A substructuring FE model for reduction of structural acoustic problems with dissipative interfaces

PhD started October 2008

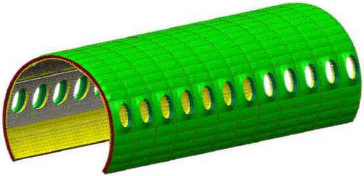
Romain RUMPLER

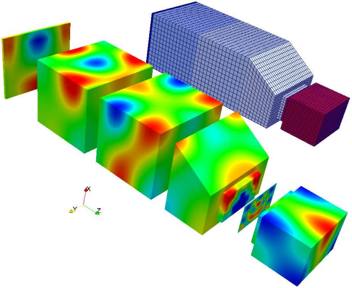
Conservatoire National des Arts et Métiers - Cnam
Future ESR Fellow at KTH (July 2010)

Motivations

- Noise reduction in the context of structural acoustic applications
- Continue W. Larbi 's work at LMSSC on dissipative interfaces modelisation
- Extend to applications including 3D modelisation (porous media)
- Develop computation-efficient methods to treat large structural acoustic problems (reduction methods)
- Implement models into efficient / open source FE software:

  FEAP (R.L. Taylor), Fortran





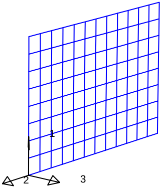
Outline

Elasto acoustic coupling: $[U_s, p_F]$

Problem equations:

$$\left(\begin{bmatrix} K_{ss} & K_{s\theta} \\ K_{\theta s} & K_{\theta\theta} \end{bmatrix} - \omega^2 \begin{bmatrix} M_{ss} & M_{s\theta} \\ M_{\theta s} & M_{\theta\theta} \end{bmatrix}\right) \begin{bmatrix} U_s \\ \theta_s \end{bmatrix} = \begin{bmatrix} f_{bs} \\ f_{b\theta} \end{bmatrix}$$

$$\left(\begin{bmatrix} K_F \end{bmatrix} - \omega^2 \begin{bmatrix} M_F \end{bmatrix}\right) \begin{bmatrix} p_F \end{bmatrix} = \begin{bmatrix} f_{bF} \end{bmatrix}$$

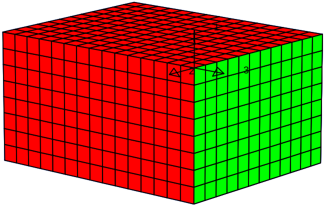


Coupling on Γ_{sF} through Neumann BC:

- $f_{bs} = f_{Fs} = C_{sF}\ p_F$
- $f_{b\theta} = f_{F\theta} = 0$
- $f_{bF} = f_{sF} = \omega^2 \rho_0 c_0^2 C_{sF}^T U_s$

  with:

  $C_{sF} = \int_{\Gamma_{sF}} (N_s)^T n N_F \, dS$



Implementation - Frequency domain:

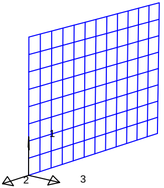
$$\left(\begin{bmatrix} K_{ss} & K_{s\theta} & -C_{sF} \\ K_{\theta s} & K_{\theta\theta} & 0 \\ 0 & 0 & K_F \end{bmatrix} - \omega^2 \begin{bmatrix} M_{ss} & M_{s\theta} & 0 \\ M_{\theta s} & M_{\theta\theta} & 0 \\ C_{sF}^T & 0 & K_F \end{bmatrix}\right) \begin{bmatrix} U_s \\ \theta_s \\ p_F \end{bmatrix} = \begin{bmatrix} f_{bs} \\ f_{b\theta} \\ f_{bF} \end{bmatrix}$$

Elasto acoustic coupling: $[U_s, p_F]$

Problem equations:

$$\left(\begin{bmatrix} K_{ss} & K_{s\theta} \\ K_{\theta s} & K_{\theta\theta} \end{bmatrix} - \omega^2 \begin{bmatrix} M_{ss} & M_{s\theta} \\ M_{\theta s} & M_{\theta\theta} \end{bmatrix}\right) \begin{bmatrix} U_s \\ \theta_s \end{bmatrix} = \begin{bmatrix} f_{bs} \\ f_{b\theta} \end{bmatrix}$$

$$\left(\begin{bmatrix} K_F \end{bmatrix} - \omega^2 \begin{bmatrix} M_F \end{bmatrix}\right) \begin{bmatrix} p_F \end{bmatrix} = \begin{bmatrix} f_{bF} \end{bmatrix}$$

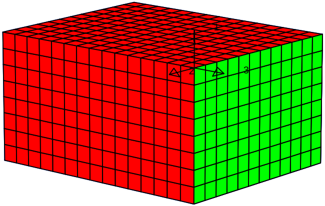


Coupling on Γ_{sF} through Neumann BC:

- $f_{bs} = f_{Fs} = C_{sF}\ p_F$
- $f_{b\theta} = f_{F\theta} = 0$
- $f_{bF} = f_{sF} = \omega^2 \rho_0 c_0^2 C_{sF}^T U_s$

  with:

  $C_{sF} = \int_{\Gamma_{sF}} (N_s)^T n N_F \, dS$



Implementation - Frequency domain:

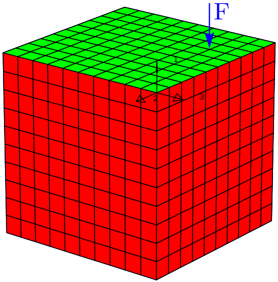
$$\left(\begin{bmatrix} 0 & 0 & -C_{sF} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \omega^2 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ C_{sF}^T & 0 & 0 \end{bmatrix}\right) \begin{bmatrix} U_s \\ \theta_s \\ p_F \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

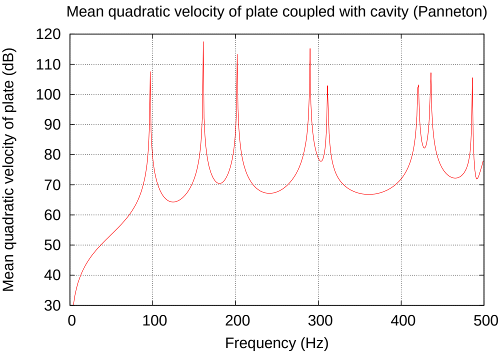
Frequency response of a cavity coupled with a plate

Elasto-acoustic coupled cavity:

- $(1m * 1m * 1m)$ cavity
- $(10 * 10 * 10)$ acoustic elements
- $(1m * 1m)$ plate on one face
- $(10 * 10)$ shell elements
- $(10 * 10)$ quad coupling elts
- Normal unit excitation force

Coupled cavity mesh:





[R. Panneton, *Modélisation numérique tridimensionnelle par éléments finis des milieux poroélastiques*, Ph.D Thesis, 1996]

Us-Uf porous element implementation

Implementation - Frequency domain:

$$\left(\begin{bmatrix} K_{ss} & K_{sf} \\ K_{fs} & K_{ff} \end{bmatrix} - \omega^2 \begin{bmatrix} M_{ss} & M_{sf} \\ M_{fs} & M_{ff} \end{bmatrix}\right)\begin{bmatrix} u_s \\ u_f \end{bmatrix} = \begin{bmatrix} f_{bs} \\ f_{bf} \end{bmatrix}$$

with:

- $M_{ss} = \tilde{\rho}_{ss} \int_{\Omega_P} N_s^T N_s dV$
- $M_{ff} = \tilde{\rho}_{ff} \int_{\Omega_P} N_f^T N_f dV$
- $M_{sf} = \tilde{\rho}_{sf} \int_{\Omega_P} N_s^T N_f dV = M_{fs}^T$
- $K_{ss} = \int_{\Omega_P} (\nabla N_s)^T \tilde{D}_s \nabla N_s dV$
- $K_{ff} = \int_{\Omega_P} (\nabla N_f)^T \tilde{D}_f \nabla N_f dV$
- $K_{sf} = \int_{\Omega_P} (\nabla N_s)^T \tilde{Q} \nabla N_f dV = K_{sf}^T$
- f_{bs} Neumann BC on frame
- f_{bf} Neumann BC on fluid phase

Material parameters:

- In vacuo frame: Es, ν, ρ_s
- Fluid: ρ_0, γ, Pr, η, P_0
- Biot porous: α_∞, ϕ, σ, Λ, Λ'

[J.- F. Allard, *Propagation of Sound in Porous Media: Modelling Sound Absorbing Materials*, Elsevier Science Publishers Ltd, London, 1993]

Us-Uf porous element implementation

Implementation - Frequency domain:

$$\left(\begin{bmatrix} K_{ss} & K_{sf} \\ K_{fs} & K_{ff} \end{bmatrix} - \omega^2 \begin{bmatrix} M_{ss} & M_{sf} \\ M_{fs} & M_{ff} \end{bmatrix}\right) \begin{bmatrix} u_s \\ u_f \end{bmatrix} = \begin{bmatrix} f_{bs} \\ f_{bf} \end{bmatrix}$$

with:

- $M_{ss} = \tilde{\rho}_{ss} \int_{\Omega_P} N_s^T N_s dV$
- $M_{ff} = \tilde{\rho}_{ff} \int_{\Omega_P} N_f^T N_f dV$
- $M_{sf} = \tilde{\rho}_{sf} \int_{\Omega_P} N_s^T N_f dV = M_{fs}^T$
- $K_{ss} = \int_{\Omega_P} (\nabla N_s)^T \tilde{D}_s \nabla N_s dV$
- $K_{ff} = \int_{\Omega_P} (\nabla N_f)^T \tilde{D}_f \nabla N_f dV$
- $K_{sf} = \int_{\Omega_P} (\nabla N_s)^T \tilde{Q} \nabla N_f dV = K_{sf}^T$
- f_{bs} Neumann BC on frame
- f_{bf} Neumann BC on fluid phase

Material parameters:

- In vacuo frame: Es, ν, ρ_s
- Fluid: ρ_0, γ, Pr, η, P_0
- Biot porous: α_∞, ϕ, σ, Λ, Λ'

Detailed expressions:

- $\tilde{\rho}_{ss} = \rho_s + \rho_a - i\sigma\phi^2 \frac{G(\omega)}{\omega}$
- $\tilde{\rho}_{ff} = \phi\rho_0 + \rho_a - i\sigma\phi^2 \frac{G(\omega)}{\omega}$
- $\tilde{\rho}_{sf} = -\rho_a + i\sigma\phi^2 \frac{G(\omega)}{\omega}$

  with:

  $\rho_a = \phi\rho_0 (\alpha_\infty - 1)$

  $G(\omega) = \left[1 + \frac{4i\alpha_\infty^2 \eta\rho_0\omega}{\sigma^2\Lambda^2\phi^2}\right]^{\frac{1}{2}}$

Us-Uf porous element implementation

Implementation - Frequency domain:

$$\left(\begin{bmatrix} K_{ss} & K_{sf} \\ K_{fs} & K_{ff} \end{bmatrix} - \omega^2 \begin{bmatrix} M_{ss} & M_{sf} \\ M_{fs} & M_{ff} \end{bmatrix} \right) \begin{bmatrix} u_s \\ u_f \end{bmatrix} = \begin{bmatrix} f_{bs} \\ f_{bf} \end{bmatrix}$$

with:

- $M_{ss} = \tilde{\rho}_{ss} \int_{\Omega_P} N_s^T N_s dV$
- $M_{ff} = \tilde{\rho}_{ff} \int_{\Omega_P} N_f^T N_f dV$
- $M_{sf} = \tilde{\rho}_{sf} \int_{\Omega_P} N_s^T N_f dV = M_{fs}^T$
- $K_{ss} = \int_{\Omega_P} (\nabla N_s)^T \tilde{D}_s \nabla N_s dV$
- $K_{ff} = \int_{\Omega_P} (\nabla N_f)^T \tilde{D}_f \nabla N_f dV$
- $K_{sf} = \int_{\Omega_P} (\nabla N_s)^T \tilde{Q} \nabla N_f dV = K_{sf}^T$
- f_{bs} Neumann BC on frame
- f_{bf} Neumann BC on fluid phase

Material parameters:

- In vacuo frame: Es, ν, ρ_s
- Fluid: ρ_0, γ, Pr, η, P_0
- Biot porous: α_∞, ϕ, σ, Λ, Λ'

Detailed expressions:

- $\sigma_s = \tilde{D}_s \varepsilon_s + \tilde{Q} \varepsilon_f$
- $\sigma_f = \tilde{Q}^T \varepsilon_s + \tilde{D}_f \varepsilon_f$

with:

$\tilde{D}_s \left(Es, \nu, \phi, \gamma, P_0, \Lambda, \Lambda', \sigma, \alpha_\infty, \eta, \rho_0, \omega, Pr\right)$

$\tilde{D}_f \left(\phi, \gamma, P_0, \Lambda, \Lambda', \sigma, \alpha_\infty, \eta, \rho_0, \omega, Pr\right)$

$\tilde{Q} \left(\phi, \gamma, P_0, \Lambda, \Lambda', \sigma, \alpha_\infty, \eta, \rho_0, \omega, Pr\right)$

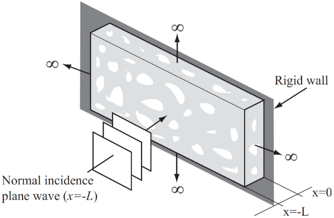
Normal incidence impedance of porous covered wall

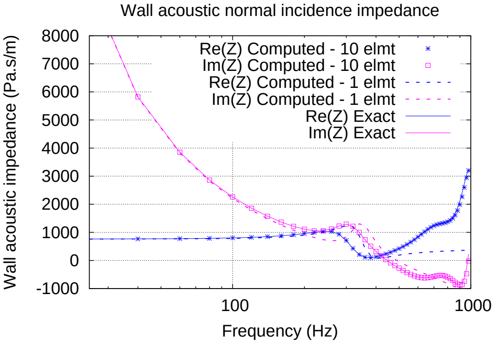
Poroelastic layer on rigid wall:

- Material properties:
- Infinite lateral dimensions
- Unit plane wave excitation / Normal incidence
- 7.62cm thick porous layer

Frame	Fluid	Porous
$Es = 800.8kPa$	$c_0 = 343m/s$	$\phi = 0.9$
$\nu_s = 0.4$	$\gamma = 1.4$	$\sigma = 25kNs/m^4$
$\rho_s = 30kg/m^3$	$Pr = 0.71$	$\alpha_\infty = 7.8$
	$\rho_0 = 1.21kg/m^3$	$\Lambda = 226\mu m$
	$\eta = 1.84 \cdot 10^{-5}Ns/m^2$	$\Lambda' = 226\mu m$

Problem illustration:





[J.-F. Deü, W. Larbi, R. Ohayon, *Vibration and transient response of structural-acoustic interior coupled systems with dissipative interface*, CMAME, 2008]

Elements implementation

- $(U_s - p_F)$ coupling element
- $(u_s - u_f)$ porous element
- $(u_s - u_f) - p_F$ coupling element
- Complete concrete car coupled problem

Implementation of reduction method

- Objectives of the reduction method
- CMS: Craig and Bampton reduction method

Applications

- CMS: Academic applications
- CMS: Application to the concrete car

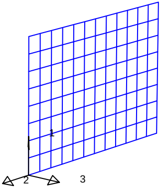
Conclusion - Pespectives

Poro acoustic coupling: $[(u_s, u_f), p_F]$

Problem equations:

$$\left(\begin{bmatrix} K_{ss} & K_{sf} \\ K_{fs} & K_{ff} \end{bmatrix} - \omega^2 \begin{bmatrix} M_{ss} & M_{sf} \\ M_{fs} & M_{ff} \end{bmatrix}\right) \begin{bmatrix} u_s \\ u_f \end{bmatrix} = \begin{bmatrix} f_{bs} \\ f_{bf} \end{bmatrix}$$

$$\left(\begin{bmatrix} K_F \end{bmatrix} - \omega^2 \begin{bmatrix} M_F \end{bmatrix}\right) \begin{bmatrix} p_F \end{bmatrix} = \begin{bmatrix} f_{bF} \end{bmatrix}$$

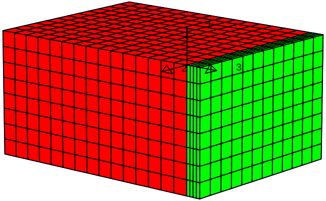


Coupling on Γ_{PF} through Neumann BC:

- $f_{bs} = f_{Fs} = (1 - \phi) C_{sF} \; p_F$
- $f_{bf} = f_{Ff} = \phi C_{fF} \; p_F$
- $f_{bF} = f_{PF} = \omega^2 \rho_0 c_0^2 \left((1 - \phi) \; C_{sF}^T u_s + \phi C_{fF}^T u_f\right)$

  with:

  $$C_{sF} = \int_{\Gamma_{PF}} (N_s)^T n N_F dS$$
  $$C_{fF} = \int_{\Gamma_{PF}} (N_f)^T n N_F dS$$



Implementation - Frequency domain:

$$\left(\begin{bmatrix} K_{ss} & K_{sf} & -(1-\phi) \; C_{sF} \\ K_{fs} & K_{ff} & -\phi C_{fF} \\ 0 & 0 & K_F \end{bmatrix} - \omega^2 \begin{bmatrix} M_{ss} & M_{sf} & 0 \\ M_{fs} & M_{ff} & 0 \\ (1-\phi) C_{sF}^T & \phi C_{fF}^T & K_F \end{bmatrix}\right) \begin{bmatrix} u_s \\ u_f \\ p_F \end{bmatrix} = \begin{bmatrix} f_{bs} \\ f_{bf} \\ f_{bF} \end{bmatrix}$$

Poro acoustic coupling: $[(u_s, u_f), p_F]$

Problem equations:

$$\left(\begin{bmatrix} K_{ss} & K_{sf} \\ K_{fs} & K_{ff} \end{bmatrix} - \omega^2 \begin{bmatrix} M_{ss} & M_{sf} \\ M_{fs} & M_{ff} \end{bmatrix}\right) \begin{bmatrix} u_s \\ u_f \end{bmatrix} = \begin{bmatrix} f_{bs} \\ f_{bf} \end{bmatrix}$$

$$\left(\left[K_F\right] - \omega^2 \left[M_F\right]\right) \left[p_F\right] = \left[f_{bF}\right]$$

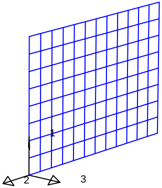
Coupling on Γ_{PF} through Neumann BC:

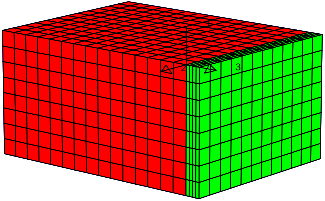
- $f_{bs} = f_{Fs} = (1-\phi) C_{sF} \; p_F$
- $f_{bf} = f_{Ff} = \phi C_{fF} \; p_F$
- $f_{bF} = f_{PF} = \omega^2 \rho_0 c_0^2 \left((1-\phi) \, C_{sF}^T u_s + \phi C_{fF}^T u_f \right)$

  with:

  $C_{sF} = \int_{\Gamma_{PF}} (N_s)^T n N_F \, dS$

  $C_{fF} = \int_{\Gamma_{PF}} (N_f)^T n N_F \, dS$





Implementation - Frequency domain:

$$\left(\begin{bmatrix} 0 & 0 & -(1-\phi)\, C_{sF} \\ 0 & 0 & -\phi C_{fF} \\ 0 & 0 & 0 \end{bmatrix} - \omega^2 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ (1-\phi) C_{sF}^T & \phi C_{fF}^T & 0 \end{bmatrix}\right) \begin{bmatrix} u_s \\ u_f \\ p_F \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

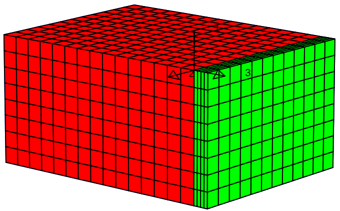
Frequency response of partly treated rigid acoustic cavity

Rigid cavity - partly covered:

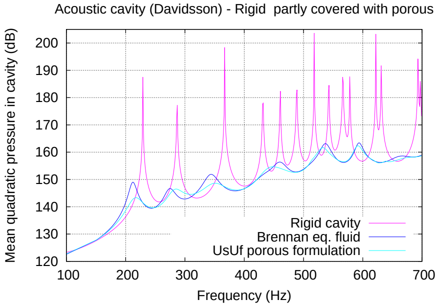
- $(0.4m * 0.6m * 0.75m)$ cavity
- $(8 * 12 * 15)$ acoustic elements
- $(0.4m * 0.6m * 0.05m)$ porous layer on one wall
- $(8 * 12 * 4)$ $u_s - u_f$ porous elts
- $(8 * 12 * 4)$ quad coupling elts
- Corner harmonic excitation

Frame	Fluid	Porous
$Es = 4.4MPa$	$c_0 = 343m/s$	$\phi = 0.94$
$\nu_s = 0$	$\gamma = 1.4$	$\sigma = 40kNs/m^4$
$\rho_s = 130kg/m^3$	$Pr = 0.71$	$\alpha_\infty = 1.06$
	$\rho_0 = 1.21kg/m^3$	$\Lambda = 56\mu m$
	$\eta = 1.84 \cdot 10^{-5} Ns/m^2$	$\Lambda' = 110\mu m$

Poro-acoustic coupled cavity mesh:



Mean quadratic pressure in cavity:

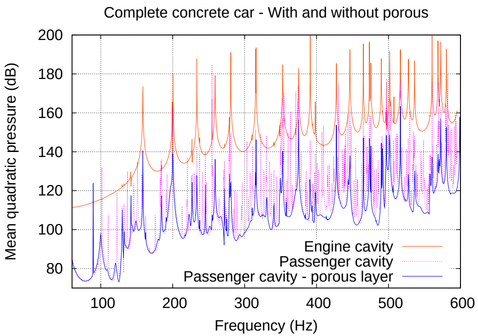


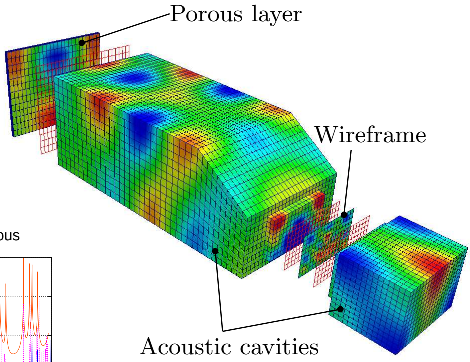
[P. Davidsson, *Structure-acoustic analysis: Finite element modelling and reduction methods*, Ph.D Thesis, 2004]

Frequency response in the EC and PC of the concrete car

Elasto-poro-acoustic problem:

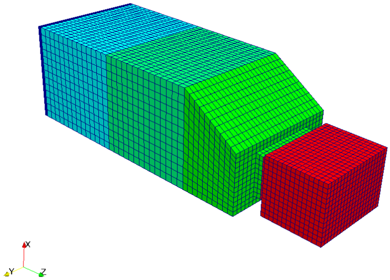
- 2 acoustic cavities: EC and PC
- A wireframe in between cavities
- A porous layer on a wall of the PC
- Corner harmonic excitation in EC
- 28500 DOFs





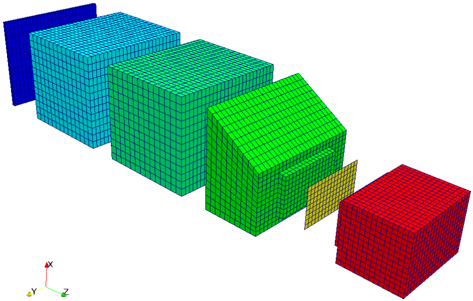
Objectives of the applied reduction method

- Complete model: 28500 DOFs
- First objective: reduction of the non dissipative part
- Reduction to interfaces
- Reduced model: 10600 DOFs (8700 for porous)



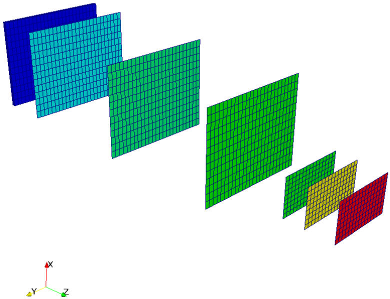
Objectives of the applied reduction method

- Complete model: 28500 DOFs
- First objective: reduction of the non dissipative part
- Reduction to interfaces
- Reduced model: 10600 DOFs (8700 for porous)



Objectives of the applied reduction method

- Complete model: 28500 DOFs
- First objective: reduction of the non dissipative part
- Reduction to interfaces
- Reduced model: 10600 DOFs (8700 for porous)



Craig and Bampton reduction method: Outline

C & B subdomain partitioning:

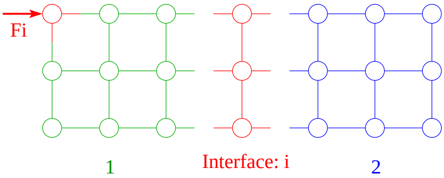
$$\left(\begin{bmatrix} K_{ii} & K_{1i}^T & K_{2i}^T \\ K_{1i} & K_{11} & 0 \\ K_{2i} & 0 & K_{22} \end{bmatrix} - \omega^2 \begin{bmatrix} M_{ii} & M_{1i}^T & M_{2i}^T \\ M_{1i} & M_{11} & 0 \\ M_{2i} & 0 & M_{22} \end{bmatrix} \right) \begin{bmatrix} u_i \\ u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} f_i \\ 0 \\ 0 \end{bmatrix}$$

C & B transformation:

$$\begin{bmatrix} u_i \\ u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} I_{ii} & 0 & 0 \\ \Psi_1 & \Phi_1 & 0 \\ \Psi_2 & 0 & \Phi_2 \end{bmatrix} \begin{bmatrix} u_i \\ \alpha_1 \\ \alpha_2 \end{bmatrix}$$

with for s=1..2:

- $\Psi_s = -K_{ii}^{-1} K_{is}$: Static modes
- Φ_s: First normal modes of fixed interface eigenvalue problem $K_{ss}\Phi_s = \omega_s^2 M_{ss}\Phi_s$
- α_s: Modal coordinates (<<nb nodal coordinates)



[R. Craig, M. Bampton, *Coupling of substructures for dynamic analysis*, AIAA, 1968]

Craig and Bampton reduction method: Outline

Resulting system for n substructures:

$$\left(\begin{bmatrix} K_{ii_\alpha} & 0 & \dots & 0 \\ 0 & \omega_1^2 & 0 & 0 \\ \vdots & 0 & \ddots & \vdots \\ 0 & 0 & \dots & \omega_n^2 \end{bmatrix} - \omega^2 \begin{bmatrix} M_{ii_\alpha} & M_{1i_\alpha}^T & \dots & M_{ni_\alpha}^T \\ M_{1i_\alpha} & I_1 & 0 & 0 \\ \vdots & 0 & \ddots & \vdots \\ M_{ni_\alpha} & 0 & \dots & I_n \end{bmatrix} \right) \begin{bmatrix} u_i \\ \alpha_1 \\ \vdots \\ \alpha_n \end{bmatrix} = \begin{bmatrix} f_i \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

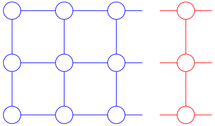
With:

- $K_{ii_\alpha} = K_{ii} + \sum_{s=1}^{n} K_{si}^T \Psi_s$
- $M_{ii_\alpha} = M_{ii} + \sum_{s=1}^{n} \left[M_{si}^T \Psi_s + \Psi_s^T \left(M_{si} + M_{ss} \Psi_s \right) \right]$
- $M_{si_\alpha} = \left[\Phi_s^T \left(M_{si} + M_{ss} \Psi_s \right) \right]$

C & B implementation in FEAP

Resulting system for subdomain n with its boundary interface:

$$\left(\begin{bmatrix} \begin{pmatrix} K_{ii_n}+ \\ K_{ni}^T\Psi_n \end{pmatrix} & 0 \\ 0 & \omega_n^2 \end{bmatrix} - \omega^2 \begin{bmatrix} M_{ii_n} + \begin{bmatrix} M_{ni}^T\Psi_n + \\ \Psi_n^T\left(M_{ni} + M_{nn}\Psi_n\right) \end{bmatrix} & \begin{bmatrix} \Phi_n^T \begin{pmatrix} M_{ni}+ \\ M_{nn}\Psi_n \end{pmatrix} \end{bmatrix}^T \\ \left[\Phi_n^T\left(M_{ni} + M_{nn}\Psi_n\right)\right] & I_n \end{bmatrix}\right) \begin{bmatrix} p_{i_n} \\ \alpha_n \end{bmatrix} = \begin{bmatrix} f_{i_n} \\ 0 \end{bmatrix} \quad \begin{matrix} (1) \\ (2) \end{matrix}$$



Computation steps:

- (2) $\Rightarrow \alpha_n = \omega^2\left(\omega_n^2 - \omega^2 I_n\right)^{-1}\left[\Phi_n^T\left(M_{ni} + M_{nn}\Psi_n\right)\right] p_{i_n}$
- (1) $\Rightarrow \begin{bmatrix} \left(K_{ii_n} + K_{ni}^T\Psi_n\right) - \omega^2\left[M_{ii_n} + M_{ni}^T\Psi_n + \Psi_n^T\left(M_{ni} + M_{nn}\Psi_n\right)\right] \\ -\omega^2\left[\left[\Phi_n^T\left(M_{ni} + M_{nn}\Psi_n\right)\right]^T\omega^2\left(\omega_n^2 - \omega^2 I_n\right)^{-1}\left[\Phi_n^T\left(M_{ni} + M_{nn}\Psi_n\right)\right]\right] \end{bmatrix} p_{i_n} = f_{i_n}$

Solving steps:

- Solve (1) $\Rightarrow p_{i_n}$
- Compute $p_n = \left[\Phi_n\omega^2\left(\omega_n^2 - \omega^2 I_n\right)^{-1}\left[\Phi_n^T\left(M_{ni} + M_{nn}\Psi_n\right)\right] + \Psi_n\right] p_{i_n}$

C & B implementation in FEAP

Problem equations:

- Solve $\begin{bmatrix} \left(K_{ii_n} + K_{ni}^T\Psi_n\right) - \omega^2\left[M_{ii_n} + M_{ni}^T\Psi_n + \Psi_n^T\left(M_{ni} + M_{nn}\Psi_n\right)\right] \\ -\omega^2\left[\left[\Phi_n^T\left(M_{ni} + M_{nn}\Psi_n\right)\right]^T \omega^2\left(\omega_n^2 - \omega^2 I_n\right)^{-1}\left[\Phi_n^T\left(M_{ni} + M_{nn}\Psi_n\right)\right]\right] \end{bmatrix} p_{i_n} = f_{i_n}$
- Compute $p_n = \left[\Phi_n\omega^2\left(\omega_n^2 - \omega^2 I_n\right)^{-1}\left[\Phi_n^T\left(M_{ni} + M_{nn}\Psi_n\right)\right] + \Psi_n\right] p_{i_n}$

Steps of algorithm:

1. Loop on subdomains n:
   - Compute K_{nn} & M_{nn}
   - Eigenvalue problem: ω_n^2, Φ_n
   - Loop on interface dofs
     $K_{ni} \Rightarrow \Psi_n = -K_{nn}^{-1}K_{ni}$
     $M_{ni} \Rightarrow A_n = M_{ni} + M_{nn}\Psi_n$
   - Compute $B_n = \Phi_n^T A_n$
   - Update:
     $K_{ii_\alpha} = K_{ii\alpha} + K_{ni}^T\Psi_n$
     $M_{ii\alpha} = M_{ii\alpha} + M_{ni}^T\Psi_n + \Psi_n^T A_n$

2. Interface sized problem:
   - Compute K_{ii} & M_{ii}
   - Update:
     $K_{ii_\alpha} = K_{ii_\alpha} + K_{ii}$
     $M_{ii_\alpha} = M_{ii_\alpha} + M_{ii}$

3. Loop on ω:
   - $K_{FRF}(\omega) = K_{ii_\alpha} - \omega^2 M_{ii_\alpha} - \omega^4\sum_n\left[B_n^T\left(\omega_n^2 - \omega^2 I_n\right)^{-1} B_n\right]$
   - Solve $K_{FRF}(\omega)\ p_i = f_i \Rightarrow p_i$
   - $p_n = \left[\Phi_n\omega^2\left(\omega_n^2 - \omega^2 I_n\right)^{-1} B_n + \Psi_n\right] p_{i_n}$

Elements implementation

- $(U_s - p_F)$ coupling element
- $(u_s - u_f)$ porous element
- $(u_s - u_f) - p_F$ coupling element
- Complete concrete car coupled problem

Implementation of reduction method

- Objectives of the reduction method
- CMS: Craig and Bampton reduction method

Applications

- CMS: Academic applications
- CMS: Application to the concrete car

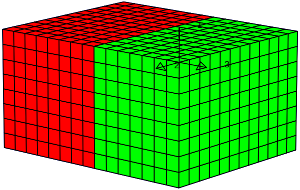
Conclusion - Pespectives

C & B reduction applied to enclosed cavity

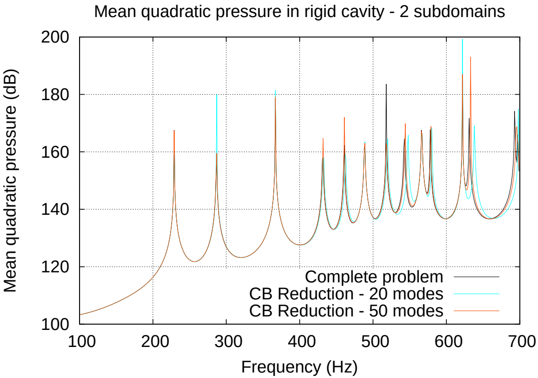
Rigid cavity:

- $(0.4m * 0.6m * 0.75m)$ cavity
- $(8 * 12 * 15)$ acoustic elements
- Decomposed in 2 subdomains
- Comparison 20/50 normal modes
- Corner harmonic excitation

Cavity mesh:



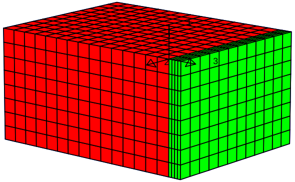
Mean quadratic pressure in cavity:



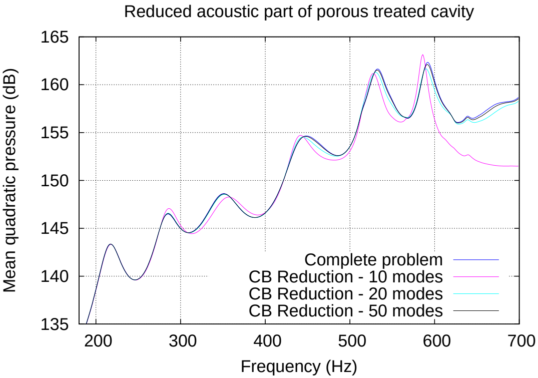
Reduction of acoustic part of porous treated cavity

Porous treated cavity:

- $(0.4m * 0.6m * 0.75m)$ cavity
- $(8 * 12 * 15)$ acoustic elements
- $(0.4m * 0.6m * 0.05m)$ porous layer on one wall
- $(8 * 12 * 4)$ hexahedric elements for porous
- Acoustic part reduced only
- Comparison 10/20/50 normal modes
- Corner harmonic excitation



Mean quadratic pressure in cavity:

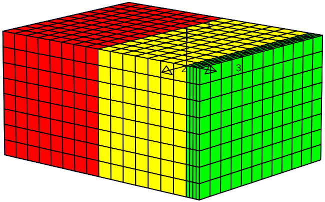


Reduction of acoustic part of porous treated cavity

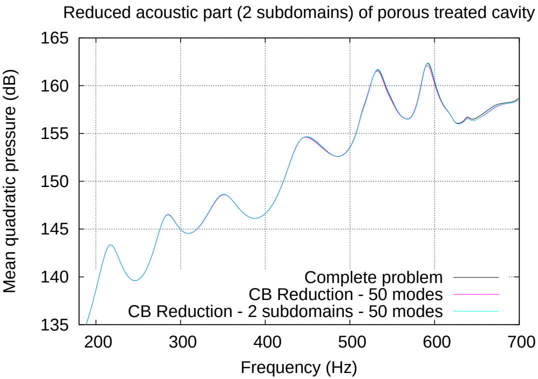
Porous treated cavity:

- Same cavity covered with porous layer
- 2 acoustic subdomains for reduction of acoustic part
- Comparison 50 normal modes
- Corner harmonic excitation

Cavity and porous layer mesh:



Mean quadratic pressure in cavity:

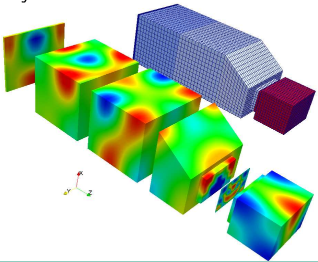


Reduction method applied to the concrete car

Reduced concrete car problem:

- 4 reduced acoustic components: EC(1) PC(3)
- Wireframe
- A 5cm porous layer on one wall
- FRF range $[0, 600]$ Hz

Cavity mesh:

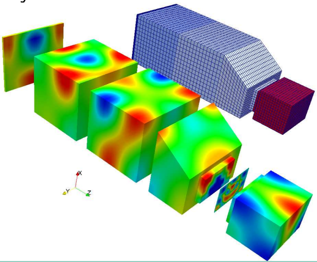


Reduction method applied to the concrete car

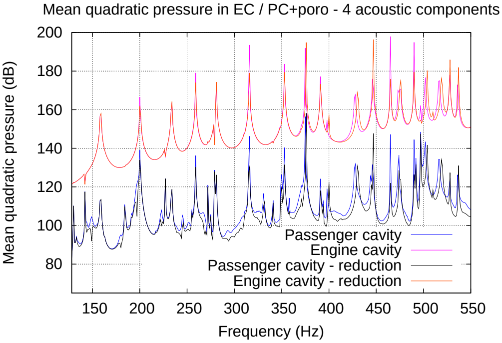
Reduced concrete car problem:

- 4 reduced acoustic components: EC(1) PC(3)
- Wireframe
- A 5cm porous layer on one wall
- FRF range $[0, 600]$ Hz
- EC: 50 modes $\Rightarrow$ 740 Hz
- PC: 75 modes $\Rightarrow$ 640 Hz

Cavity mesh:



Mean quadratic pressure in EC and PC:

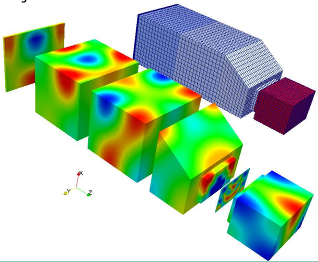


Reduction method applied to the concrete car

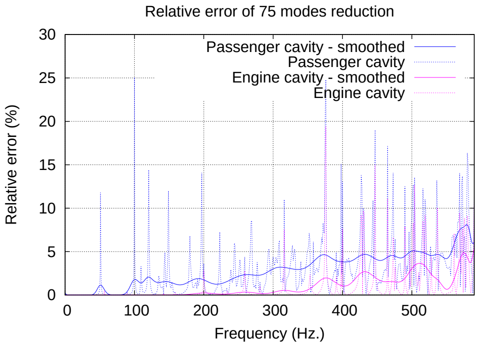
Reduced concrete car problem:

- 4 reduced acoustic components: EC(1) PC(3)
- Wireframe
- A 5cm porous layer on one wall
- FRF range $[0, 600]$ Hz
- EC: 50 modes $\Rightarrow$ 740 Hz
- PC: 75 modes $\Rightarrow$ 640 Hz

Cavity mesh:



Error on mean quadratic pressure:

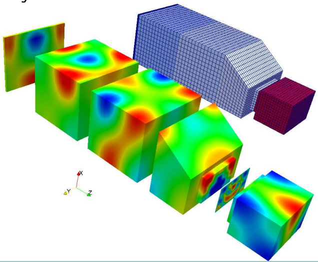


Reduction method applied to the concrete car

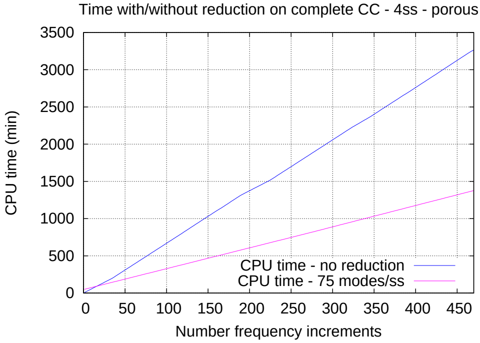
Reduced concrete car problem:

- 4 reduced acoustic components: EC(1) PC(3)
- Wireframe
- A 5cm porous layer on one wall
- FRF range $[0, 600]$ Hz
- EC: 50 modes $\Rightarrow$ 740 Hz
- PC: 75 modes $\Rightarrow$ 640 Hz

Cavity mesh:



CPU computation time:

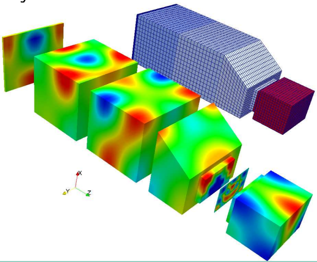


Reduction method applied to the concrete car

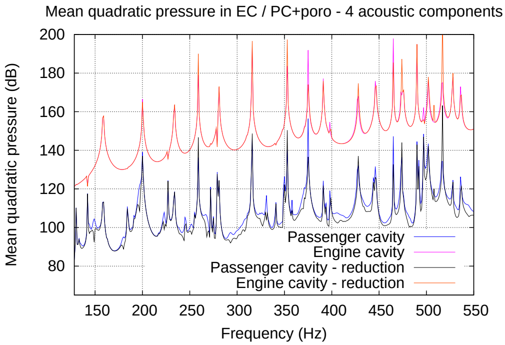
Reduced concrete car problem:

- 4 reduced acoustic components: EC(1) PC(3)
- Wireframe
- A 5cm porous layer on one wall
- FRF range $[0, 600]$ Hz
- EC: 150 modes $\Rightarrow$ 1200 Hz
- PC: 250 modes $\Rightarrow$ 1000 Hz

Cavity mesh:



Mean quadratic pressure in EC and PC:

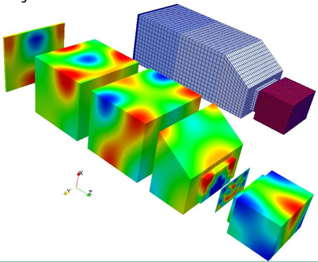


Reduction method applied to the concrete car

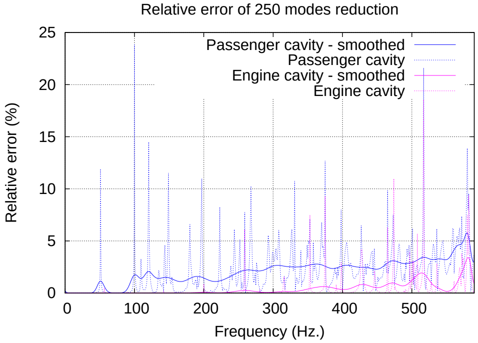
Reduced concrete car problem:

- 4 reduced acoustic components: EC(1) PC(3)
- Wireframe
- A 5cm porous layer on one wall
- FRF range $[0, 600]$ Hz
- EC: 150 modes $\Rightarrow$ 1200 Hz
- PC: 250 modes $\Rightarrow$ 1000 Hz

Cavity mesh:



Error on mean quadratic pressure:

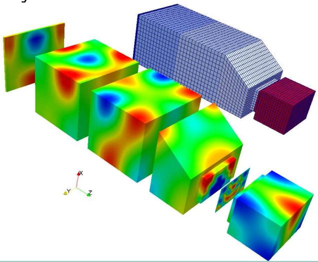


Reduction method applied to the concrete car

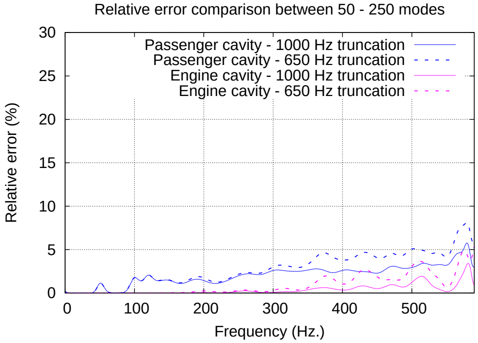
Reduced concrete car problem:

- 4 reduced acoustic components: EC(1) PC(3)
- Wireframe
- A 5cm porous layer on one wall
- FRF range $[0, 600]$ Hz
- EC: 150 modes $\Rightarrow$ 1200 Hz
- PC: 250 modes $\Rightarrow$ 1000 Hz

Cavity mesh:



Improvement on error:

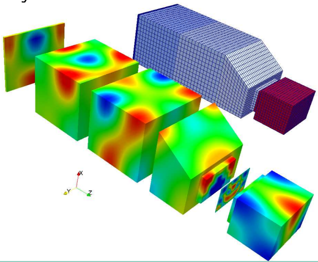


Reduction method applied to the concrete car

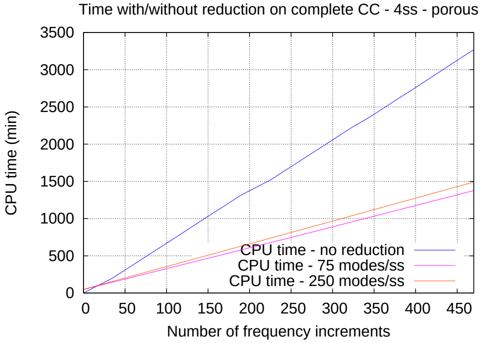
Reduced concrete car problem:

- 4 reduced acoustic components: EC(1) PC(3)
- Wireframe
- A 5cm porous layer on one wall
- FRF range $[0, 600]$ Hz
- EC: 150 modes $\Rightarrow$ 1200 Hz
- PC: 250 modes $\Rightarrow$ 1000 Hz

Cavity mesh:

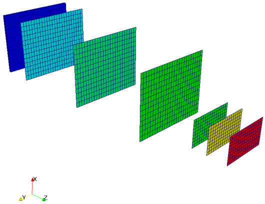


CPU computation time:



Conclusion and perspectives

- Basic tools for coupled problems implemented (acoustic, porous and coupling elements)
- Efficient implementation of a reduction method for non-dissipative part of the problem
- Reduction method applied to the Smart-Structure Concrete car model



- Porous media and interfaces responsible of remaining dofs
- Further reduce interfaces with dofs condensation
- Work with Matlab on methods for reduction of the porous media