 0 \\ 0 & M \end{bmatrix} \qquad A = \begin{bmatrix} 0 & \mathrm{I} \\ -K & -R \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ L \end{bmatrix}$$

$$\boxed{\begin{aligned} \dot{x} &= A(p)\,x + B(p)\,u \\ y &= C(p)\,x + D(p)\,u \end{aligned}}$$

How to get reduced models for new parameters ?



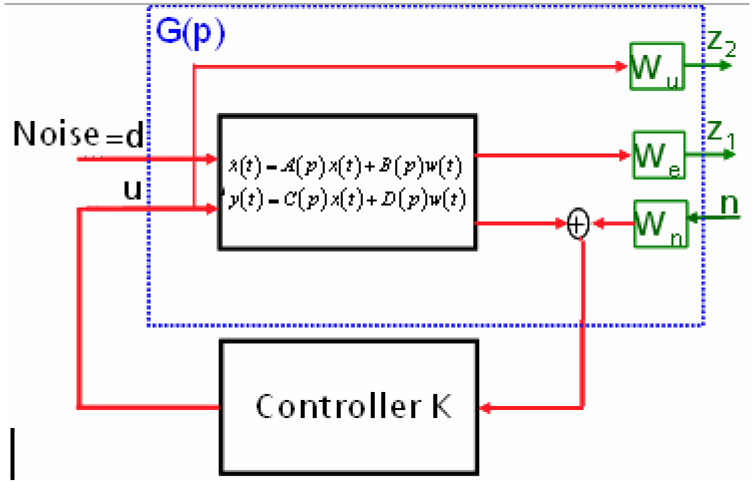
- Needs another ANSYS license
- Needs calculations taking minutes to hours
- Does not use reductions created before

➔NO !

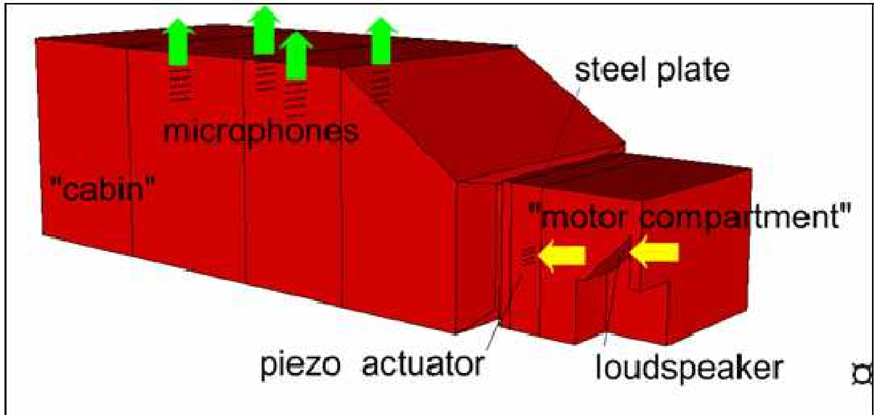
New reduced model is **interpolated** from existing ones

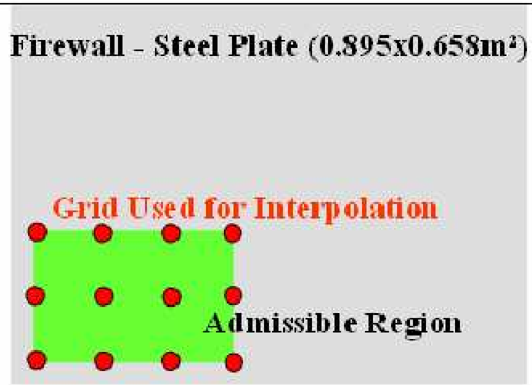
- without ANSYS
- within fractions of a second

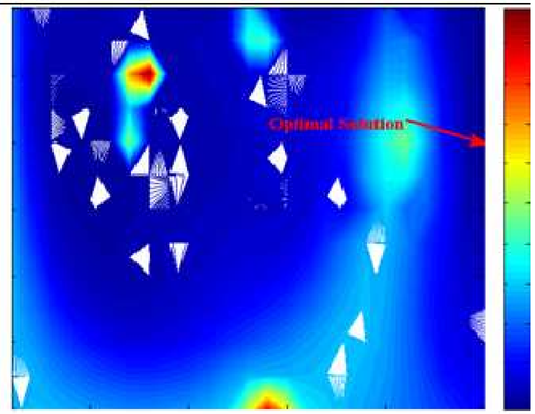
Step 2: Form optimal H2 controller



- Work done by colleague A. Wirsen
- Weighting functions put emphasis on range from 20 to 200 Hz
- To be published at ECCM 2010 (also Paris)







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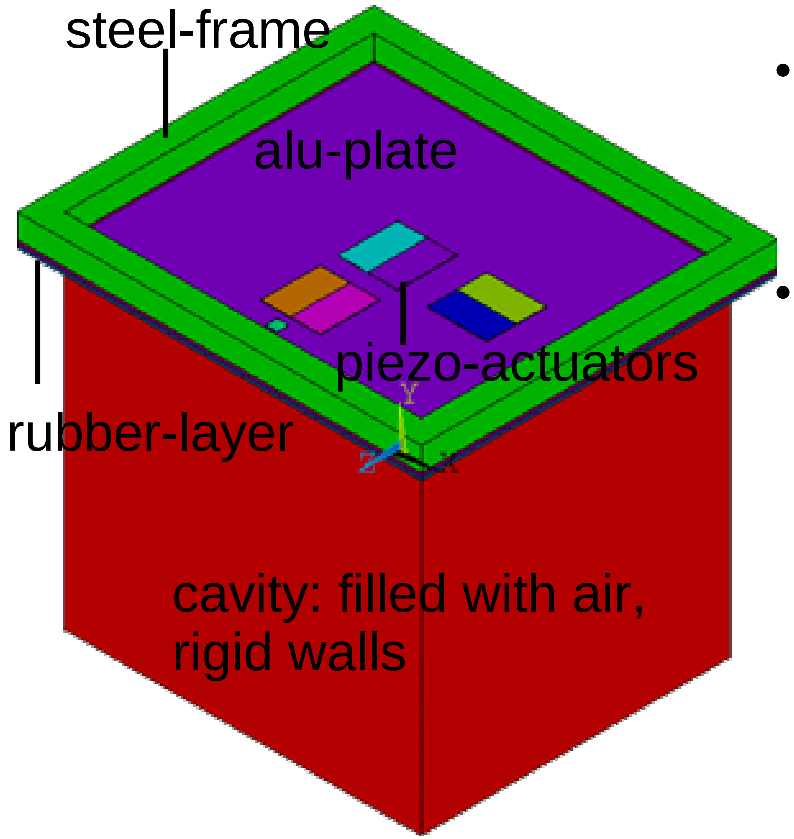
LBF acoustic aquarium

LBF acoustic aquarium



- Matthias Kurch (Fraunhofer LBF-Darmstadt) provides Ansys FE-model and measurements of “acoustic aquarium” via Tiziana and me.

- Plans: Common paper on fitting damping and stiffness of screw-rubber layer to measurements using parametric reduced model. (LBF: M. Kurch, ITWM: J. Mohring, S. Lefteriu, A. Wirsen)

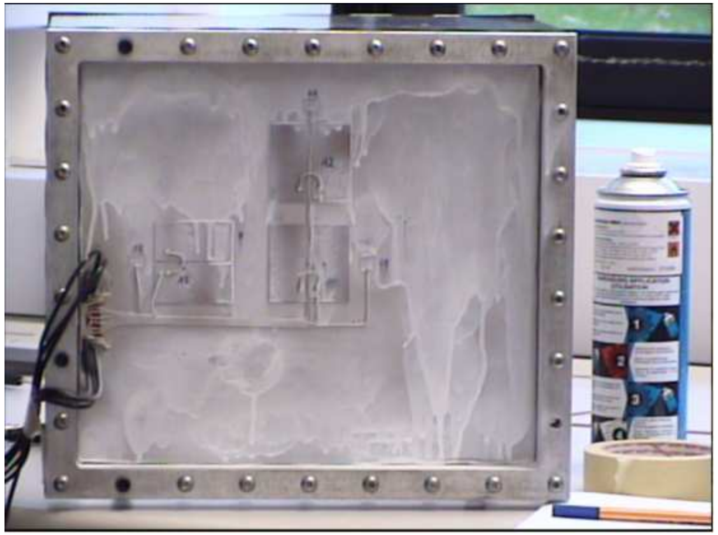
LBF acoustic aquarium : aluminum-plate

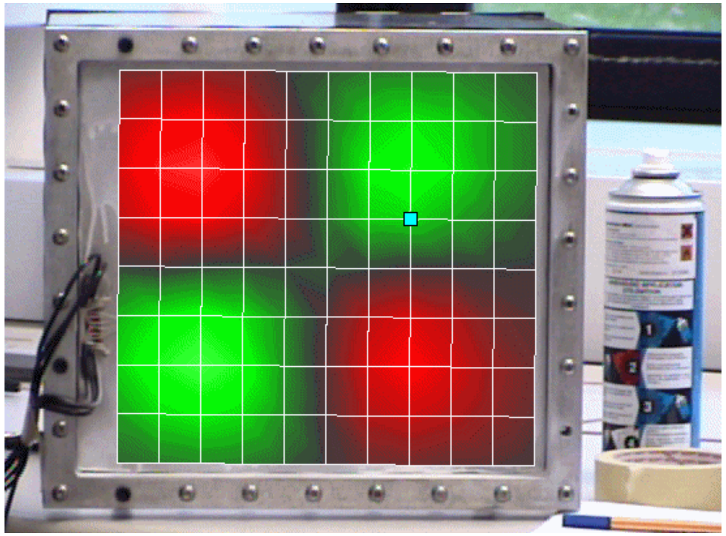
- Dimensions: 300x280x2 mm^3
- 3 x 2 actuators (A1, A2, A3)
- PIC 155 50x25x0.5
- 3 sensors (S1, S2, S3)
- PIC 155 10x10x0.1

With permission of M. Kurch (LBF)

LBF acoustic aquarium: laser-scanning-vibrometer

- 99 measurement positions
- 3 measurements per excitation: microphone, vibrometer, piezo-sensor S2
- 298 frequency responses
- Hammer excitation





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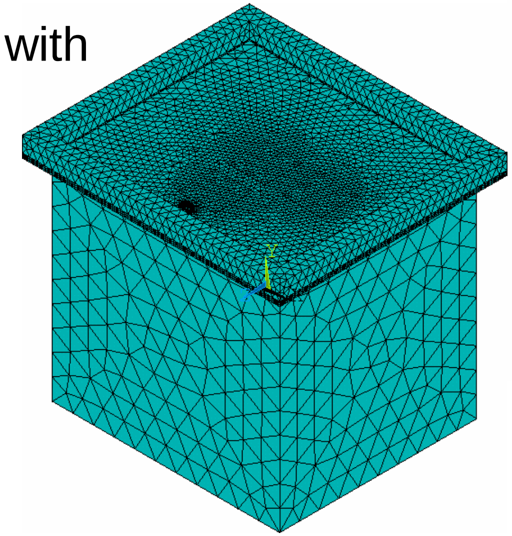
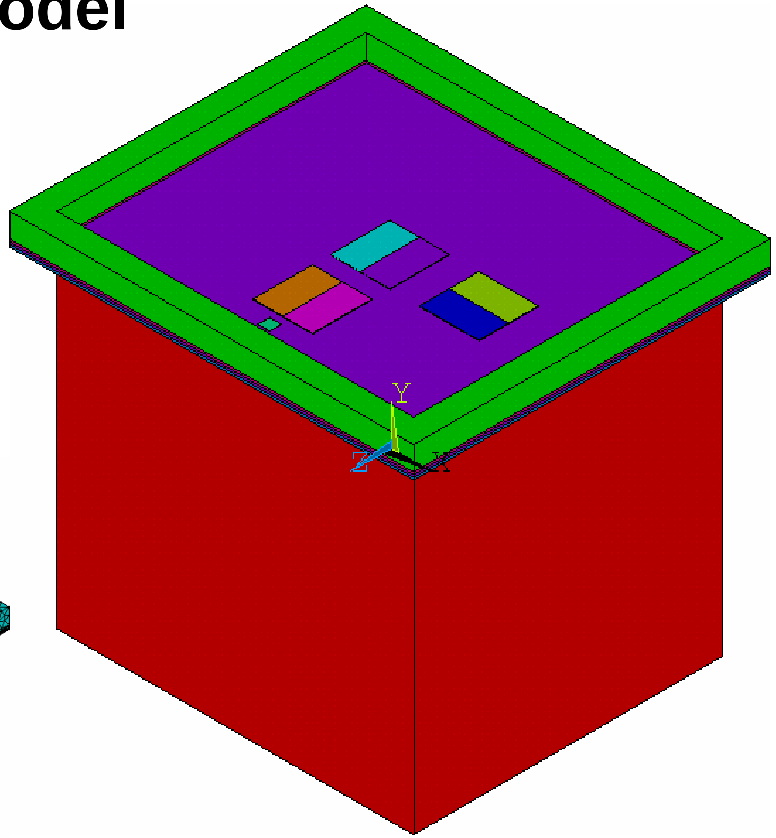
LBF acoustic aquarium: finite element model

- Aluminum-plate, steel-frame, rubber-layer: SOLID186
- Actuators and sensors: SOLID226
- Air volume: FLUID30
- Measurement positions coincide with nodes of FE-mesh
- 35537 nodes





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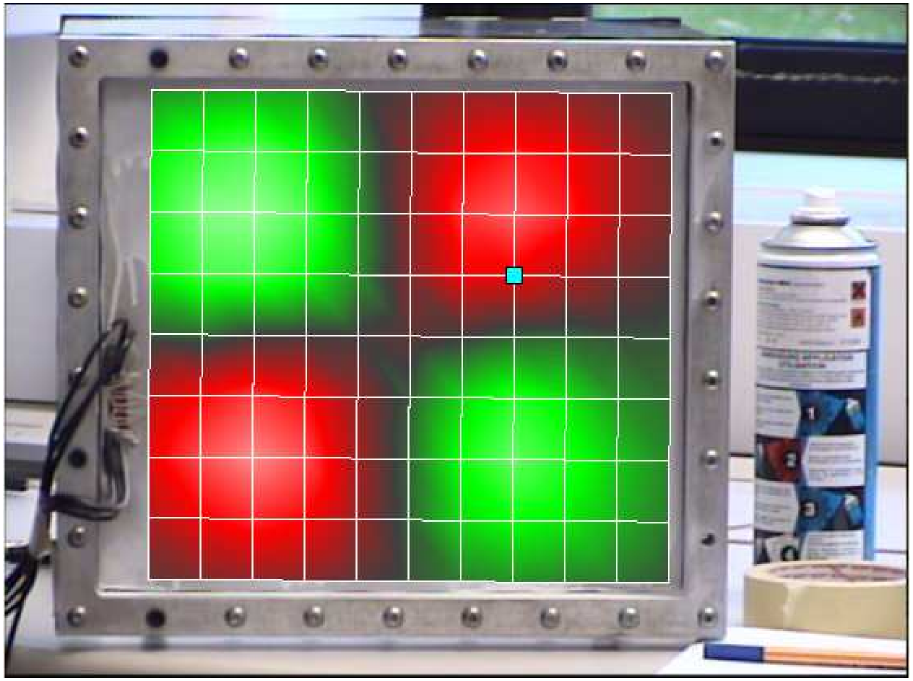
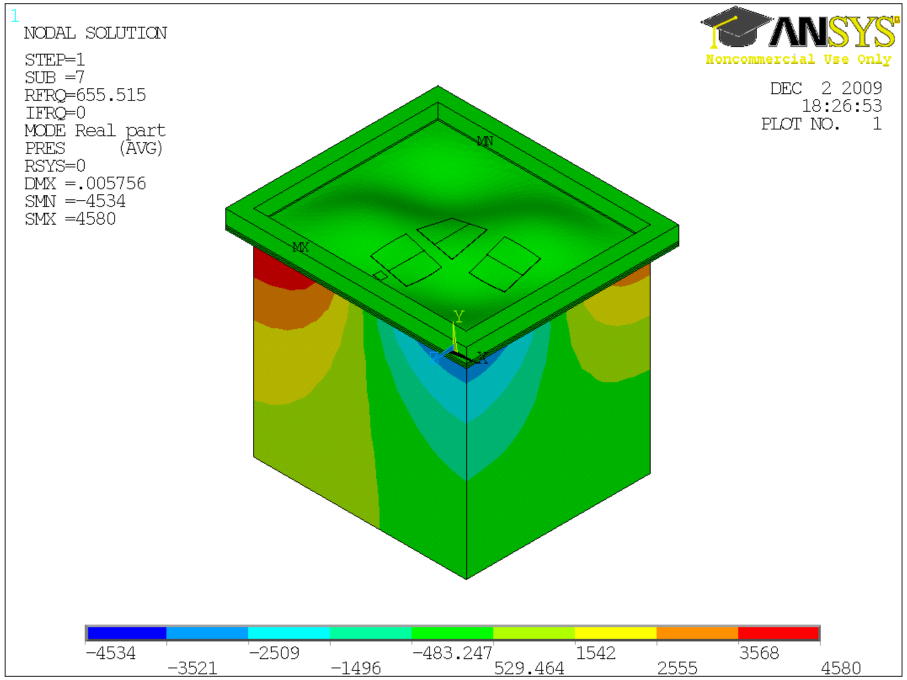
LBF acoustic aquarium: need for fitting parameters of screw-rubber-layer



621,30 Hz



655,51 Hz

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Numerical improvements in Fraunhofer model reduction toolbox

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Numerical improvements

Parallel version

- Depending on number of parameters and order of polynomial we have to build and reduce several 10 or 100 FE models
- This can be done in parallel without any communication, if enough cores and Ansys licenses are available
- Full parametric LMS car model takes about **45 min** (12 evaluations, 32 cores, 5 Ansys licenses)

Numerical improvements

Efficient reduction of thermal problems with many inputs (area sources)

- Continuation of common work with **Julian**
- **Goal:** reduce $E\dot{x} = Ax + Bu,\ y = Cx$
- Instead of standard moment matching, i.e. computing

$$V = \mathrm{orth}\left[A^{-1}B, A^{-1}EA^{-1}B, \left(A^{-1}E\right)^2 A^{-1}B, \cdots \right]$$

form incomplete **SVD** of operator T **without forming matrix** explicitly

$$T \approx USV^*$$

$$T\xi = \left[A^{-1}B, A^{-1}EA^{-1}B, \cdots, \left(A^{-1}E\right)^k A^{-1}B \right] \xi$$

U essential states to project on.

New theoretical results on parametric reduction via matrix matching

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Idea of matrix matching

FE-models for selected parameters

$$\left(E(p_1),A(p_1),B(p_1),C(p_1)\right) \quad \ldots \quad \left(E(p_n),A(p_n),B(p_n),C(p_n)\right)$$

Independent reduction

Reduced models

$$\left(\tilde{E}_1,\tilde{A}_1,\tilde{B}_1,\tilde{C}_1\right) \quad \ldots \quad \left(\tilde{E}_k,\tilde{A}_k,\tilde{B}_k,\tilde{C}_k\right)$$

Bring realizations close to first one by solving gen. Sylvester eqns. for similarity transform

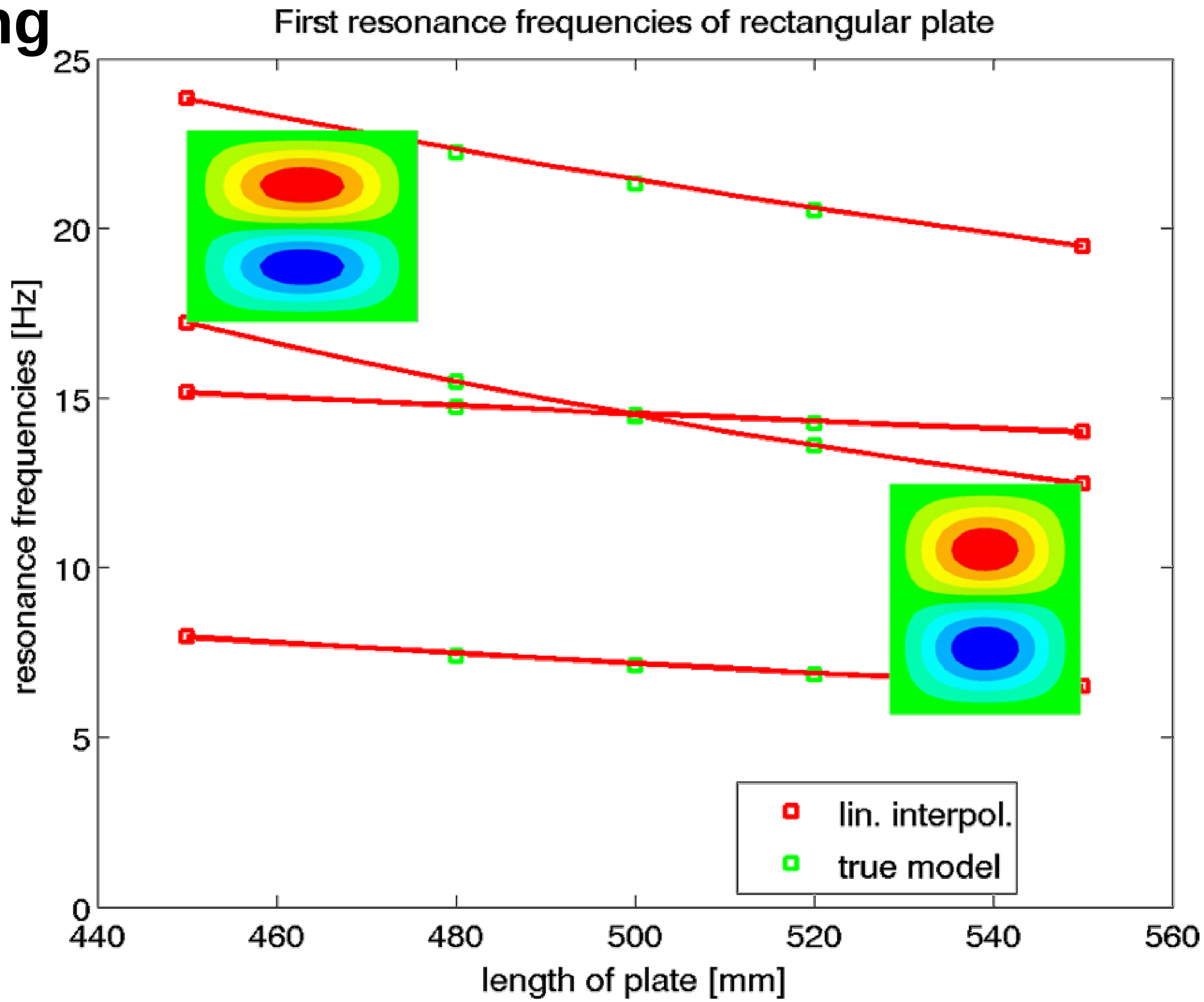
$$\left(\breve{E}_1,\breve{A}_1,\breve{B}_1,\breve{C}_1\right) \quad \ldots \quad \left(T_k^{-1}\tilde{E}_kT_k,\underbrace{T_k^{-1}\tilde{A}_kT_k}_{\breve{A}_k},T_k^{-1}\tilde{B}_k,\tilde{C}_kT_k\right)$$

Interpolate matrices

$$\breve{A}(p)=\sum_i \omega_i(p)\,\breve{A}_i$$

Success of matrix matching

- **Poles move correctly with the parameters, even in case of eigenvalue-crossing**
- FE-mesh may change topology



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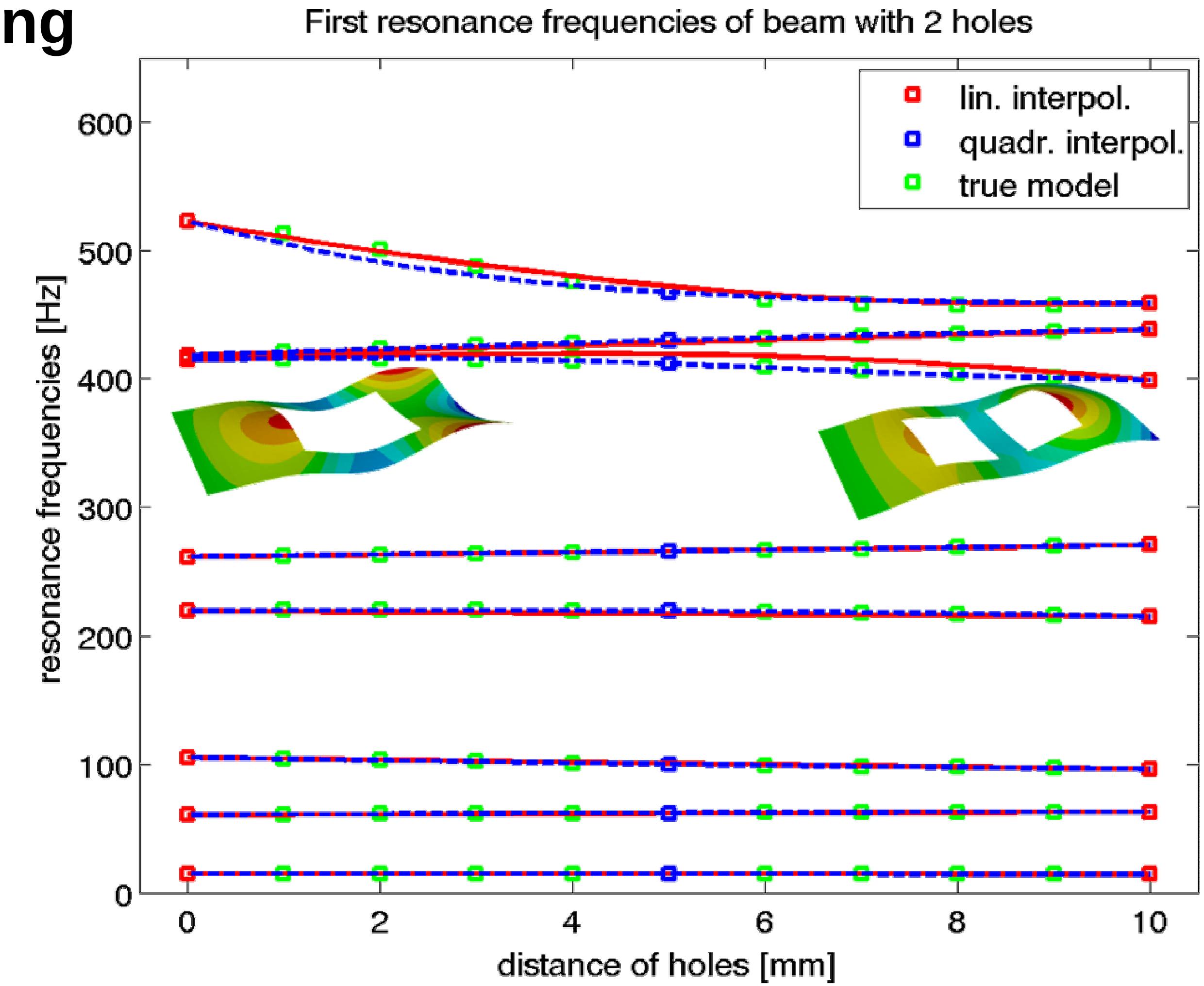

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Choosing realizations optimal for interpolation

For illustration

Consider linear interpolation of one-parametric class of standard state space systems

$$\Sigma_p = \left(A(p), B(p), C(p)\right): \quad \dot{x} = A(p)x + B(p)u,\ y = C(p)x, \quad p \in [0,1]$$

Models are already reduced, i.e. typical state space dimension < 100.

Quality of interpolation depends strongly on chosen realization

Example 1

Assume $\Sigma_1 = \Sigma_0 = (A,B,C)$. Expect $\Sigma_{0.5} = \Sigma_0 = \Sigma_1$.

Keep Σ_0 , but replace realization of Σ_1 by equivalent one:

$\Sigma_1' = \left(X^{-1}AX, X^{-1}B, CX\right)$ with $X = -I$, i.e. $\Sigma_1' = \left(A, -B, -C\right)$

Linear interpolation of matrices yields nonsense:

$$\Sigma_{0.5}' = 0.5\left(A,B,C\right) + 0.5\left(A,-B.-C\right) = (A,0,0)$$

Choosing realizations optimal for interpolation

For illustration

Consider linear interpolation of one-parametric class of standard state space systems

$$\Sigma_p = \left(A(p), B(p), C(p)\right): \quad \dot{x} = A(p)x + B(p)u,\ y = C(p)x, \quad p \in [0,1]$$

Models are already reduced, i.e. typical state space dimension < 100.

Quality of interpolation depends strongly on chosen realization

Example 2

Assume $\Sigma_1 = \Sigma_0 = \left(\begin{bmatrix} \alpha & 0 \\ 0 & \beta \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right)$. Expect $\Sigma_{0.5} = \Sigma_0 = \Sigma_1$

Keep Σ_0, but replace realization of Σ_1 by equivalent one:

$$\Sigma_1' = \left(\begin{bmatrix} \beta & 0 \\ 0 & \alpha \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \right)$$

Linear interp. is not minimal

$$\Sigma_{0.5}' = \left(\begin{bmatrix} (\alpha+\beta)/2 & 0 \\ 0 & (\alpha+\beta)/2 \end{bmatrix}, \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}, \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix} \right)$$

Choosing realizations optimal for interpolation

Known :
Interpolating transfer functions works fine for *s* away from poles

$$H_i(s) = C_i M_i B_i, \quad M_i = (sI - A_i)^{-1}, \quad i = 1,2$$

$$\tilde{H}_p(s) = (1-p)\,H_0(s) + pH_1(s), \quad \text{i.e. with } \Delta B = B_1 - B_0$$

$$= (1-p)\,C_0 M_0 B_0 + p\left(C_0 + \Delta C\right)\left(M_0 + \Delta M\right)\left(B_0 + \Delta B\right)$$

Idea: compare this interpolation with transfer-function of interpolated matrices at "save" *s*

$$H_p(s) = \left(C_0 + p\Delta C\right)\left(M_0 + p\Delta M\right)\left(B_0 + p\Delta B\right)$$

$$\tilde{H}_p(s) - H_p(s)$$

$$= p(1-p)\underbrace{\left(C_0 \Delta M \Delta B + \Delta C M_0 \Delta B + \Delta C \Delta M B_0\right)}_{\Delta} + O\left(\Delta \cdot^3\right)$$

Difference can be estimated

$$\|\Delta\| \le \alpha \|M_1 - M_0\|^2 + \beta \|B_1 - B_0\|^2 + \gamma \|C_1 - C_0\|^2,$$

$$\text{with} \quad \alpha = \frac{\|B_0\|\|C_0\|}{\|M_0\|}, \beta = \frac{\|M_0\|\|C_0\|}{\|B_0\|} \text{ etc.}$$

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Choosing realizations optimal for interpolation

Idea: Minimize bound for difference over all equi-valent realizations of Σ_1

$$\tilde{J}(X) = \alpha \left\| X^{-1} M_1 X - M_0 \right\|^2 + \beta \left\| X^{-1} B_1 - B_0 \right\|^2 + \gamma \left\| C_1 X - C_0 \right\|^2$$

For (almost) unitary *X* this is (almost) equivalent to minimizing

$$J(X) = \alpha \left\| M_1 X - X M_0 \right\|^2 + \beta \left\| B_1 - X B_0 \right\|^2 + \gamma \left\| C_1 X - C_0 \right\|^2$$

Claiming necessary cond. with Frobenius-norm leads to **generalized Sylvester equation** for *X*

$$J'(X)[H] = 0 \quad \forall H$$

$$\|A\| = \|A\|_F = tr\left(A^* A\right), \quad \|AB\| = \|BA\|, \quad \|A\| = \left\|A^*\right\|$$

$$\left(\alpha M_1^* M_1 + \gamma C_1^* C_1\right) X + X \left(\alpha M_0 M_0^* + \beta B_0 B_0^*\right)$$
$$-\alpha\left(M_1 X\, M_0^* + M_1^* X\, M_0\right) = \beta B_1 B_0^* + \gamma C_1^* C_0$$

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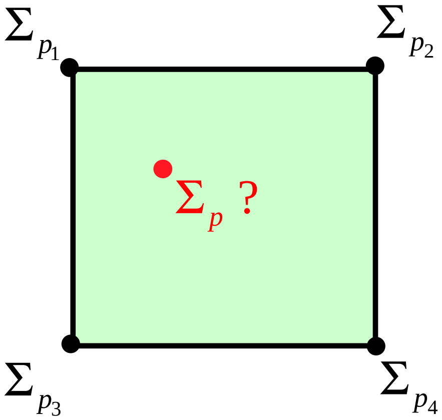
Open questions

Stability



- Assume Σ_{p_i} are **stable**. Are so the interpolated systems?
- Answered by Sanda

Open questions

Unitarity

- True error bound

$$\tilde{J}(X) = \alpha \left\| X^{-1} M_1 X - M_0 \right\|^2 + \beta \left\| X^{-1} B_1 - B_0 \right\|^2 + \gamma \left\| C_1 X - C_0 \right\|^2$$

and the one used for finding optimal Transformation X

$$J(X) = \alpha \left\| M_1 X - X M_0 \right\|^2 + \beta \left\| B_1 - X B_0 \right\|^2 + \gamma \left\| C_1 X - C_0 \right\|^2$$

coincide only, if X is **unitary**.

- If some modes enter/leave with changing parameters, then X might become singular
- Robust construction of X by claiming unitarity explicitly
- Answered by Sanda

Open questions

Influence of interpolation scheme ?

- global polynomial
- piecewise polynomial
- spline
- Answered by Sanda

How to scale different i/o's (e.g. pressures, voltages)

- under investigation

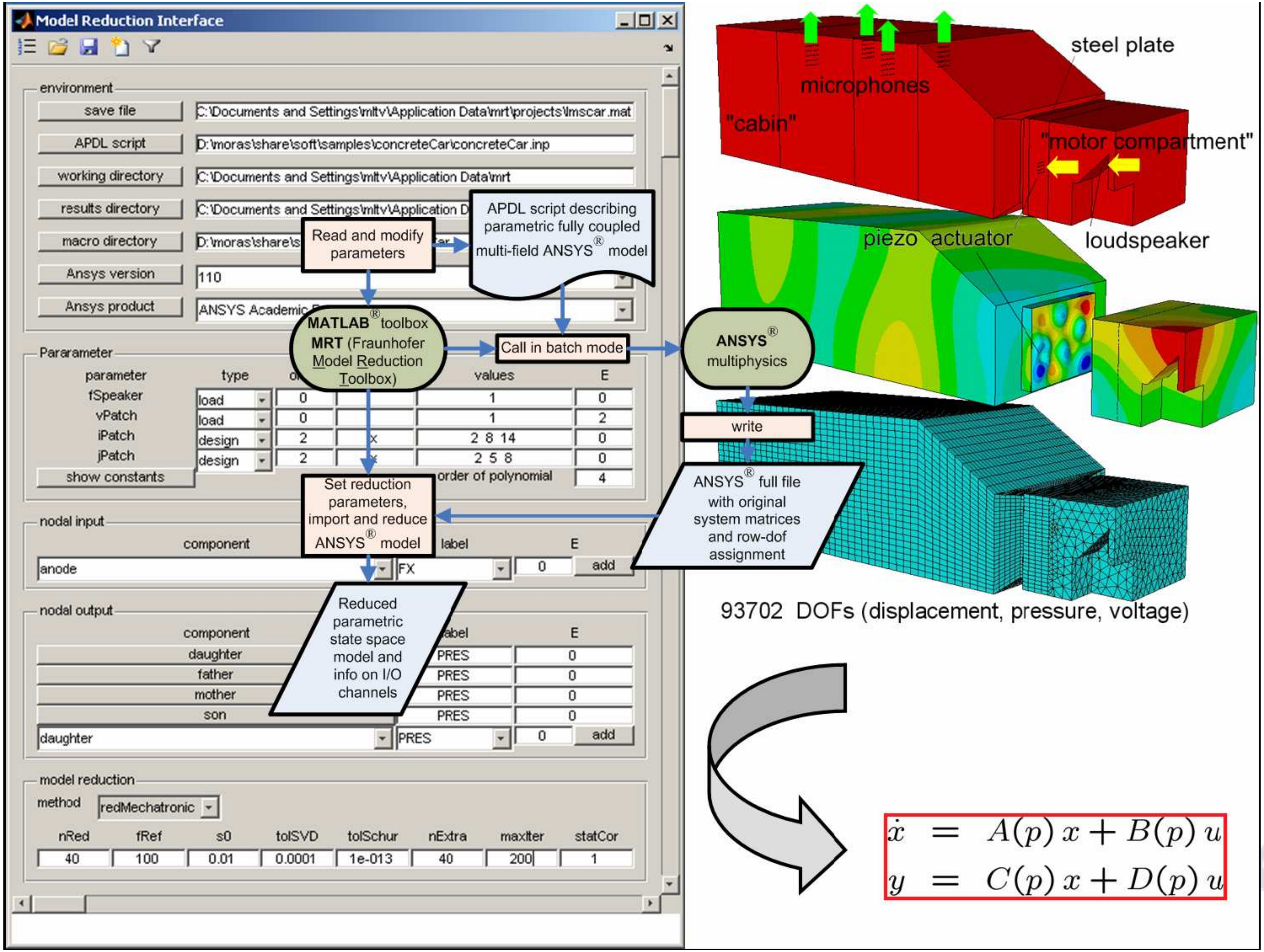
To be continued by Sanda

Introduction

It is demonstrated

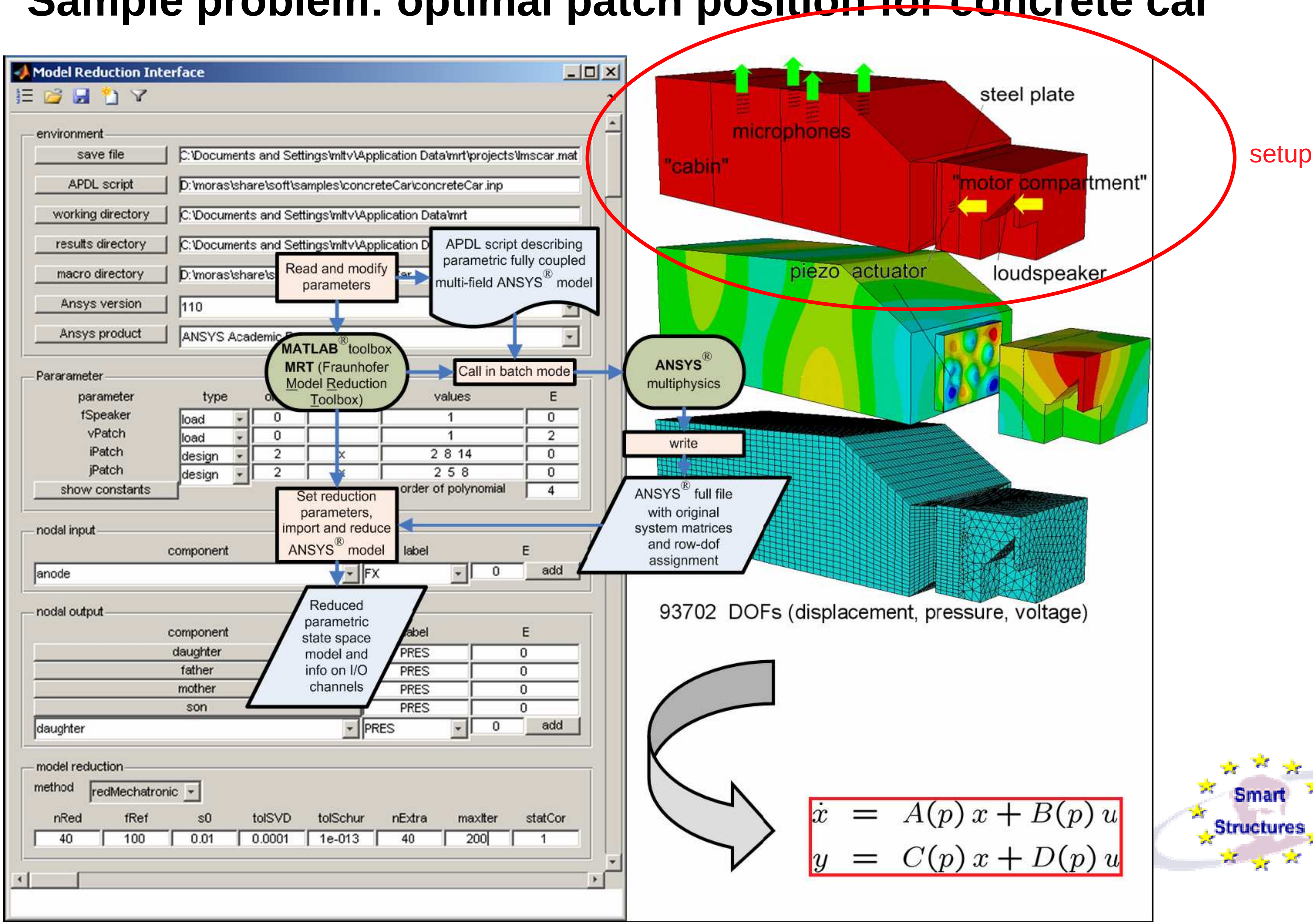
- How to optimize a smart structures application modeled by ANSYS®
  (e.g. optimal position of piezo patch on firewall)
- which requires many expensive evaluations (e.g. frequency responses)
- by parametric reduced models
- using our Model Reduction Toolbox for ANSYS® / Matlab®

Sample problem: optimal patch position for concrete car

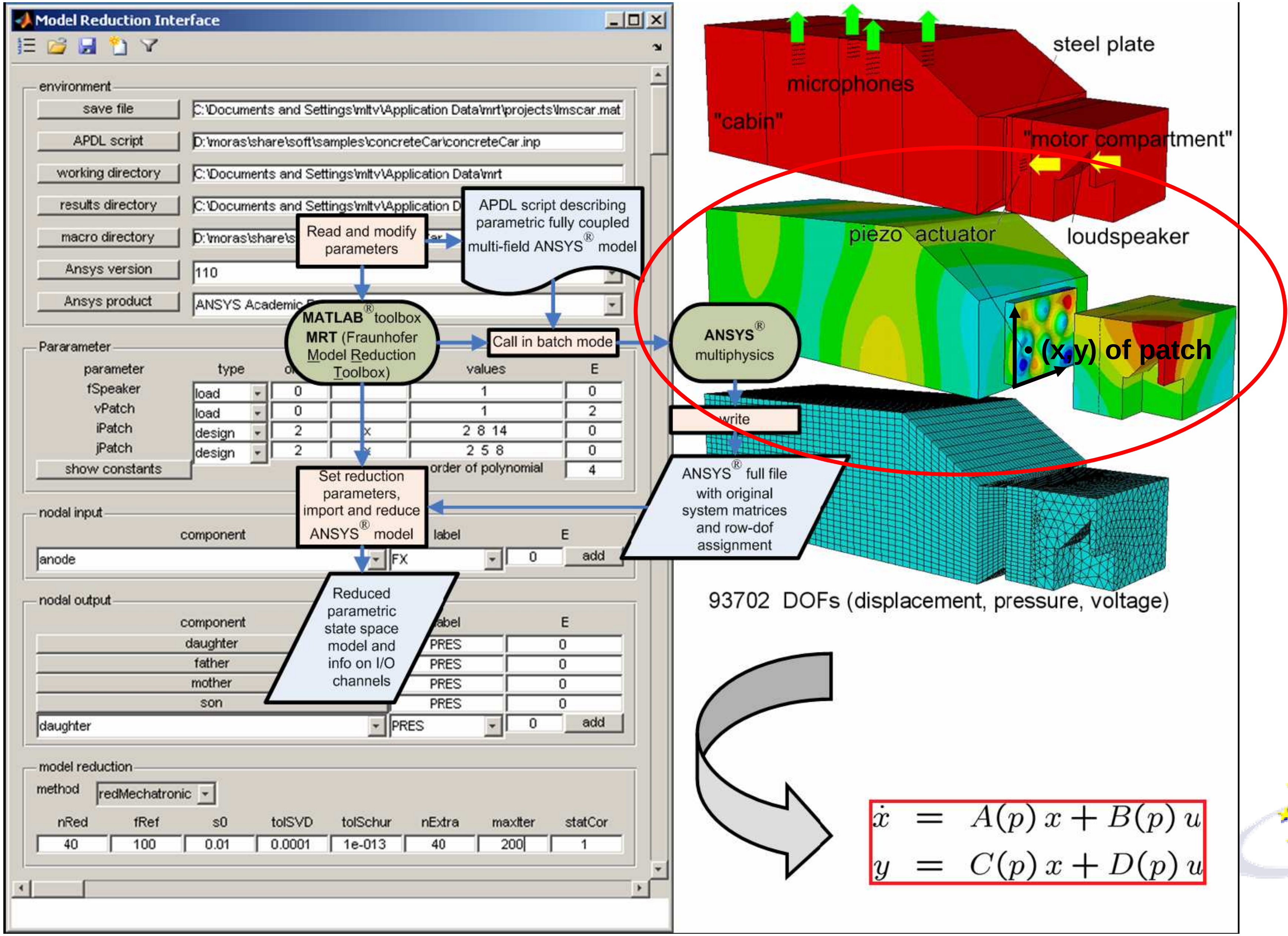


$$
\boxed{\begin{aligned} \dot{x} &= A(p)\,x + B(p)\,u \\ y &= C(p)\,x + D(p)\,u \end{aligned}}
$$

Sample problem: optimal patch position for concrete car



Sample problem: optimal patch position for concrete car



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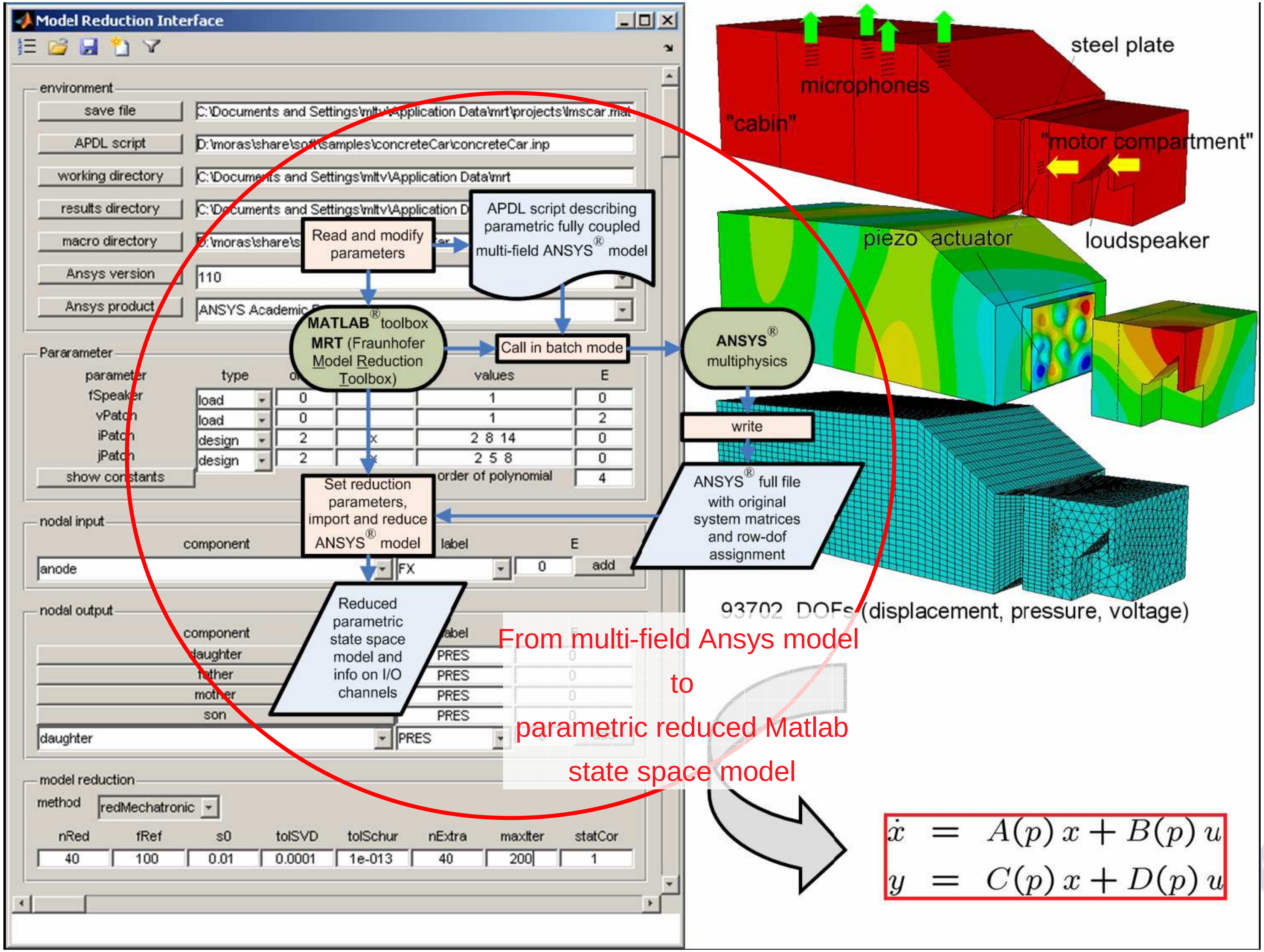
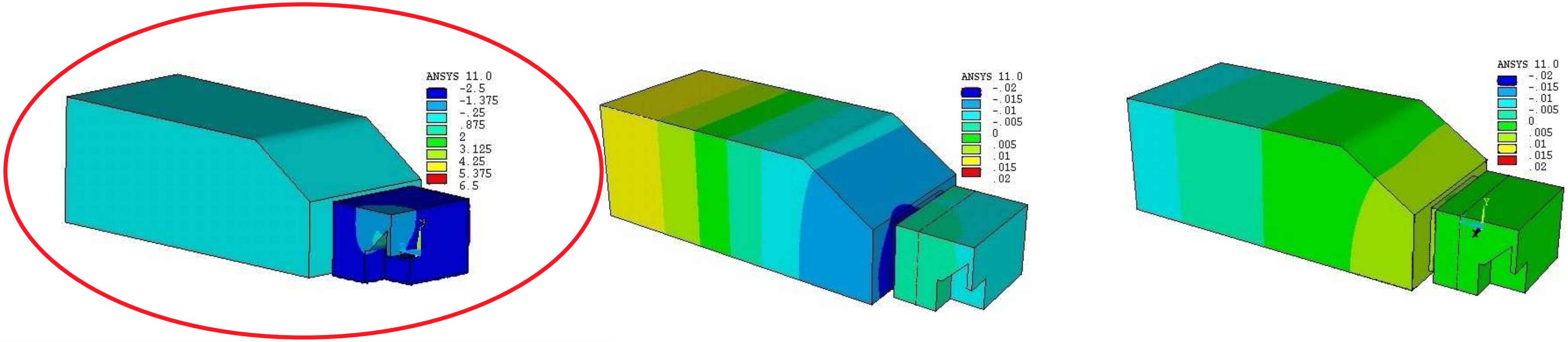
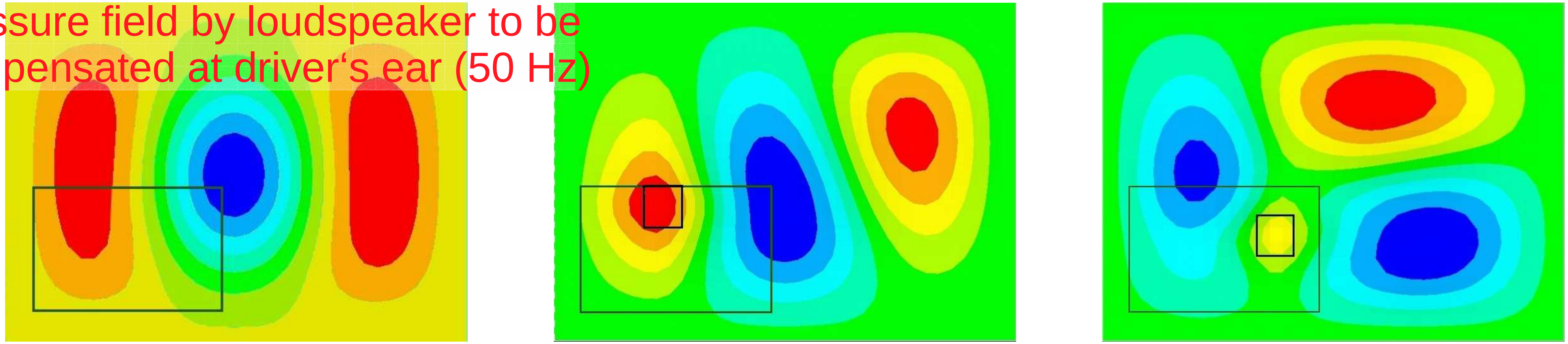


Influence of patch position on efficiency



Pressure field by loudspeaker to be compensated at driver's ear (50 Hz)



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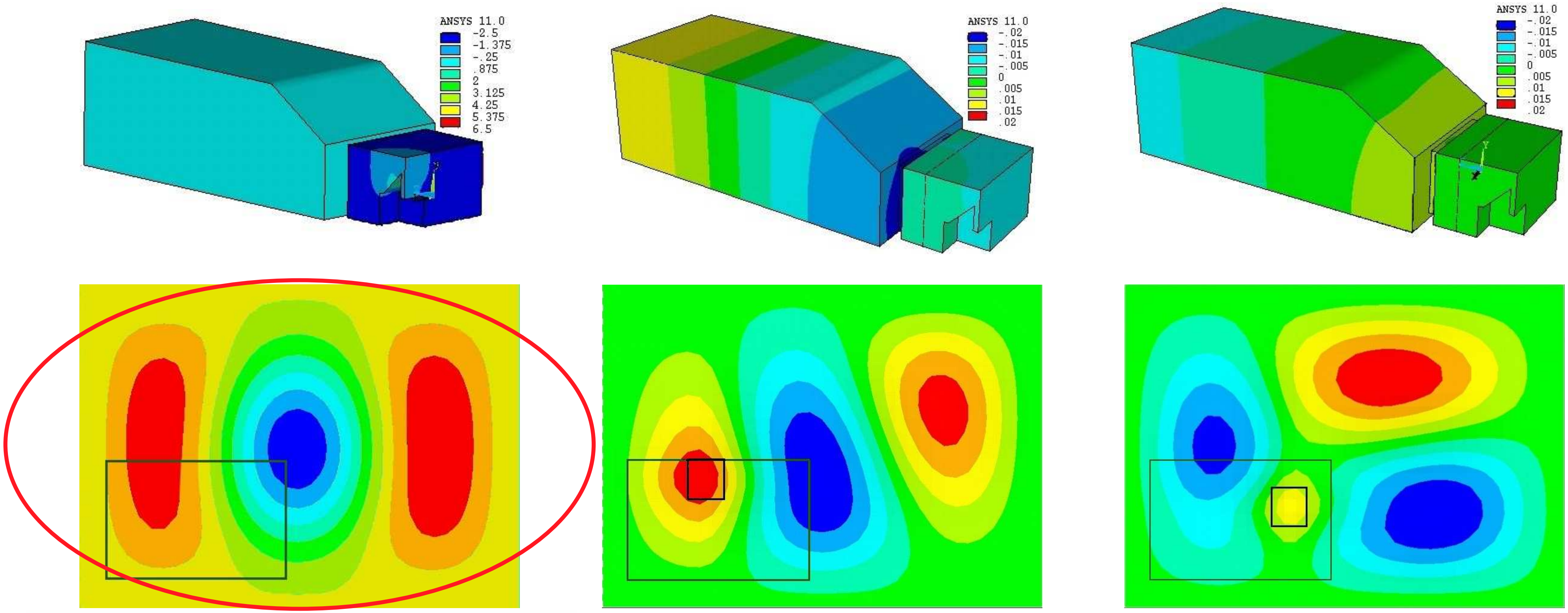
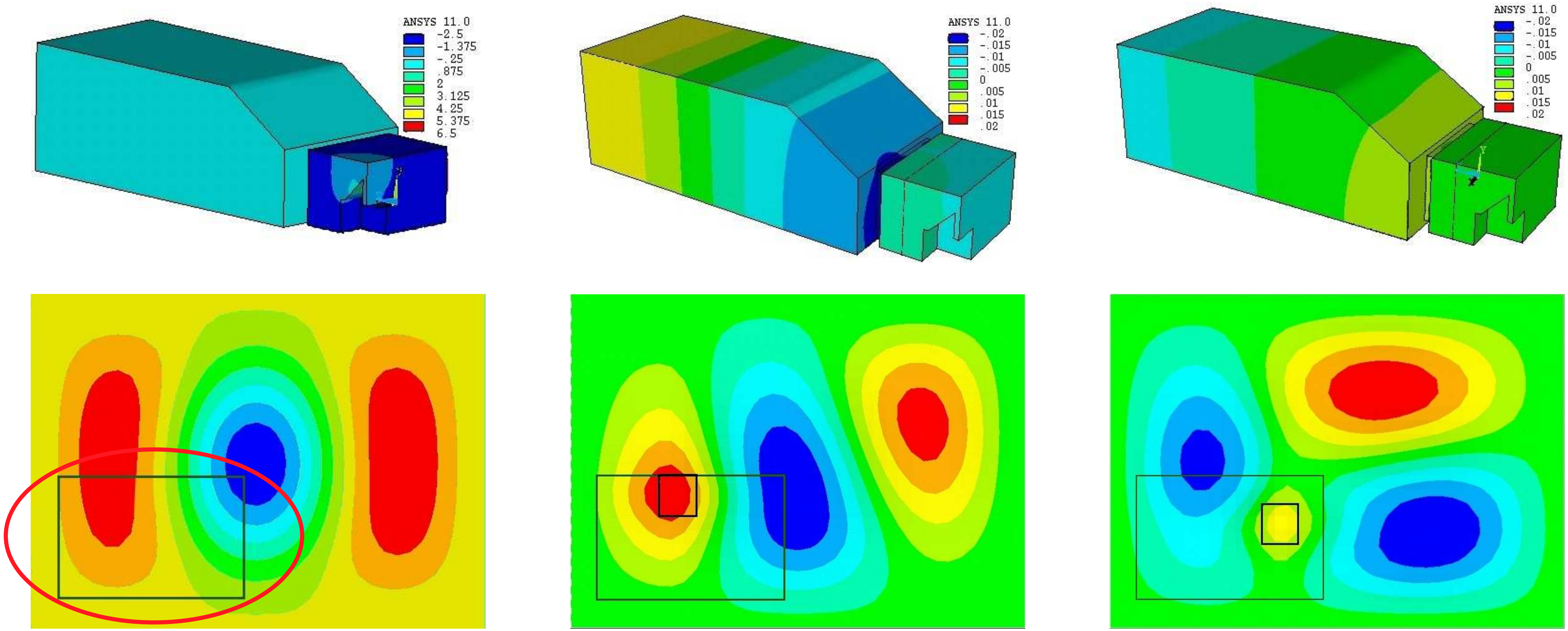
Influence of patch position on efficiency



Associated normal displacement of plate (firewall)

Influence of patch position on efficiency



Admissible patch positions

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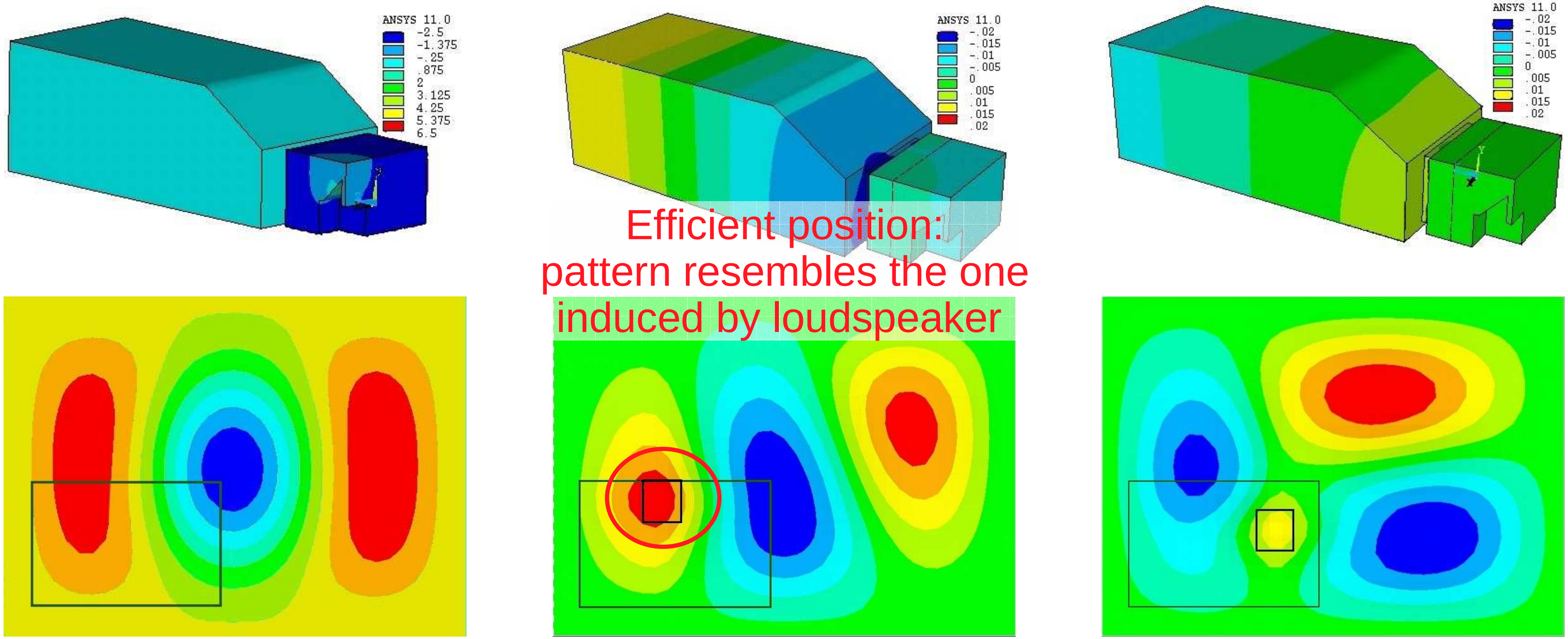
Influence of patch position on efficiency



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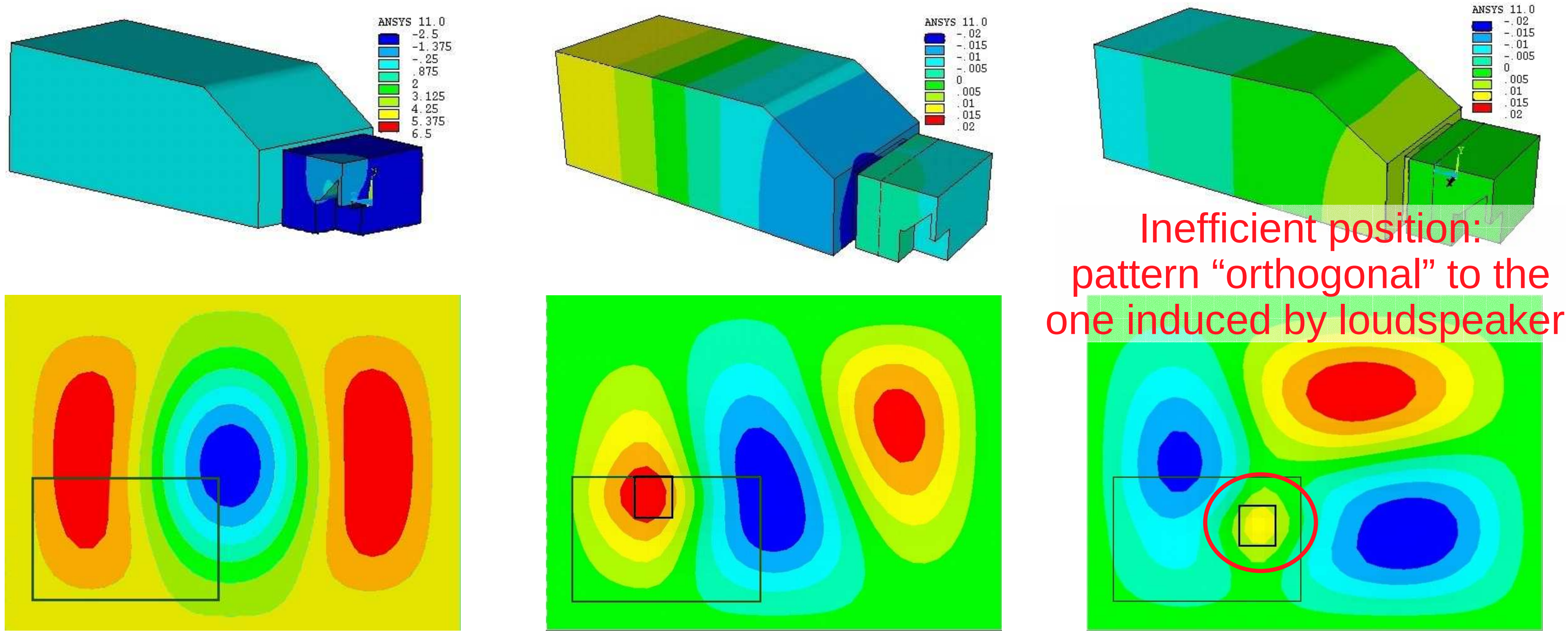
Influence of patch position on efficiency



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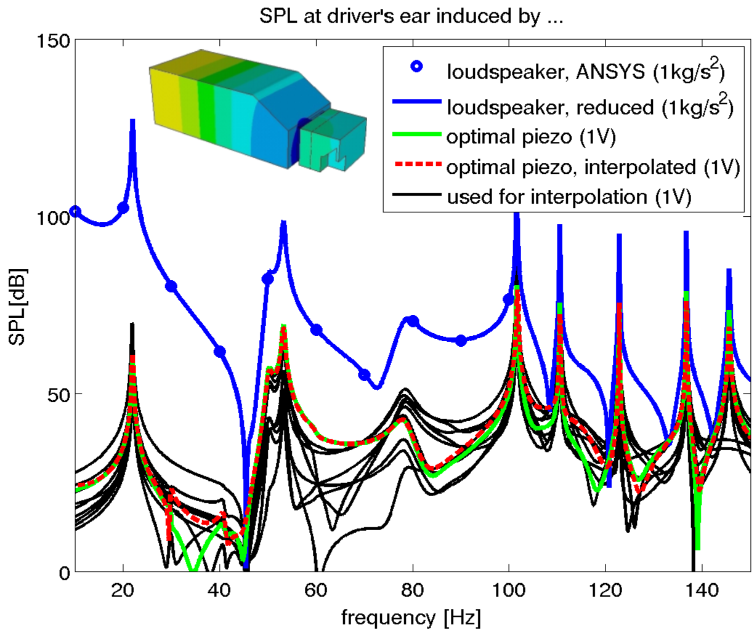
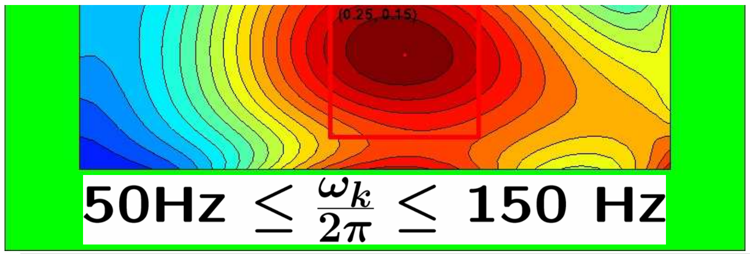
Need for parametric reduced model





- In general, patch position must be **uniformly effective** on a whole **frequency range**
- **Evaluating position** requires computing **frequency response** of transfer function **loudspeaker ➔ driver's ear**
- **ANSYS®** requires **160s** for single frequency (Xeon 2.5 GHz)

➔ **We need reduced model**

- **Many** different **patch positions** must be tried: several **local minima**, **Newton iteration** about each minimum

➔ **We need parametric model** which can be evaluated at new positions **without creating and reducing new ANSYS® models**

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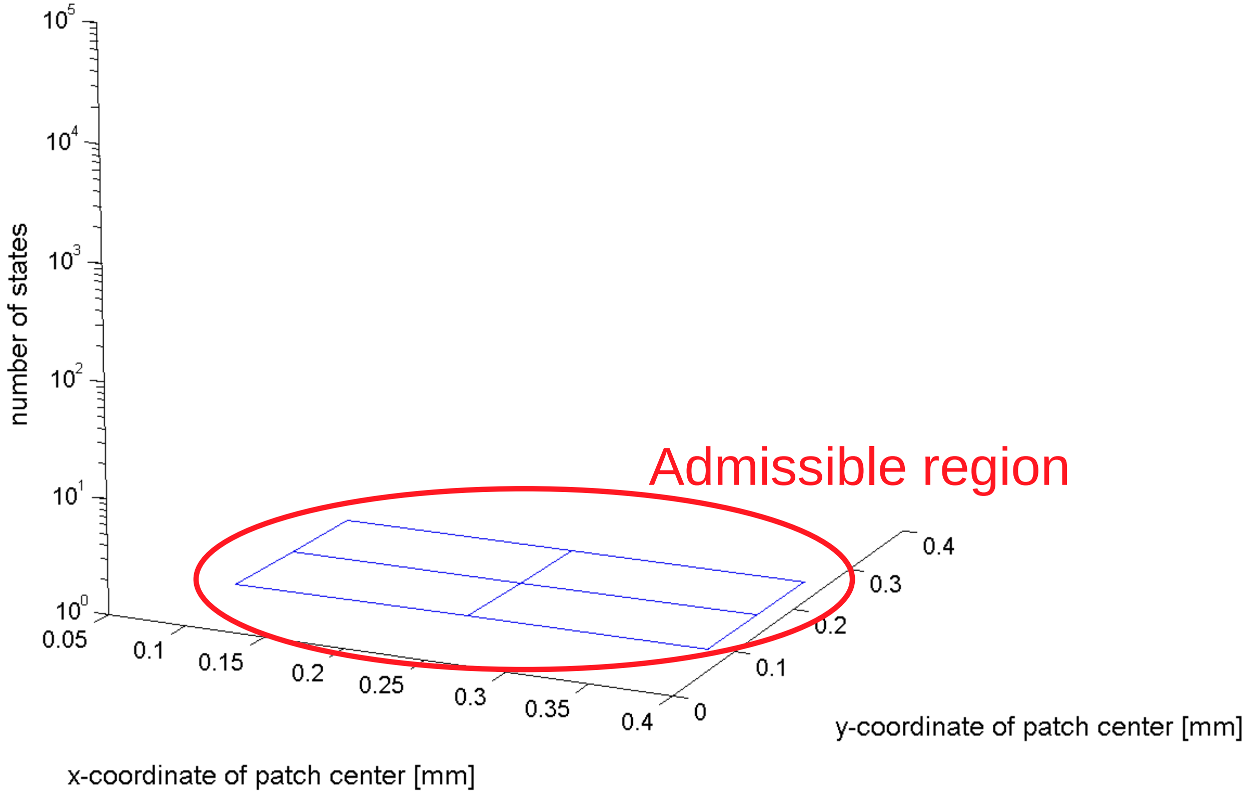
Parametric reduced model



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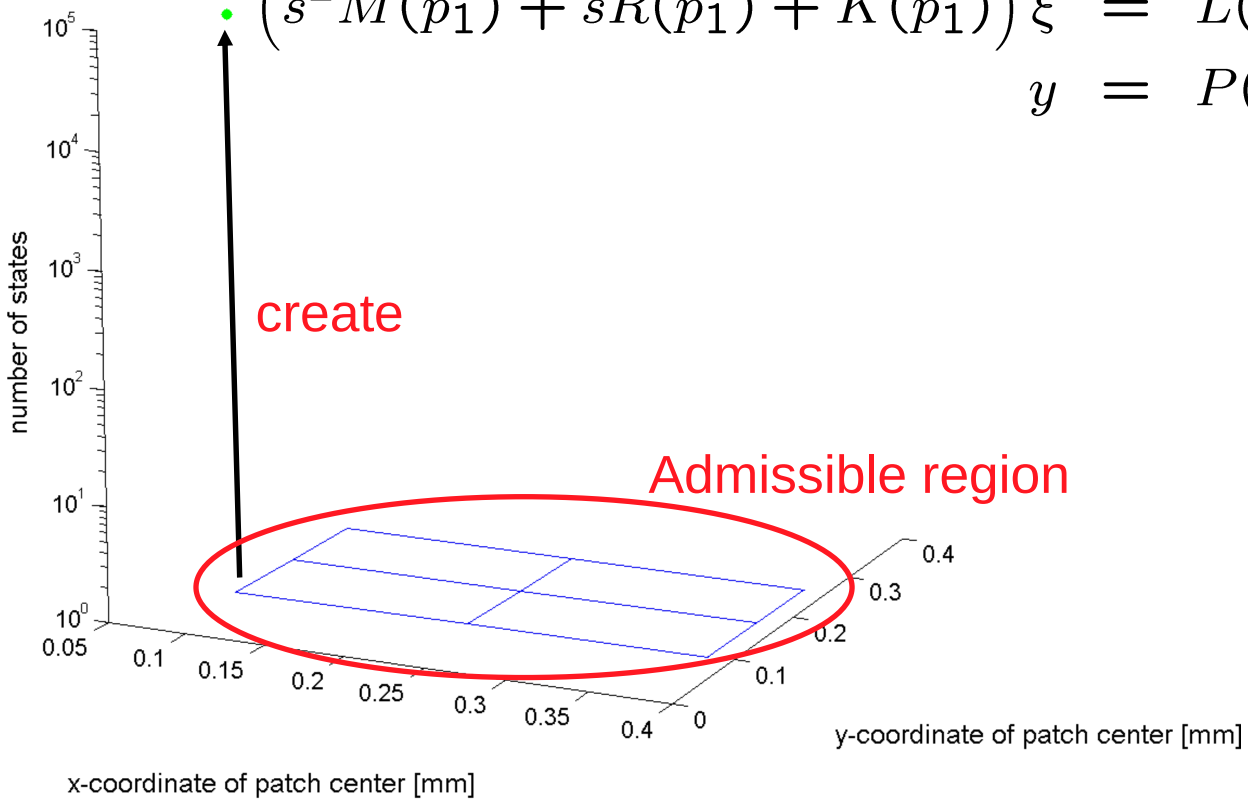
Parametric reduced model

Multi-field Ansys model for parameters $p_1 = (x_1, y_1)$

$$
\begin{aligned}
\left(s^2 M(p_1) + sR(p_1) + K(p_1)\right)\xi &= L(p_1)u \\
y &= P(p_1)\xi
\end{aligned}
$$



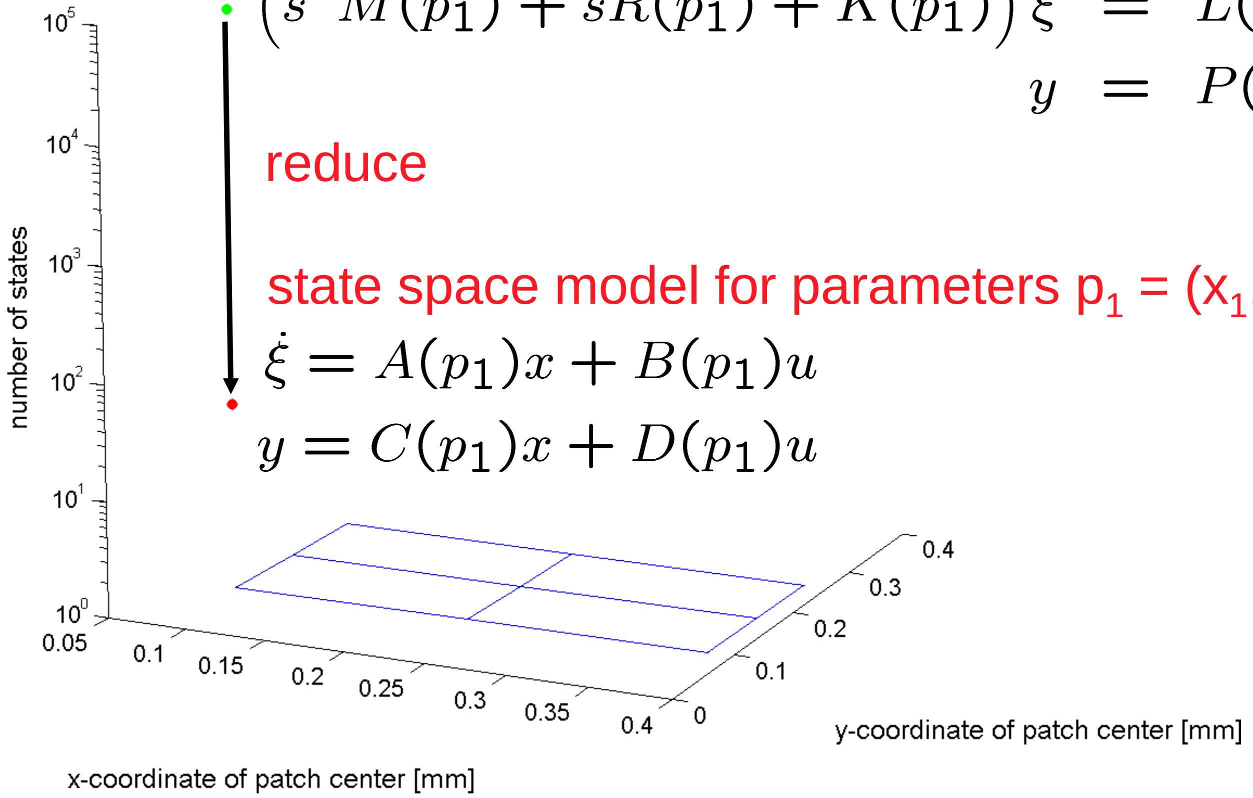
Parametric reduced model

$$\left(s^2 M(p_1) + sR(p_1) + K(p_1)\right)\xi = L(p_1)u$$
$$y = P(p_1)\xi$$

reduce

state space model for parameters $p_1 = (x_1, y_1)$

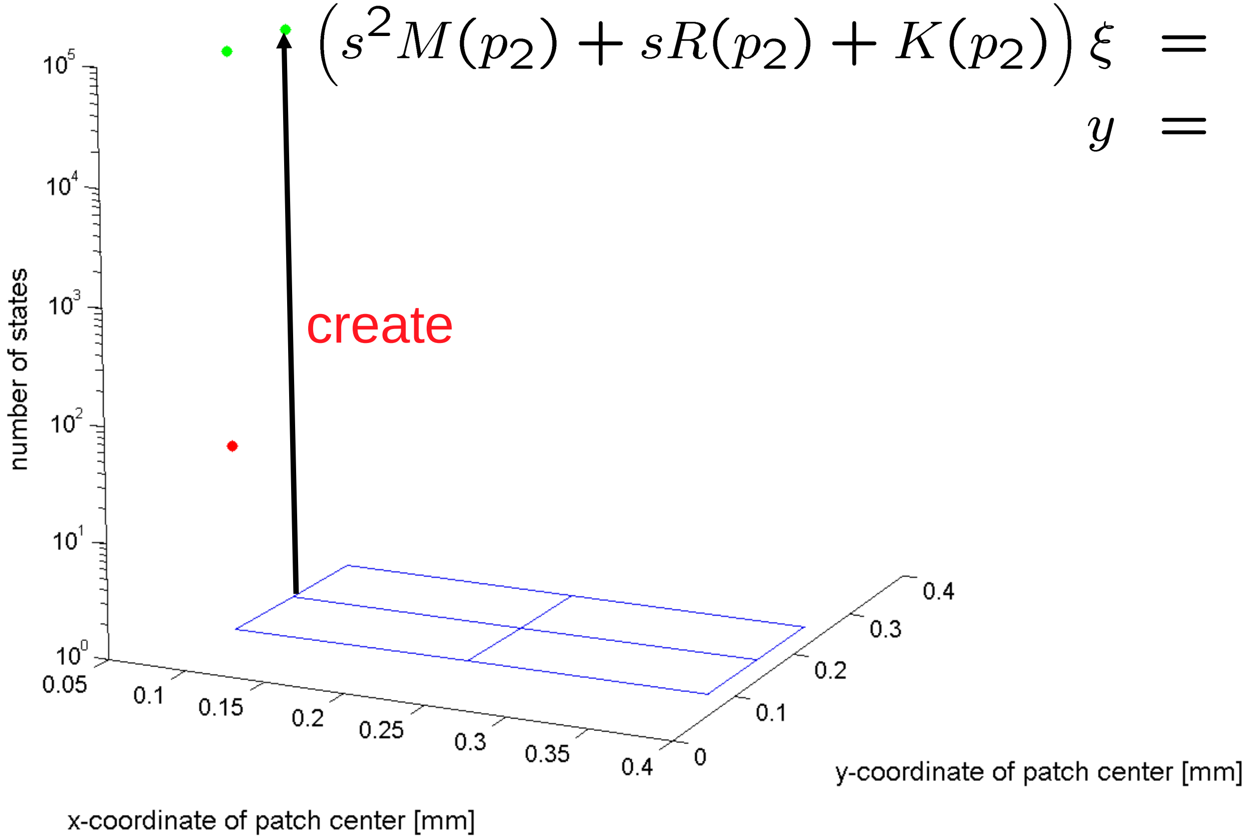
$$\dot{\xi} = A(p_1)x + B(p_1)u$$
$$y = C(p_1)x + D(p_1)u$$



Parametric reduced model

Multi-field Ansys model for parameters $p_2 = (x_2, y_2)$

$$\left(s^2 M(p_2) + sR(p_2) + K(p_2)\right)\xi = L(p_2)u$$
$$y = P(p_2)\xi$$



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Parametric reduced model

$$\left(s^2 M(p_2) + sR(p_2) + K(p_2)\right)\xi \;=\; L(p_2)u$$
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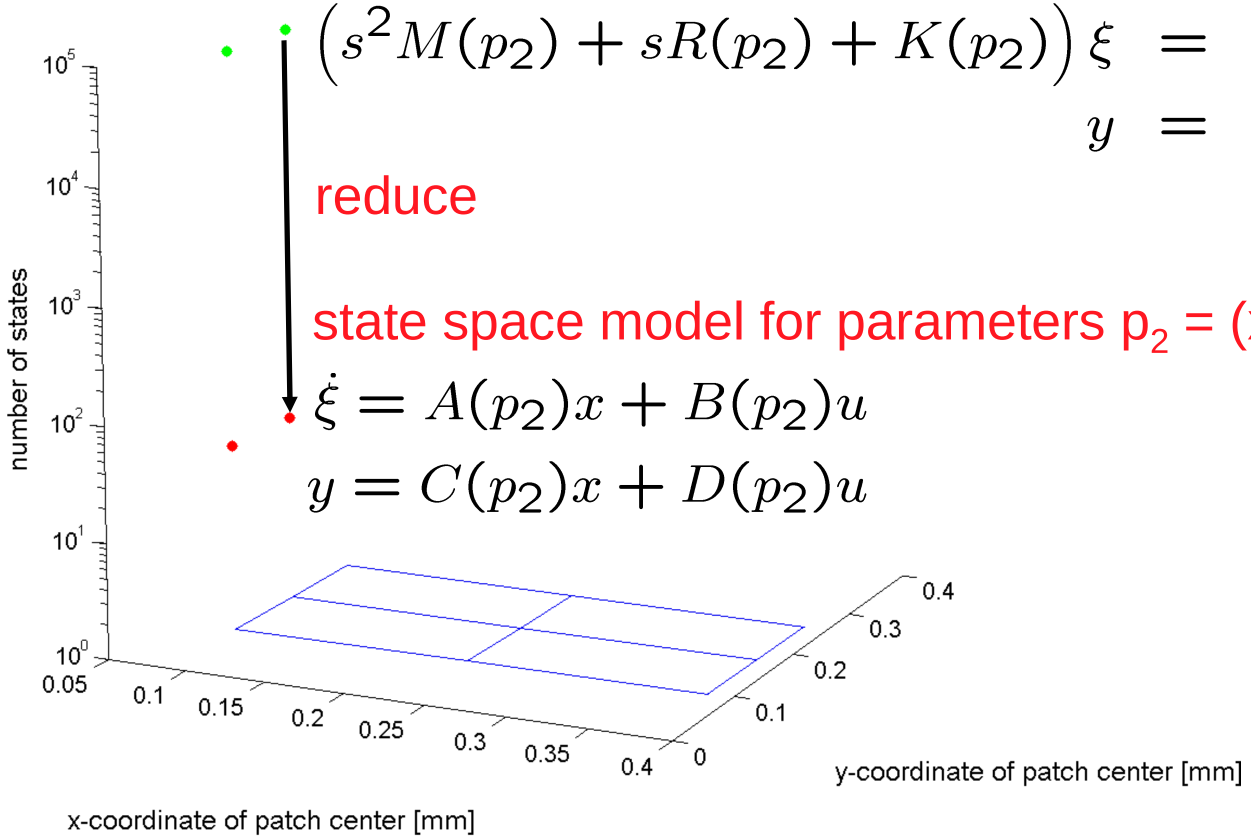
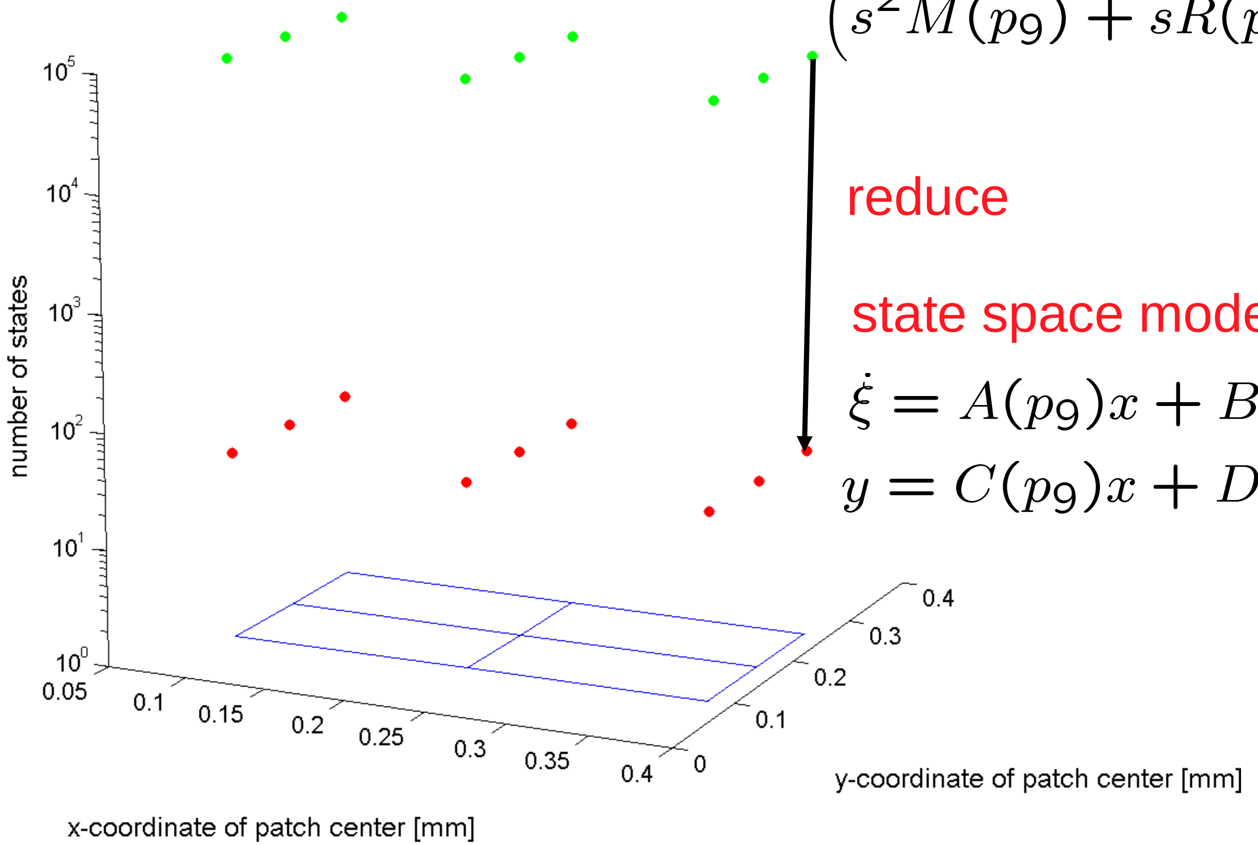


Parametric reduced model



Multi-field Ansys model for parameters $p_9 = (x_9,y_9)$

$$\begin{aligned}\left(s^2M(p_9)+sR(p_9)+K(p_9)\right)\xi &= L(p_9)u\\ y &= P(p_9)\xi\end{aligned}$$

reduce

state space model for parameters $p_9 = (x_9,y_9)$

$$\dot{\xi} = A(p_9)x + B(p_9)u$$
$$y = C(p_9)x + D(p_9)u$$

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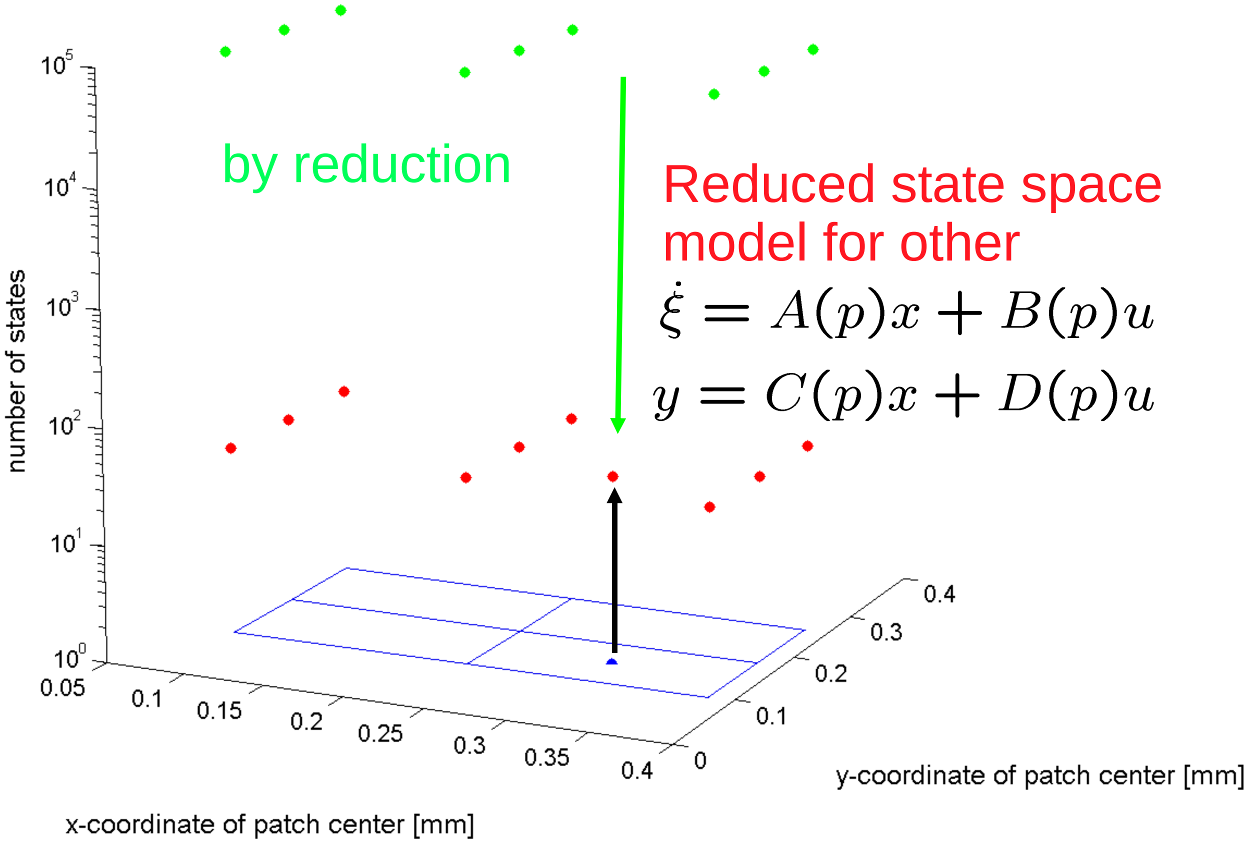
Parametric reduced model

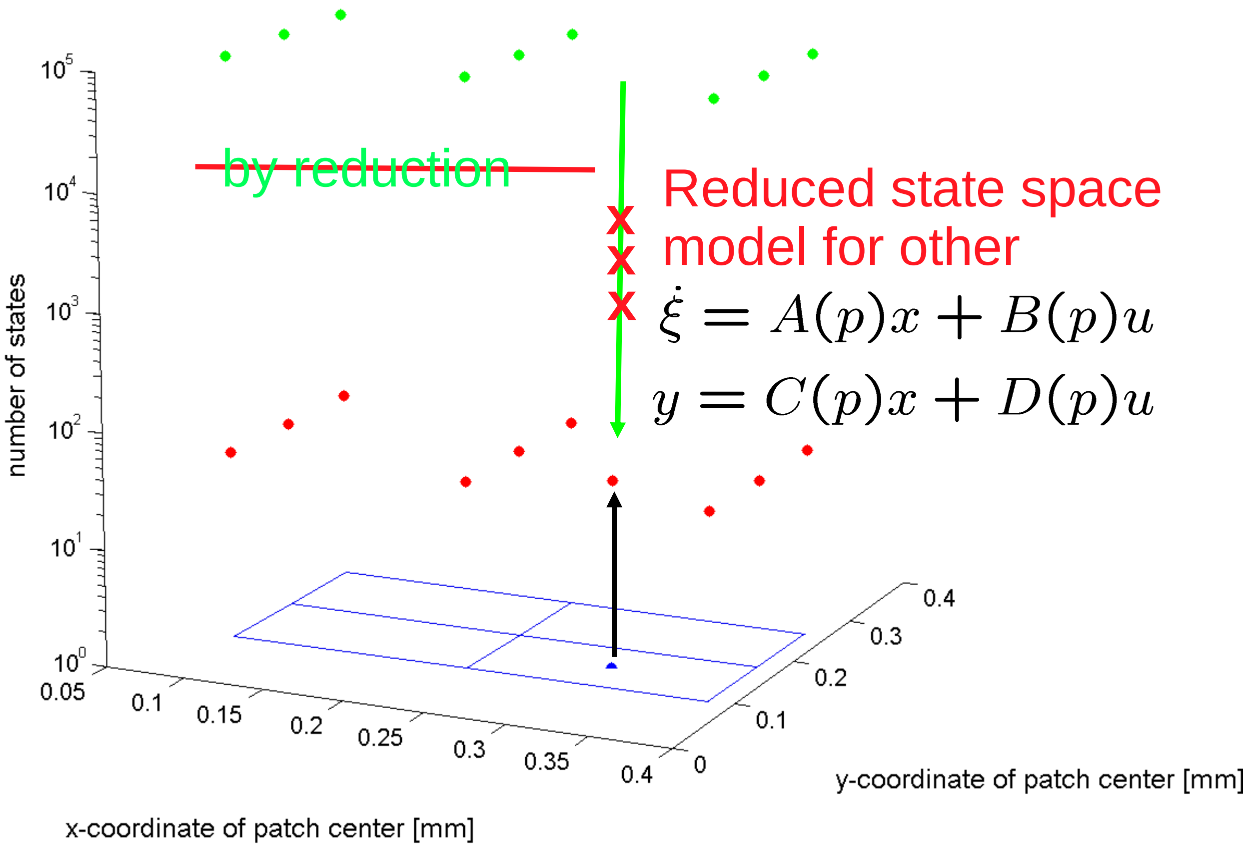


Reduced state space model for other

$$\dot{\xi} = A(p)x + B(p)u$$

$$y = C(p)x + D(p)u$$

Parametric reduced model



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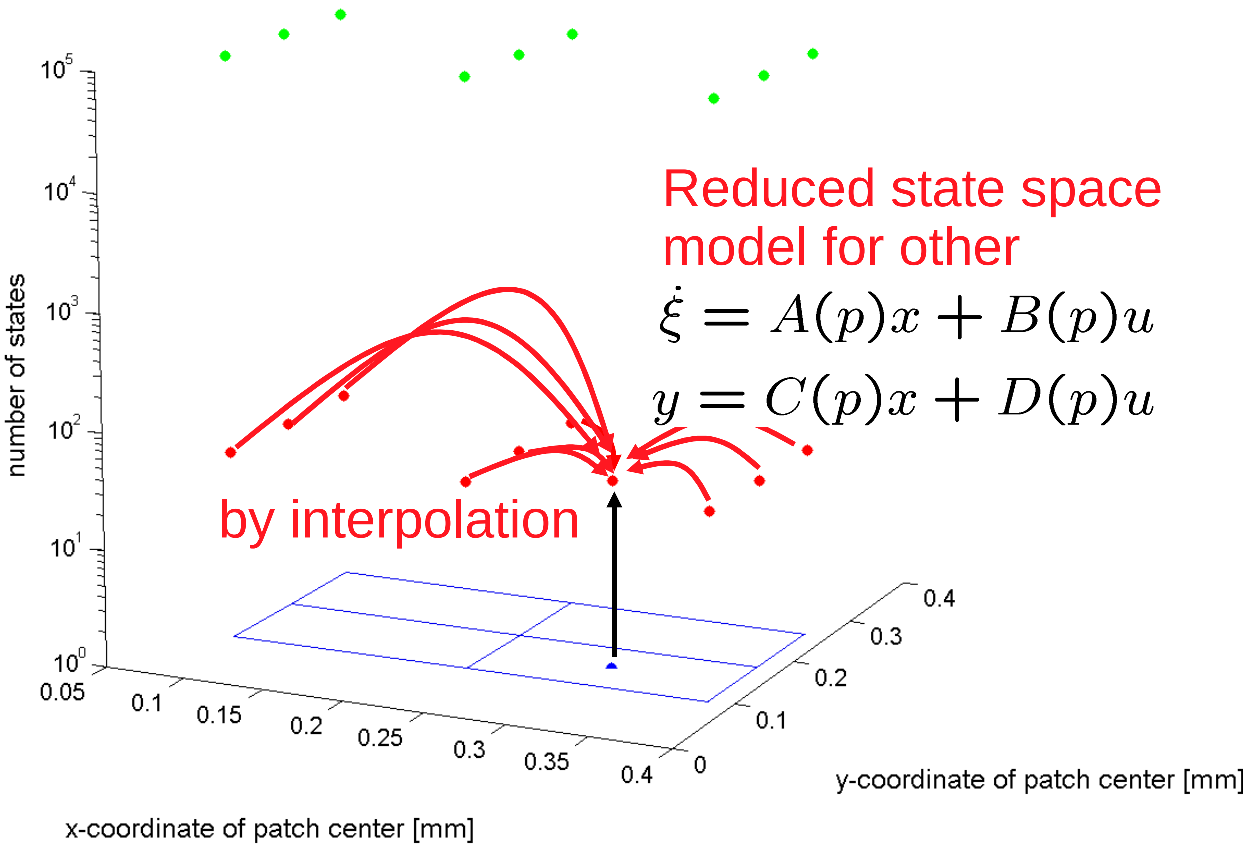
Parametric reduced model



Challenge 1: reduction of multi-field FE models

- Consider multi-field Ansys model of concrete car for fixed parameter set

$$\left(s^2 M + sR + K\right)\xi = Lu \qquad y = C\begin{bmatrix} \xi \\ s\xi \end{bmatrix}$$

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- Consider multi-field Ansys model of concrete car for fixed parameter set

$$\left(s^2 M + sR + K\right)\xi = Lu \qquad y = C\begin{bmatrix} \xi \\ s\xi \end{bmatrix}$$

- Built-in reduction schemes of Ansys and Matlab fail due to non-symmetric/singular matrices or non-Rayleigh damping and size (~100.000 DOFs), respectively.

$$M = \begin{bmatrix} M_{pp} & M_{pu} & 0 \\ 0 & M_{uu} & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad R = \begin{bmatrix} R_{pp} & 0 & 0 \\ 0 & R_{uu} & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad K = \begin{bmatrix} K_{pp} & 0 & 0 \\ K_{up} & K_{uu} & K_{uv} \\ 0 & K_{uv}^{t} & K_{vv} \end{bmatrix},$$

singular

non_Rayleigh damping

non-symmetric

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Challenge 1: reduction of multi-field FE models

Solution

- Turn 2nd order system into 1st order system

$$\begin{aligned} sEx &= Ax + Bu \\ y &= Cx + Du \end{aligned} \qquad x = \begin{bmatrix} \xi \\ s\xi \end{bmatrix} \qquad E = \begin{bmatrix} \mathrm{I} & 0 \\ 0 & M \end{bmatrix} \qquad A = \begin{bmatrix} 0 & \mathrm{I} \\ -K & -R \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ L \end{bmatrix}$$

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- Turn to shift and invert form (largest eigenvalues correspond to those of original system close to shift s_0). With $T = A - s_0 E$:

$$\Delta s\, Gx = x + Fu \quad y = Cx \quad \text{with} \quad \Delta s = s - s_0 \quad G = T^{-1}E \quad F = T^{-1}B$$

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- Project on *nxk* matrices *V,W* with *W*V=I, k<<n from modal analysis or Arnoldi process*

$$s\, G'x = \left(\mathrm{I} + s_0\, G'\right) x + F'u \quad y = C'x \quad \text{with} \quad G' = W^* G V \quad F' = W^* F \quad C' = CV$$

Challenge 1: reduction of multi-field FE models

Contribution by fellows

- **Sanda:** structure preserving reduction: Reduced model is again of 2nd order.
- **Julian:** hybrid reduction scheme for systems with many inputs and outputs combining modal truncation and moment matching.

Challenge 1: reduction of multi-field FE models

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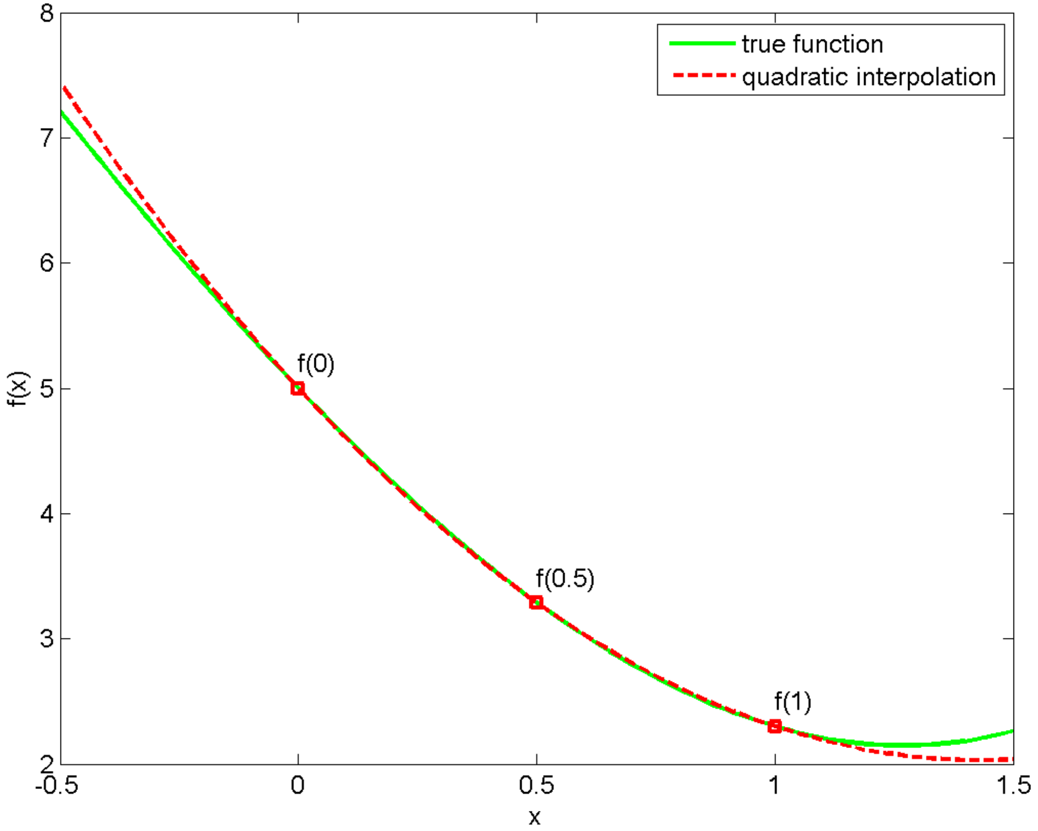
  J. Stoev and J. Mohring. Hybrid reduction of non-symmetric mimo systems. Submitted to Systems & Control Letters (Status: second revision)

Challenge 2: interpolation of reduced state space models

Polynomial interpolation

Polynomial interpolation of scalar function (example: 1 parameter *x*, 2nd order)

$$f(x) \approx f(0) + [-3f(0) + 4f(0.5) - f(1)]\, x + [2f(0) - 4f(0.5) + 2f(0)]\, x^2$$



Challenge 2: interpolation of reduced state space models

Polynomial interpolation

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System matrices may be interpolated in exactly the same way:

$$\begin{aligned} A(x) &\approx A(0)+[-3A(0)+4A(0.5)-A(1)]\,x+[2A(0)-4A(0.5)+2A(0)]\,x^2 \\ &\vdots \\ D(x) &\approx D(0)+[-3D(0)+D(0.5)-D(1)]\,x+[2D(0)-4D(0.5)+2D(0)]\,x^2 \end{aligned}$$

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Challenge 2: interpolation of reduced state space models

How to choose realizations?

- Realizations $\Sigma = (A, B, C, D)$and $\Sigma' = (T^{-1}AT, T^{-1}B, CT, D)$

  have same transfer function $H(s) = C\,(sI - A)^{-1}\,B + D$

  i.e. they describe same system.

- Interpolating badly chosen realizations may completely fail:

  Consider one-parametric family of single state SISO systems

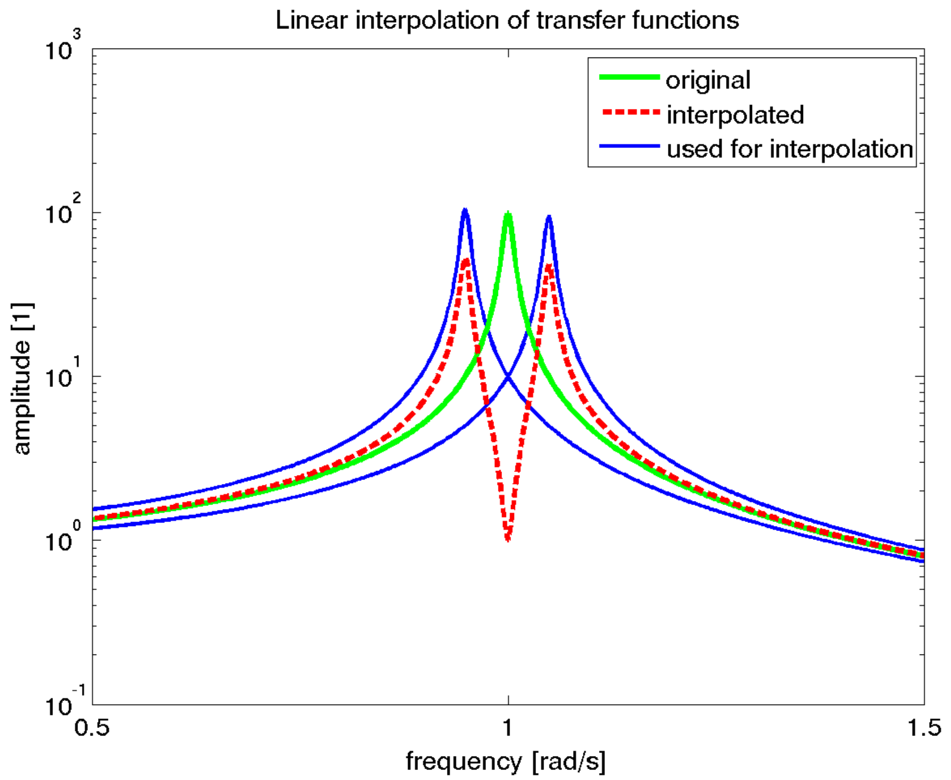
  $\Sigma(+1) = (-1, 1, 1, 0) \qquad \Sigma(-1) = (-1, -1, -1, 0)$

  Realizations of same system (T=-1), but linear interpolation gives $\Sigma(0) = (-1, 0, 0, 0)$

Challenge 2: interpolation of reduced state space models

Looking for way out

- Interpolate (unique) transfer function



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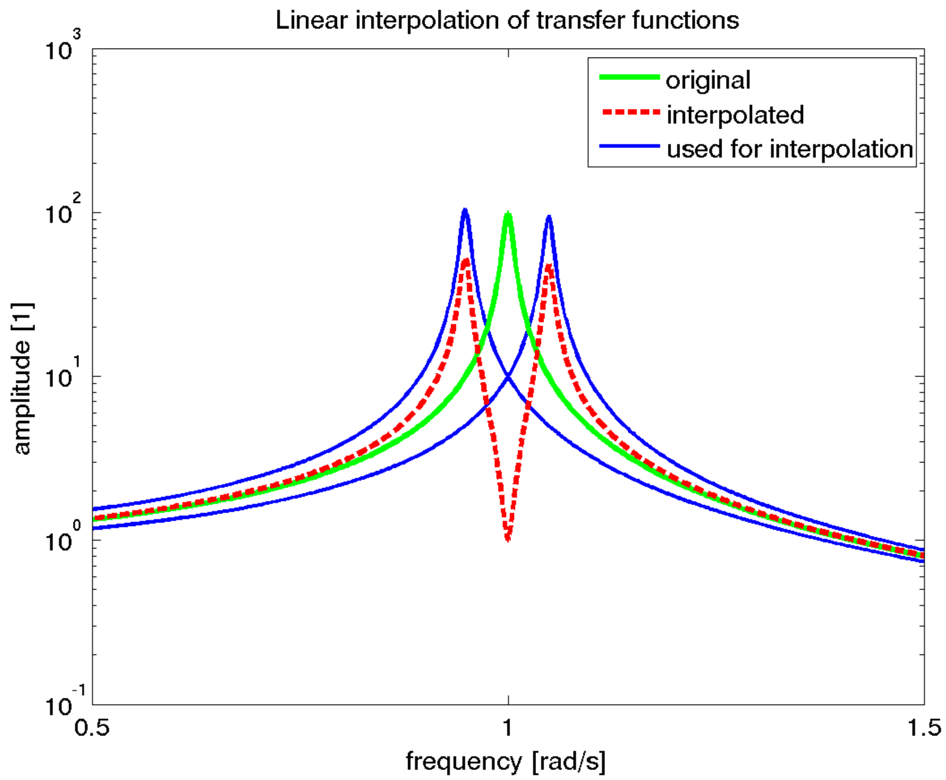
Challenge 2: interpolation of reduced state space models

Looking for way out

• ~~Interpolate (unique) transfer function~~



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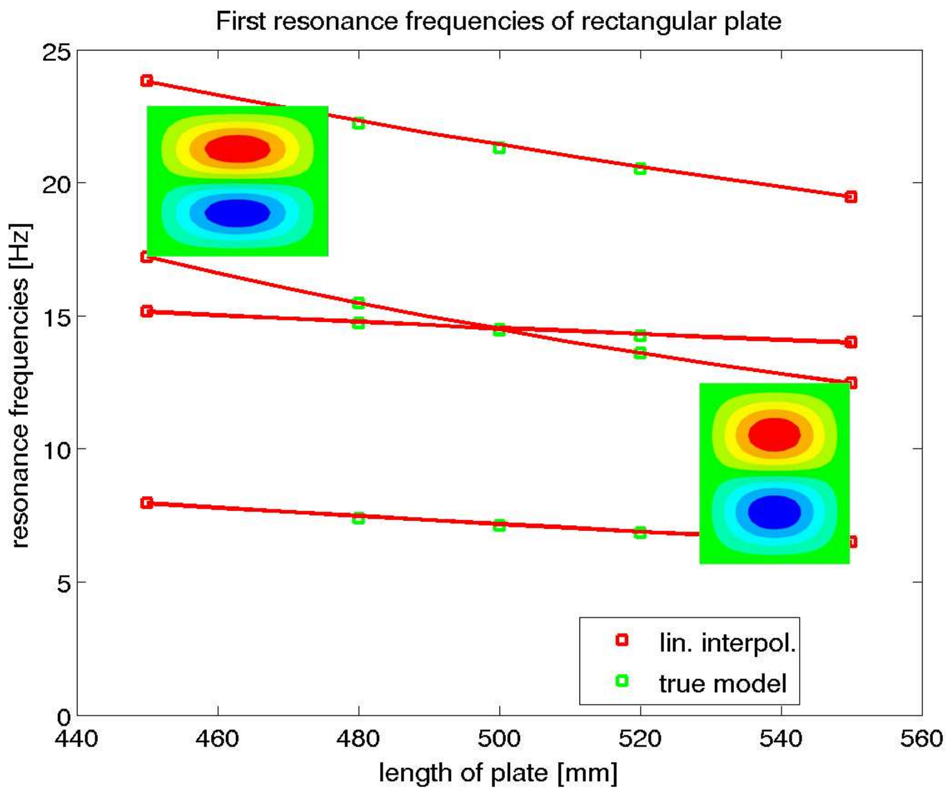

- Consider normal forms based on sorted eigenvalues:

$$A = \begin{bmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{bmatrix}$$

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Challenge 2: interpolation of reduced state space models

Looking for way out

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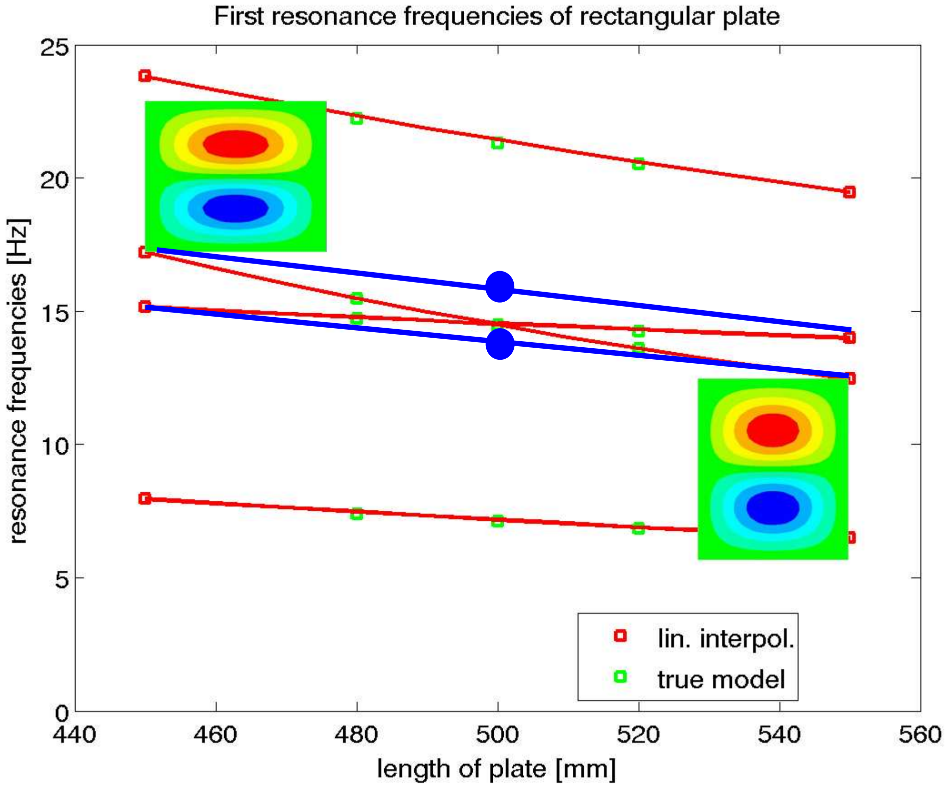

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Challenge 2: interpolation of reduced state space models

Looking for way out

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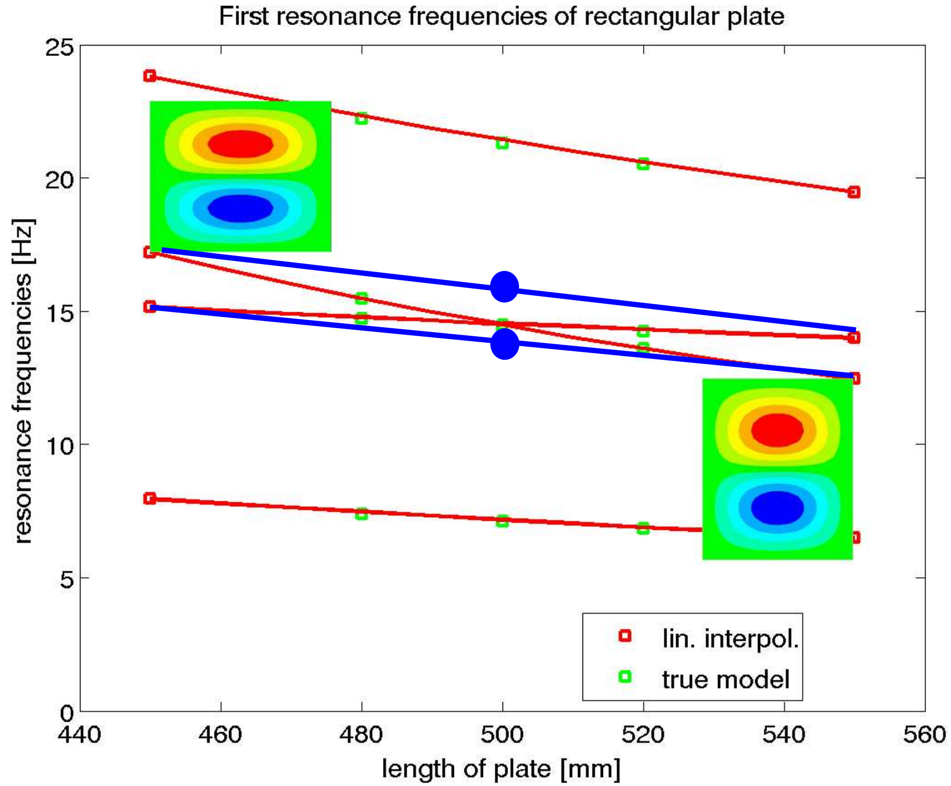

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Challenge 2: interpolation of reduced state space models

New approach

- Select arbitrary realization $\Sigma(p) = (A, B, C, D)$ for first parameter as reference
- Choose realizations $\Sigma(p') = (A', B', C', D')$ for other parameters as close as possible to reference

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- Formally: find transform *T* minimizing

$$J(T) = \alpha \left\| TA - A'T \right\|^2 + \beta \left\| TB - B' \right\|^2 + \gamma \left\| C - C'T \right\|^2$$

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$$J(T) = \alpha \left\| TA - A'T \right\|^2 + \beta \left\| TB - B' \right\|^2 + \gamma \left\| C - C'T \right\|^2$$

leading to matrix equation for *T*

$$\left(\alpha A'^* A' + \gamma C'^* C'\right) T + T \left(\alpha A A^* + \beta B B^*\right) - \alpha \left(A' T A^* + A'^* T A\right) = \beta B' B^* + \gamma C' C$$

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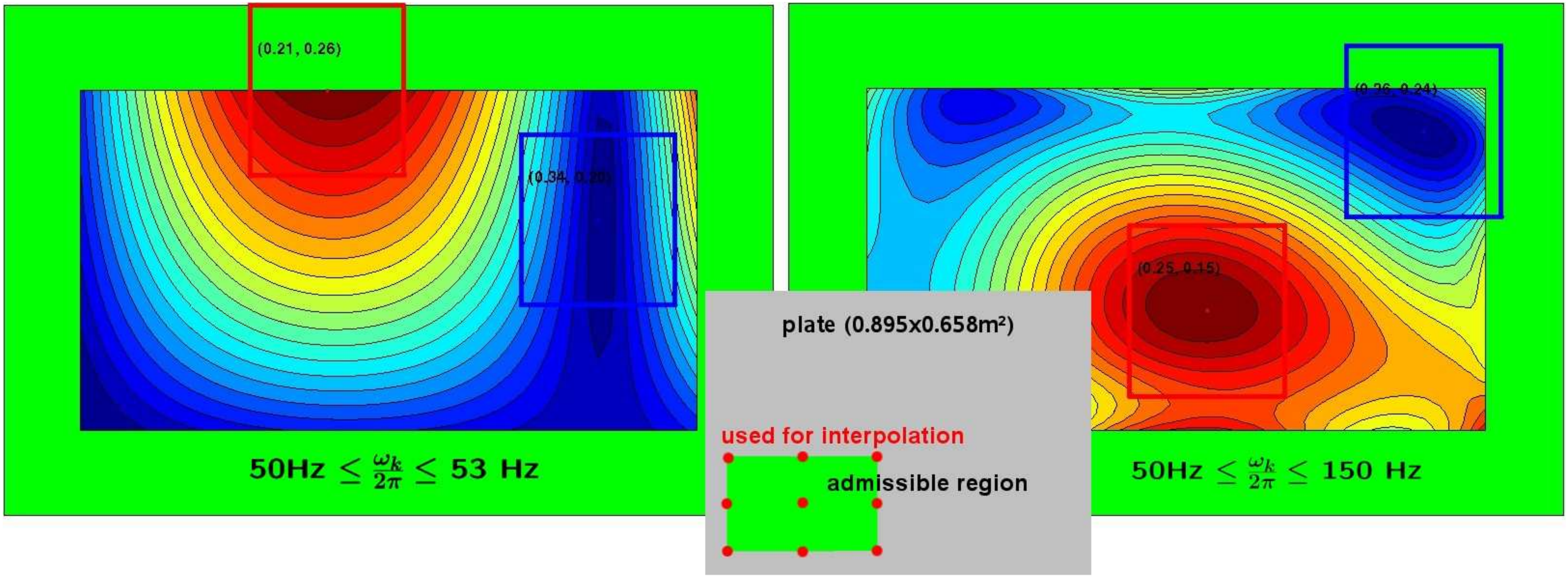
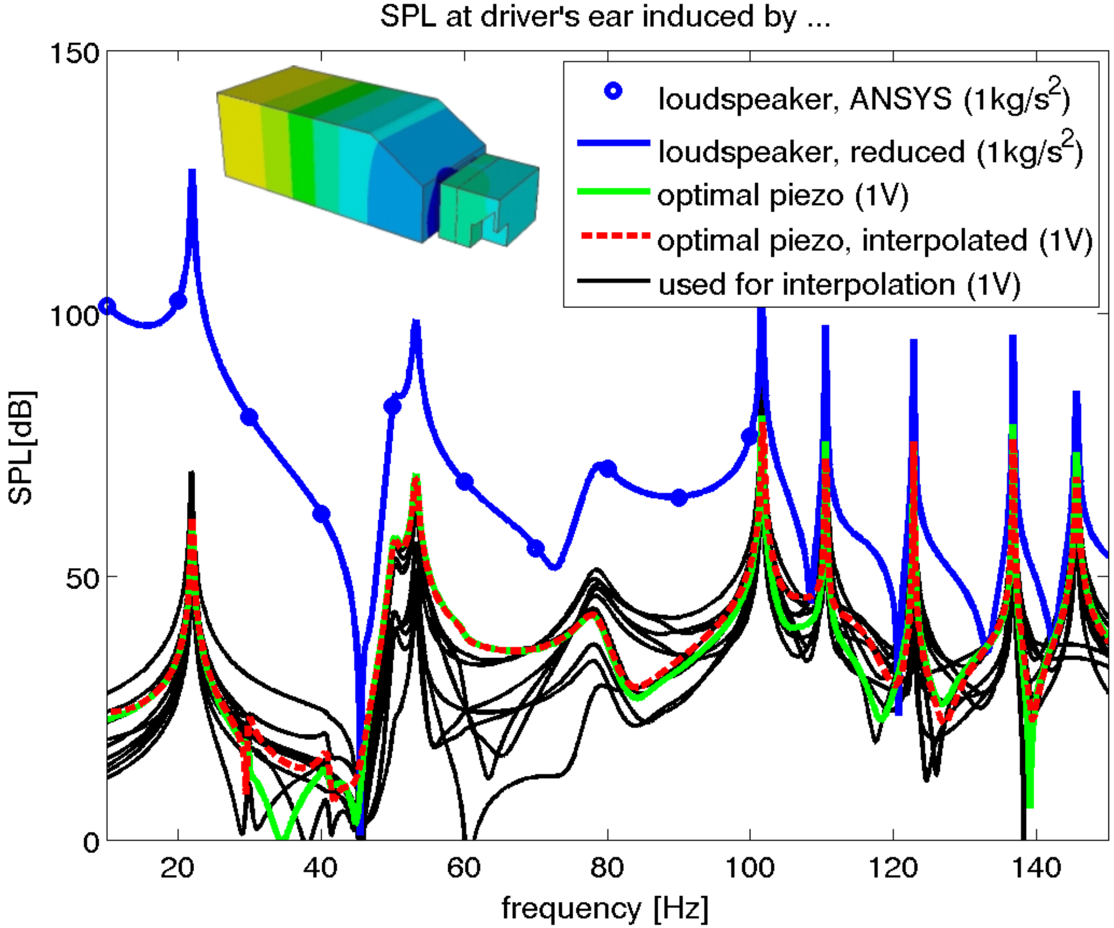
Optimal patch positions for different frequency ranges



- **ANSYS®** model is created and reduced at **9 positions**
- Reduced models are interpolated **bilinearly**
- **Efficiency function** $\varepsilon = \left(\sum_{\omega_{\min} \le \omega_k \le \omega_{\max}} \left| h_{piezo}(i\omega_k) / h_{speaker}(i\omega_k) \right|^2 \right)^{1/2}$ is evalutaed at 50x50=**2500** positions, which takes **100s** altogether

Optimal patch positions for different frequency ranges



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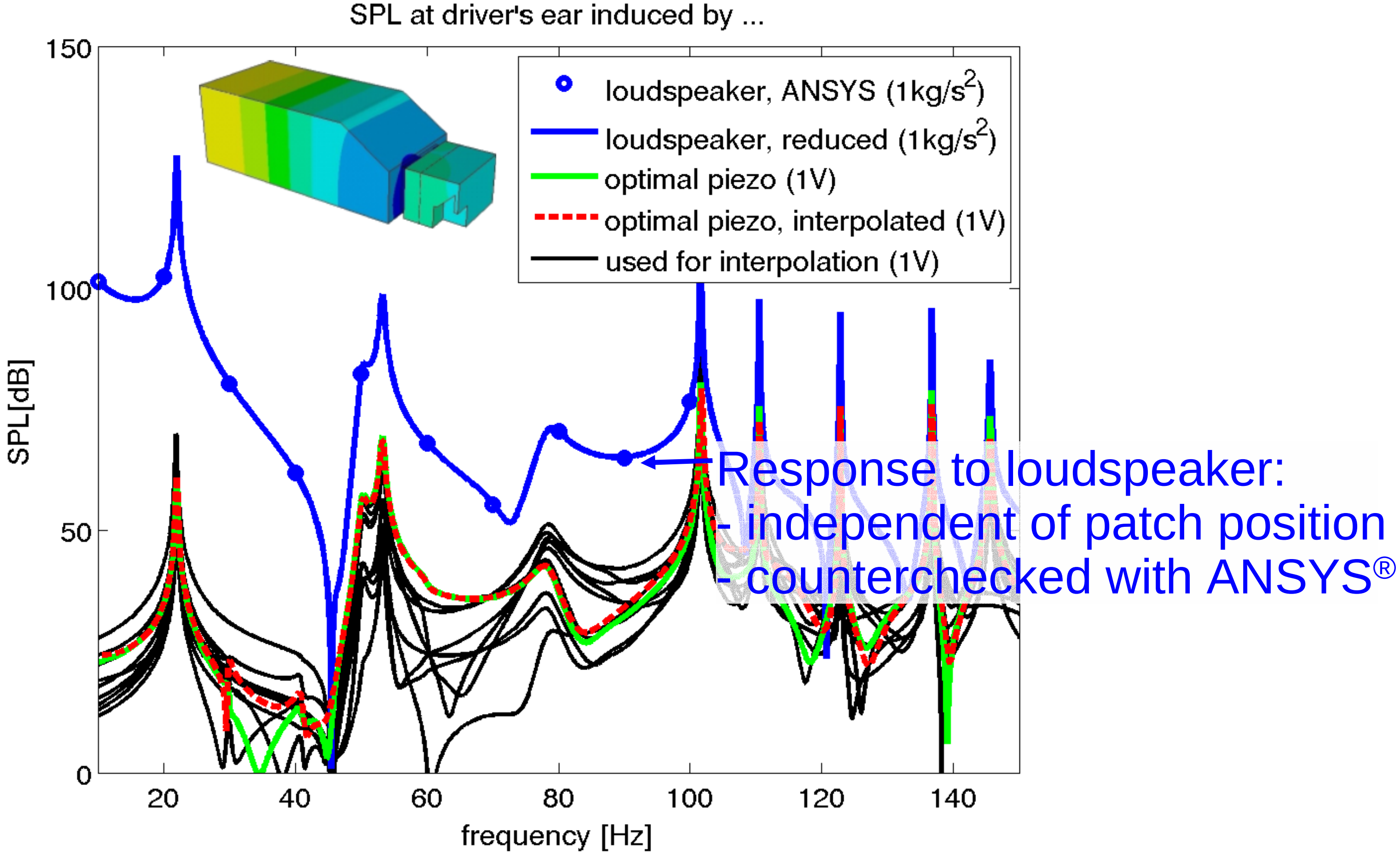
Optimal patch positions for different frequency ranges



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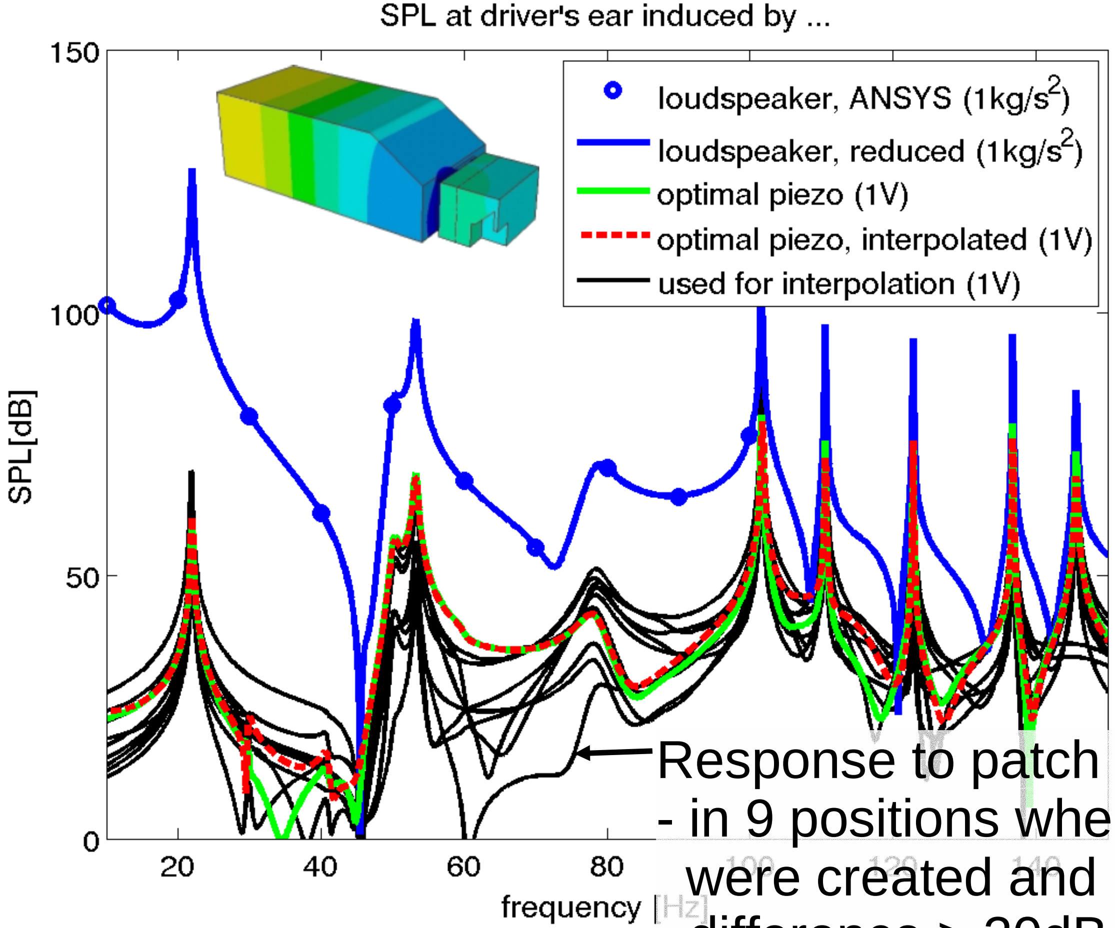
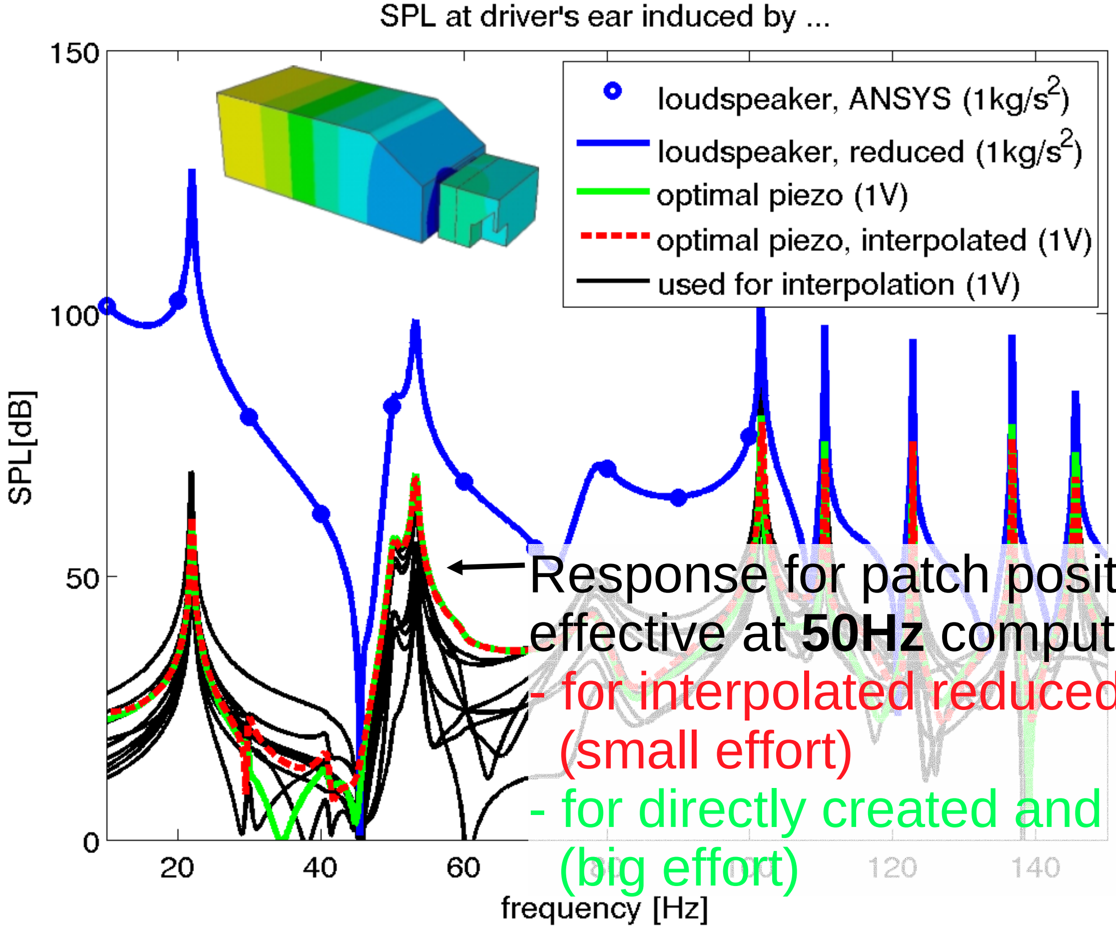


Response to patch
- in 9 positions where ANSYS® models were created and reduced
- difference > 20dB

Optimal patch positions for different frequency ranges



Response for patch position optimally effective at **50Hz** computed
- for interpolated reduced model (small effort)
- for directly created and reduced model (big effort)

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instead of Conclusion:
Request for collaboration

Are there fellows interested in optimizing their smart structures design with MRT, possibly at Kaiserslautern?

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