NONLINEAR VIBRATIONS AND STABILITY OF SHELLS WITH FLUID-STRUCTURE INTERACTION

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Outline of Presentation

•Reduce the order of models for large-amplitude (geometrically nonlinear) vibrations of fluid-filled circular cylindrical shells by using the proper orthogonal decomposition (POD) method

•Large-amplitude vibrations of rectangular plates

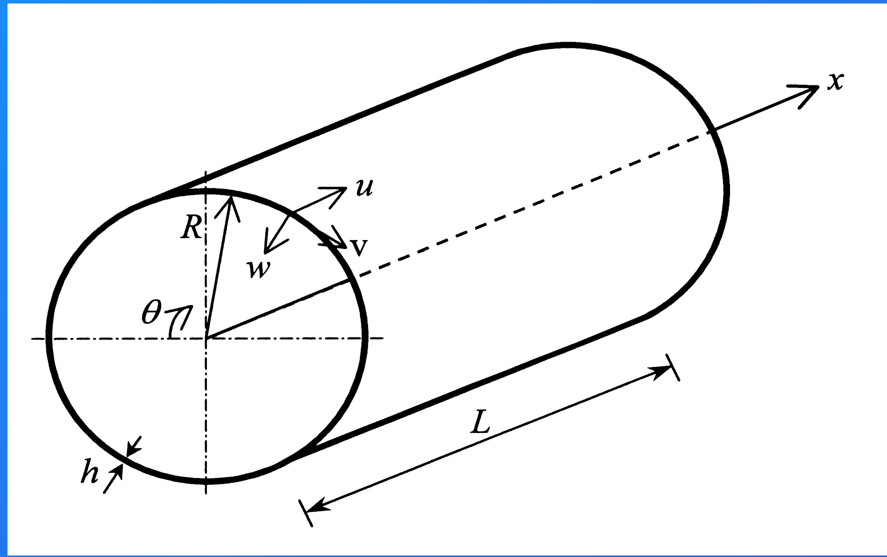
•Flutter of imperfect shells in supersonic flow

•Nonlinear vibrations and stability of circular cylindrical shells conveying flow

REDUCED-ORDER MODELS FOR NONLINEAR VIBRATIONS OF CYLINDRICAL SHELLS VIA THE PROPER ORTHOGONAL DECOMPOSITION METHOD

Donnell's nonlinear shallow-shell theory

with radial geometric imperfections w_0



$$D\nabla^4 w + c\,h\,\dot{w} + \rho\,h\,\ddot{w} = f - p + \frac{1}{R}\frac{\partial^2 F}{\partial x^2} + \frac{1}{R^2}\left[\frac{\partial^2 F}{\partial \theta^2}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w_0}{\partial x^2}\right)\right.$$

$$\left. -2\frac{\partial^2 F}{\partial x\,\partial \theta}\left(\frac{\partial^2 w}{\partial x\,\partial \theta} + \frac{\partial^2 w_0}{\partial x\,\partial \theta}\right) + \frac{\partial^2 F}{\partial x^2}\left(\frac{\partial^2 w}{\partial \theta^2} + \frac{\partial^2 w_0}{\partial \theta^2}\right)\right]$$

$$\frac{1}{E\,h}\nabla^4 F = -\frac{1}{R}\frac{\partial^2 w}{\partial x^2} + \frac{1}{R^2}\left[\left(\frac{\partial^2 w}{\partial x\,\partial \theta}\right)^2 + 2\frac{\partial^2 w}{\partial x\,\partial \theta}\frac{\partial^2 w_0}{\partial x\,\partial \theta} - \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w_0}{\partial x^2}\right)\frac{\partial^2 w}{\partial \theta^2} - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w_0}{\partial \theta^2}\right]$$

Out-of-plane boundary conditions

$$w = w_0 = 0$$

$$M_x = -D\left\{\left(\partial^2 w/\partial x^2\right) + \nu\left[\partial^2 w/(R^2\,\partial\,\theta^2)\right]\right\} = 0$$

$$\partial^2 w_0 / \partial x^2 = 0 \qquad \text{at } x = 0, L$$

In-plane boundary conditions

$N_x = 0$ and $v = 0$ $at\ x = 0, L$

Expansion of the radial displacement *w*: Conventional Galerkin approach

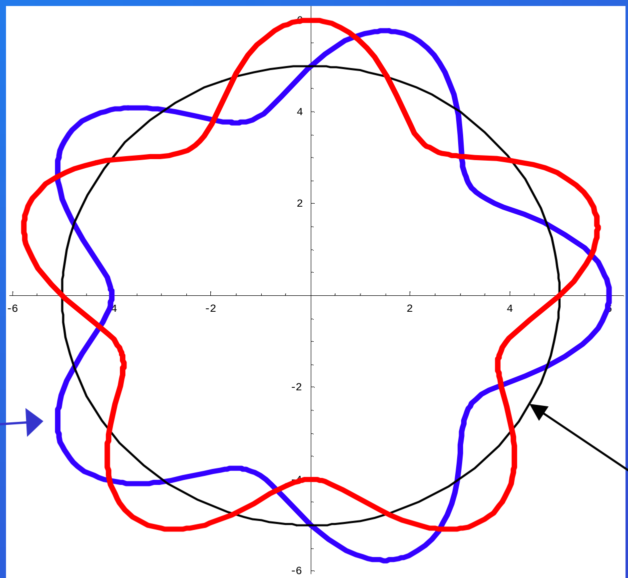
$$w(x,\theta,t)=\sum_{m=1}^{3}\sum_{k=1}^{3}\left[\underbrace{A_{m,kn}(t)\cos(kn\theta)}_{\text{Driven mode}}+\underbrace{B_{m,kn}(t)\sin(kn\theta)}_{\text{Companion mode}}\right]\sin(\lambda_m x)$$

$$+\underbrace{\sum_{m=1}^{4}A_{(2m-1),0}(t)\sin(\lambda_{(2m-1)}x)}_{\text{Axisymmetric modes}}$$

eigenmodes
16 dofs

Companion mode

Driven mode



Harmonic excitation

Circular shell

PROPER ORTHOGONAL DECOMPOSITION (POD) METHOD

Conventional Galerkin solution

$$w(\xi, t) = \sum_{i=1}^{K} q_i(t)\,\varphi_i(\xi)$$

dofs

Basis functions (eigenmodes)

Generalized coordinates

POD solution

Reduced dofs

$$w(\xi, t) = \sum_{i=1}^{\tilde{K}} a_i(t)\,\psi_i(\xi)$$

Proper orthogonal modes

Proper orthogonal coordinates

PO modes are extracted from temporal snapshots of the response (from Galerkin solution in the present case)

Minimizing the objective function

$$\tilde{\lambda} = \left\langle \left(\psi(\xi) - \tilde{w}(\xi, t) \right)^2 \right\rangle \qquad \forall \xi \in \Omega$$

time-averaging

zero-mean response

$$\tilde{w}(\xi, t) = \left(w(\xi, t) - \overline{w}(\xi) \right)$$

$$\int_{\Omega} < \tilde{w}(\xi, t)\, \tilde{w}(\xi', t) > \psi(\xi')\, \mathrm{d}\,\xi' = \lambda\, \psi(\xi)$$

Eigenvalue problem

Projection of the proper orthogonal modes on the Galerkin modes

$$\psi(\xi) = \sum_{i=1}^{K} \alpha_i \varphi_i(\xi)$$

$$\sum_{i=1}^{K} \varphi_i(\xi) \sum_{j=1}^{K} \sum_{k=1}^{K} \left\langle \tilde{q}_i(t) \tilde{q}_j(t) \right\rangle \alpha_k \int_{\Omega} \varphi_j(\xi') \varphi_k(\xi') \, \mathrm{d}\xi' = \lambda \sum_{i=1}^{K} \alpha_i \varphi_i(\xi)$$

zero-mean response of the i-th generalized coordinate $\tilde{q}_i = (q_i - \bar{q}_i)$

The final eigenvalue problem is

Eigenvalues: significance of PO modes

$$\mathbf{A}\,\boldsymbol{\alpha} = \lambda\,\mathbf{B}\boldsymbol{\alpha}$$

Eigenvectors: Coefficients of projection of PO mode on Galerkin modes

$$A_{ij} = \tau_i\,\tau_j \left\langle \tilde{q}_i(t)\,\tilde{q}_j(t) \right\rangle$$

$$B_{ij} = \tau_i\,\delta_{ij}$$

$$\tau_i = \int_{\Omega} \varphi_i^2(\xi)\,\mathrm{d}\,\xi$$

CASE OF WEIGHTED RESPONSES

weights

$$\mathbf{A} = \frac{p_1\,\mathbf{A}^{(1)}}{\left\|\mathbf{A}^{(1)}\right\|} + \frac{p_2\,\mathbf{A}^{(2)}}{\left\|\mathbf{A}^{(2)}\right\|}$$

$\left\|\mathbf{A}\right\| = \sqrt{\mathrm{Tr}\,(\mathbf{A}\,\mathbf{A}^{\mathrm{T}})}$ **Frobenius norm of A**

$$A_{ij}^{(1)} = \tau_i\,\tau_j \left\langle \tilde{q}_i^{(1)}(t)\,\tilde{q}_j^{(1)}(t) \right\rangle$$

$$A_{ij}^{(2)} = \tau_i\,\tau_j \left\langle \tilde{q}_i^{(2)}(t)\,\tilde{q}_j^{(2)}(t) \right\rangle$$

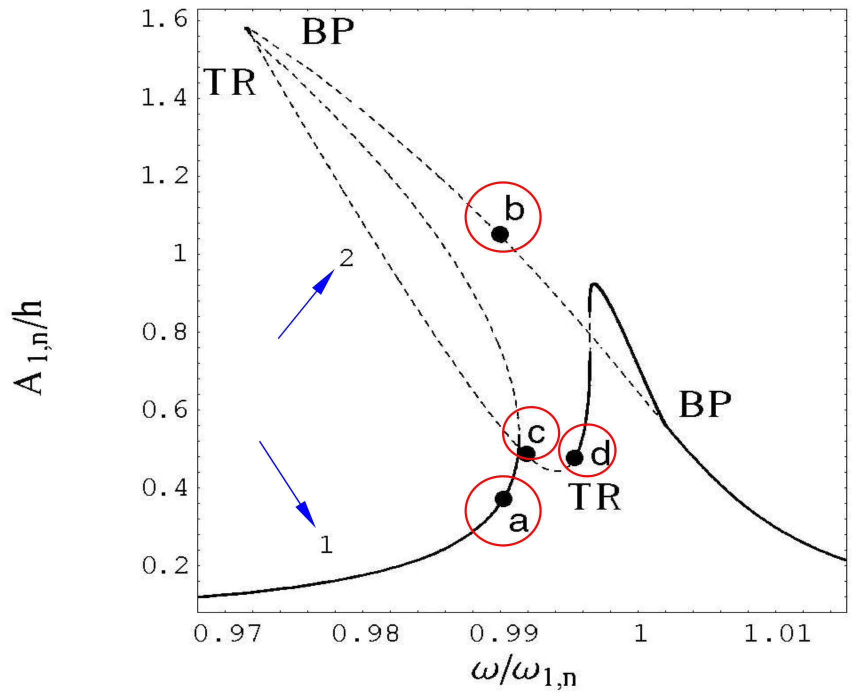
Proper orthogonal modes

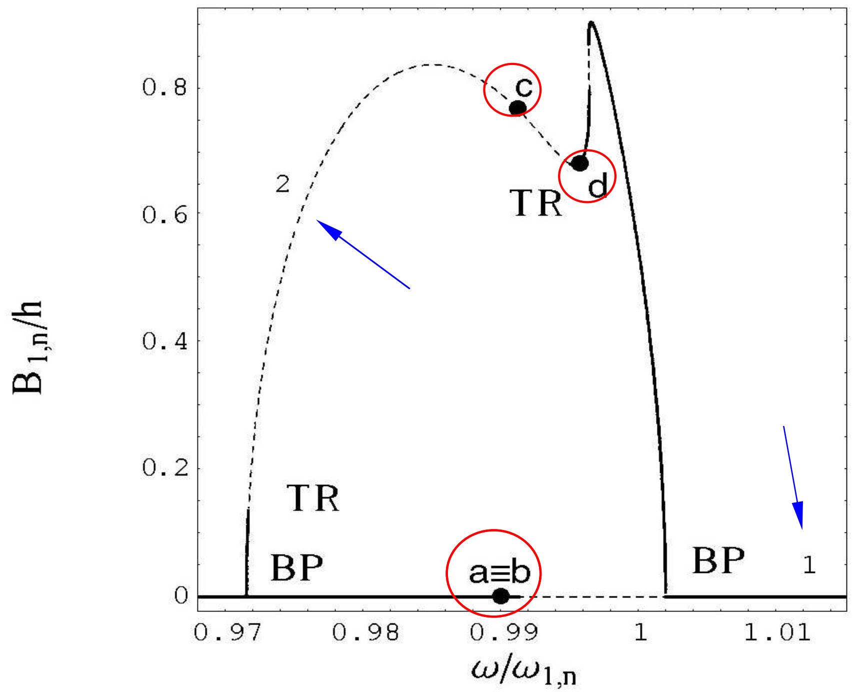
$$w(\xi,t) = \sum_{i=1}^{\tilde{K}} a_i(t) \sum_{j=1}^{K} \alpha_{j,i}\,\varphi_j(\xi) = \sum_{i=1}^{\tilde{K}} a_i(t) \sum_{m=1}^{M} \sum_{n=0}^{N} \left[\alpha_{m,n,i} \cos\,(n\theta) + \beta_{m,n,i} \sin\,(n\theta) \right] \sin(\lambda_m x)$$

Numerical Results

$L = 0.52$ m
$R = 0.1494$ m
$h = 0.519$ mm
$E = 198 * 10^{11}$ Pa
$\rho = 7800$ kg/m^3
$\rho_F = 1000$ kg/m^3 (water filled)
$\nu = 0.3$
Fundamental mode ($n = 5$, $m = 1$)
Harmonic point excitation of 3 N at $x = L/2$ and $\theta = 0$
$\zeta_{1,n} = 0.0017$
$\omega_{1,n} = 77.64$ Hz

Software: AUTO for bifurcation analysis and continuation
IMSL Fortran – DIVPAG routine





Conventional Galerkin results (16 dofs)

Maximum Amplitude of driven mode $A_{1,n}(t)$

——, stable solutions;

— — —, unstable periodic solutions

TR: Neimark-Sacker (torus) bifurcations

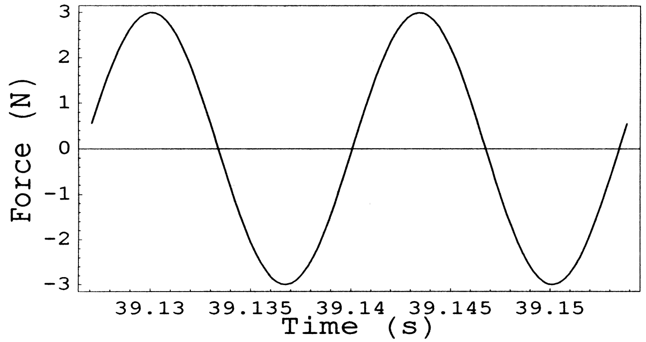
BP: Pitchfork bifurcations

Maximum Amplitude of companion mode $B_{1,n}(t)$

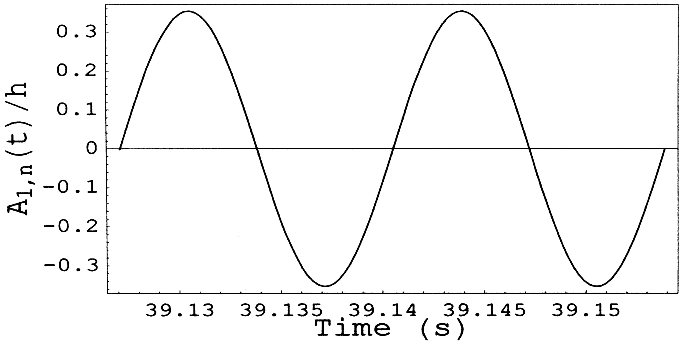
Harmonic force excitation

Time response at excitation frequency $\omega / \omega_{1,n} = 0.99$

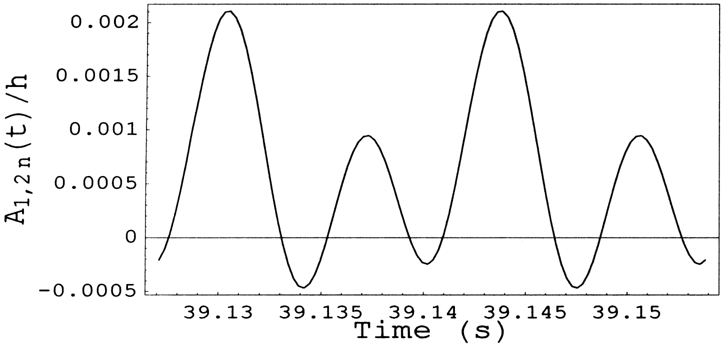
Case "a"



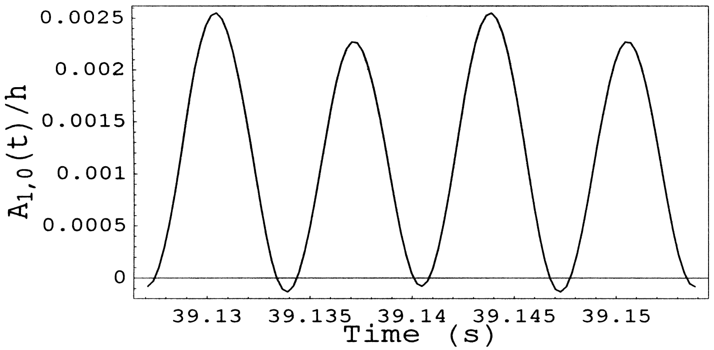
$A_{1,n}(t)$



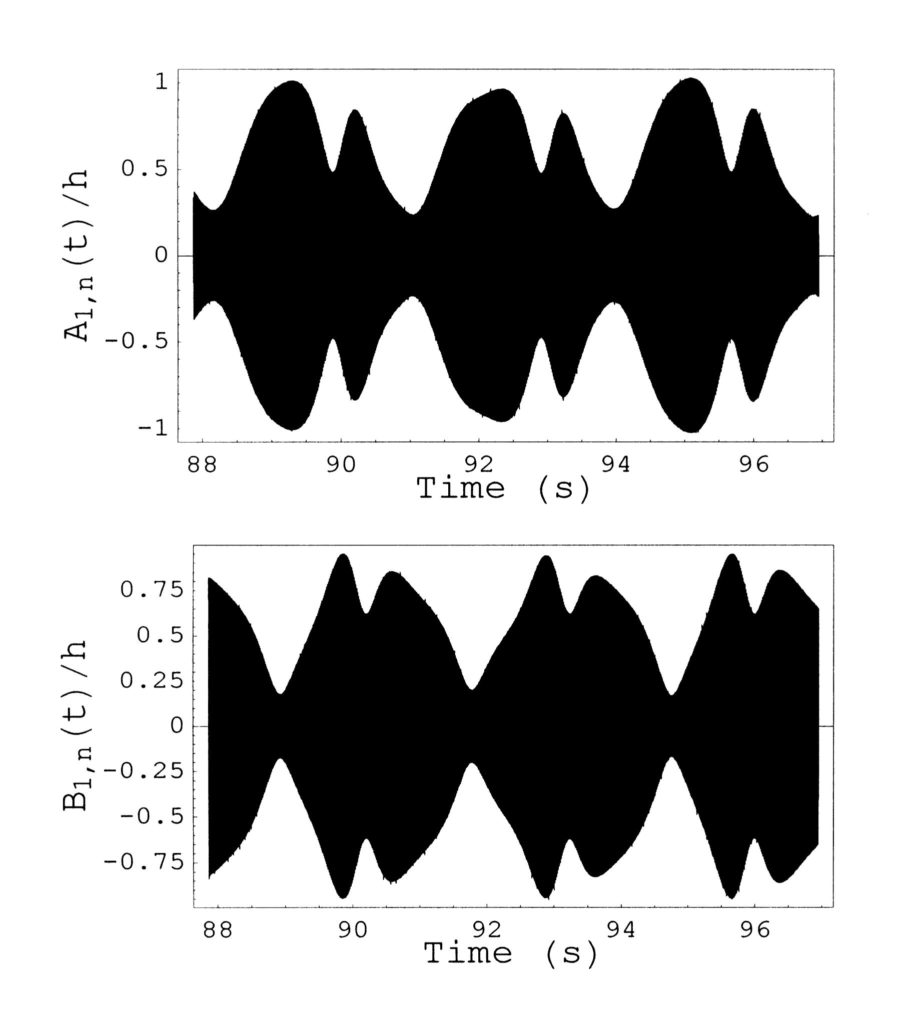
$A_{1,2n}(t)$



$A_{1,0}(t)$



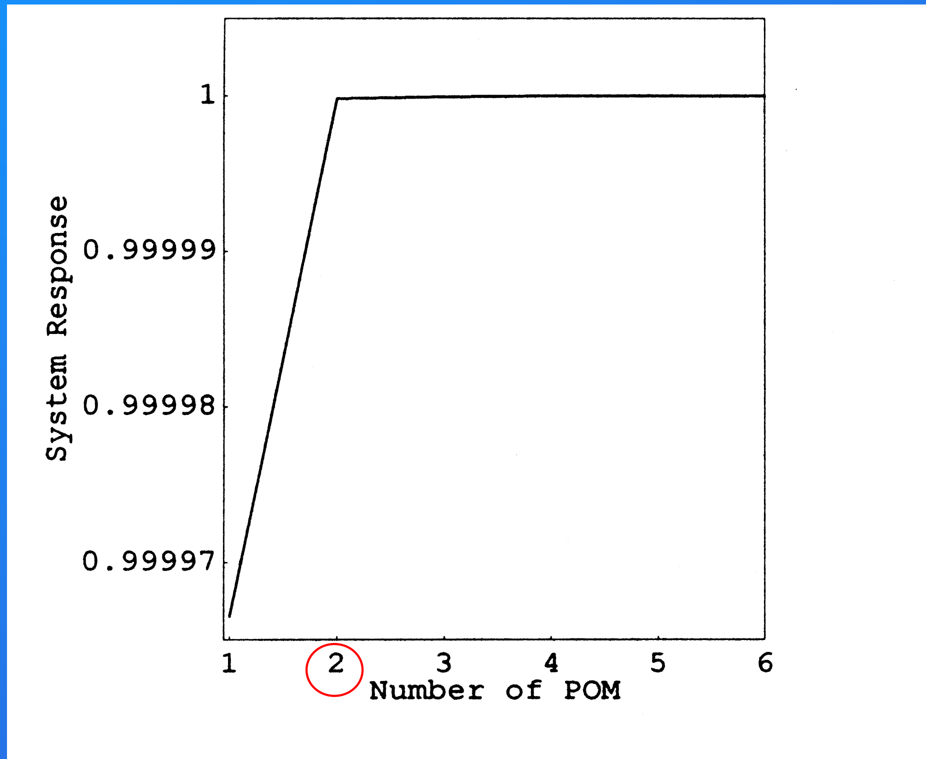
Time response at excitation frequency $\omega/\omega_{1,n}$ = 0.991 corresponding to point "c"



$A_{1,n}(t)$

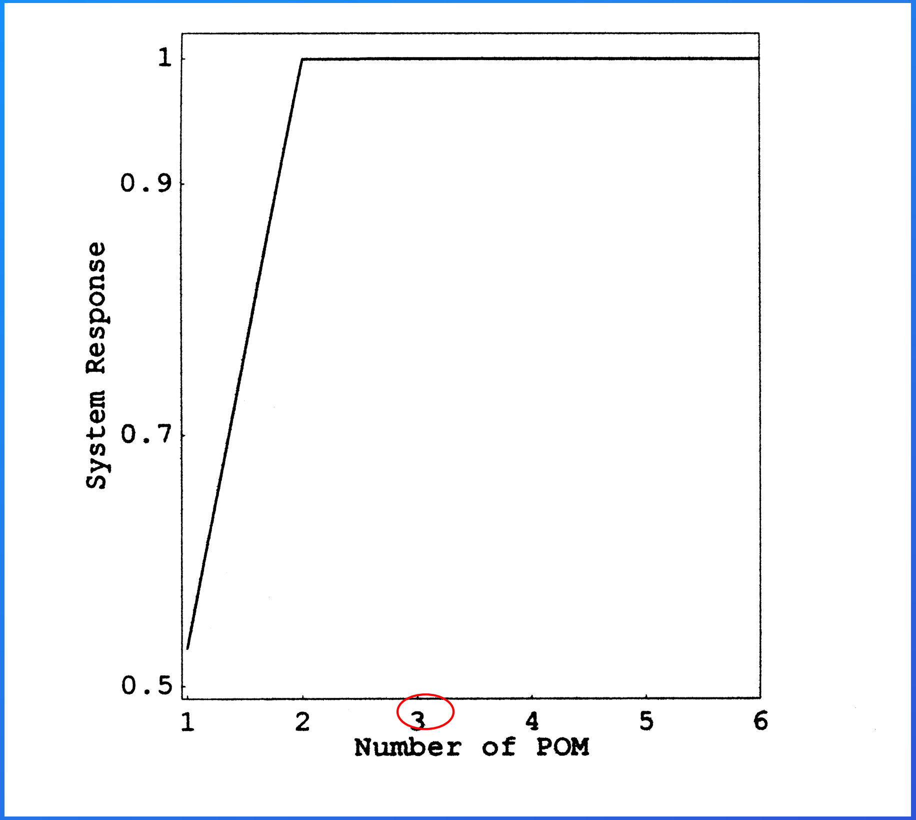
$B_{1,n}(t)$

Significance of POD eigenvalues versus the number of proper orthogonal modes; case "a"



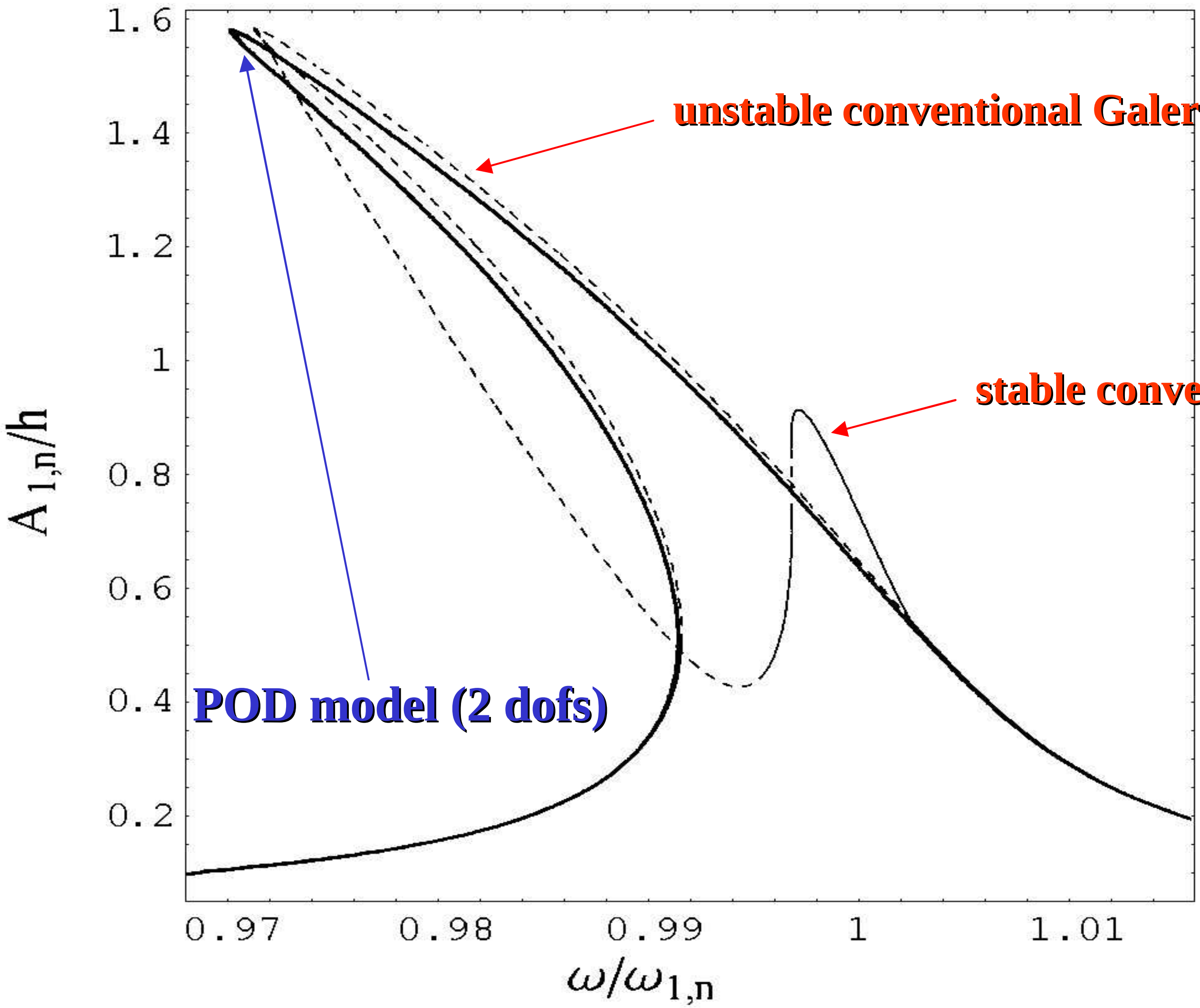
$$\sum_{i=1}^{\tilde{K}} \lambda_i \Big/ \sum_{i=1}^{K} \lambda_i \geq 0.999$$

Significance of POD eigenvalues versus the number of proper orthogonal modes; case "c"



POD model (2dofs) versus conventional Galerkin model, case “a”

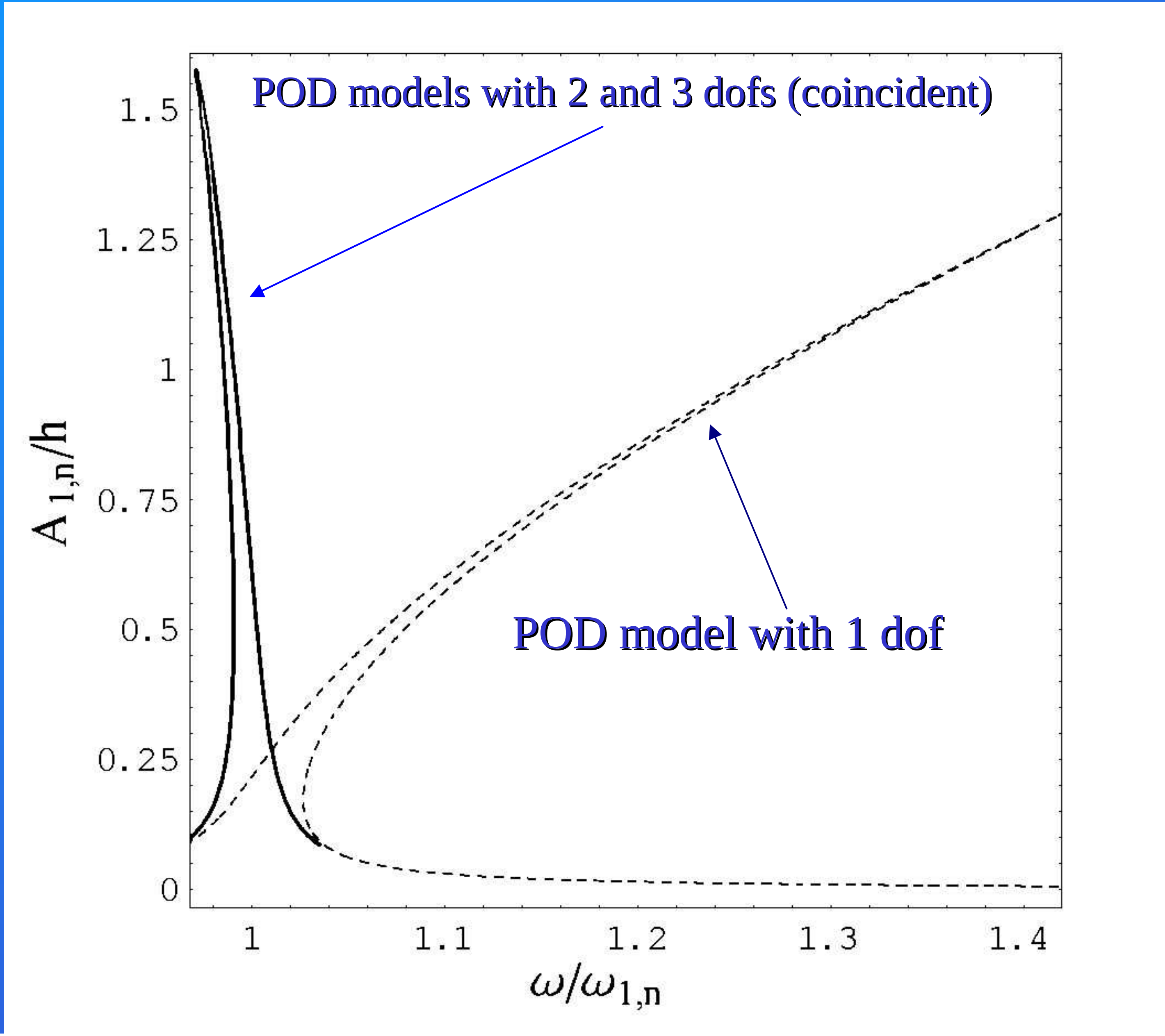
Maximum amplitude of vibration versus excitation frequency



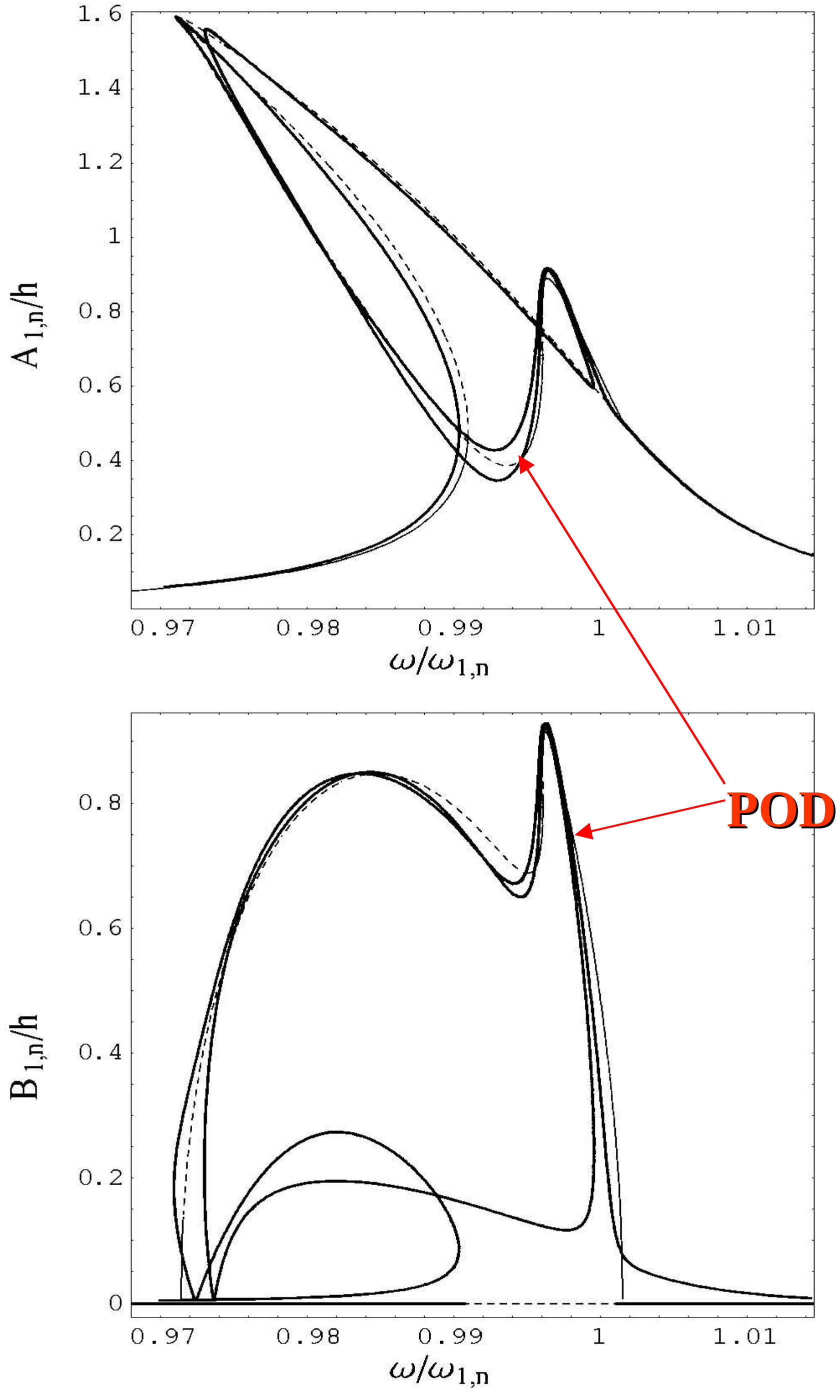
Branch 2 is not obtained

Convergence of the solution with the number of dof
POD model, case "a"

Maximum amplitude of $A_{1,n}(t)$ versus excitation frequency



POD model versus conventional Galerkin model, case “c”



Maximum amplitude of vibration versus excitation frequency

Maximum Amplitude of $A_{1,n}(t)$

—— stable conventional Galerkin solutions

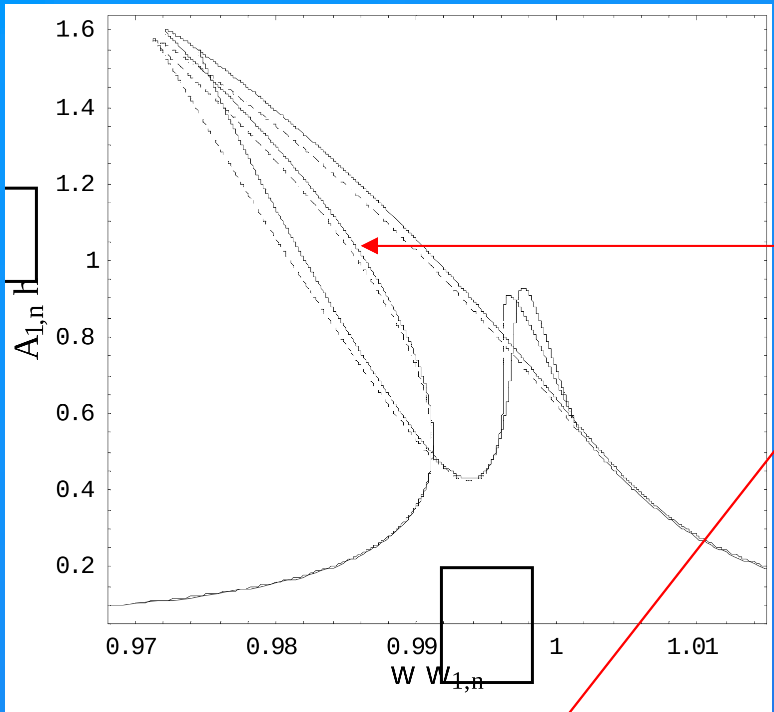
— — — unstable conventional Galerkin solutions.

POD model (3 dofs)

Maximum Amplitude of $B_{1,n}(t)$

NO PITCHFORK BIFURCATIONS

POD model versus Galerkin model, case “c” combined “-c”



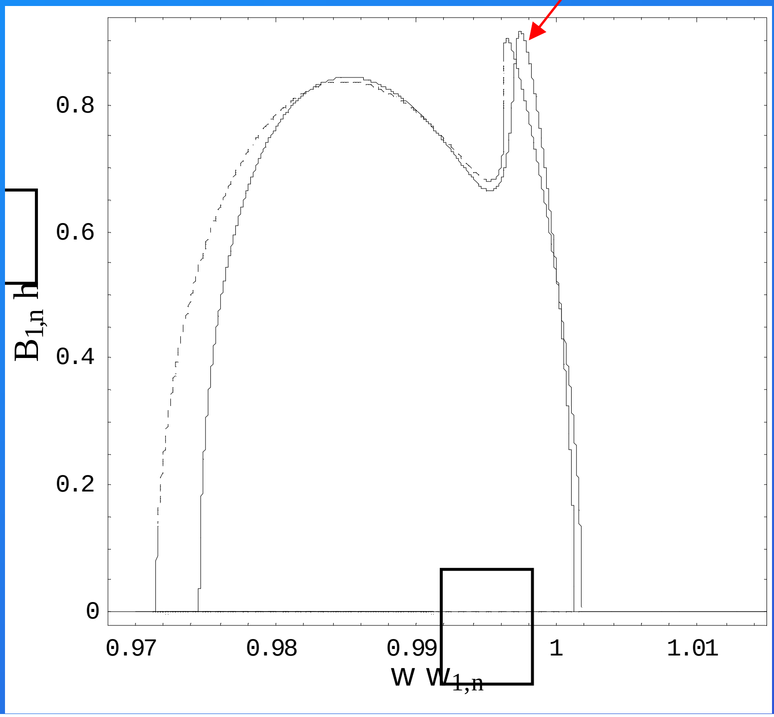
Maximum amplitude of vibration versus excitation frequency

POD model (3 dofs)

Maximum Amplitude of $A_{1,n}(t)$

—— stable conventional Galerkin solutions

— — — unstable conventional Galerkin solutions.



Maximum Amplitude of $B_{1,n}(t)$

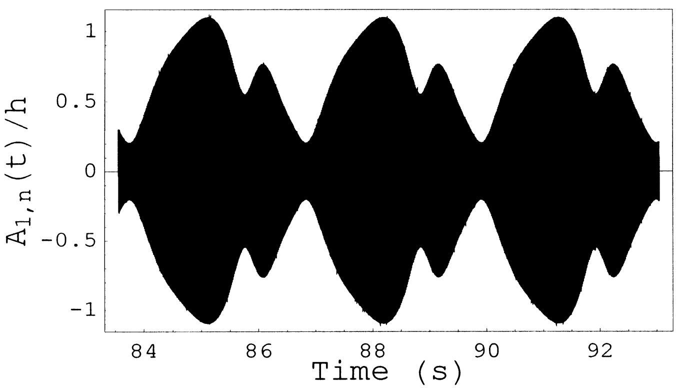
PITCHFORK BIFURCATIONS DETECTED

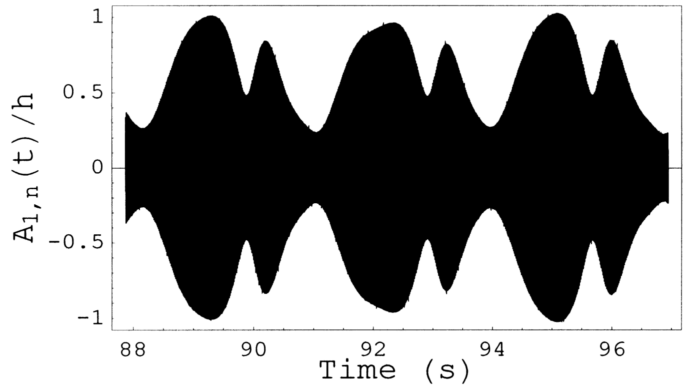
Time response at excitation frequency $\omega/\omega_{1,n}$ = 0.991

case "c"

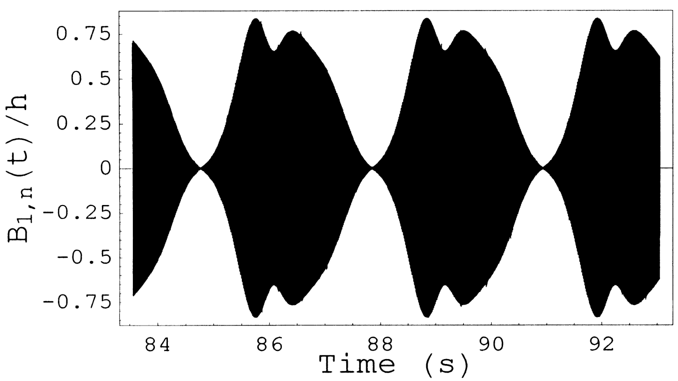
POD model

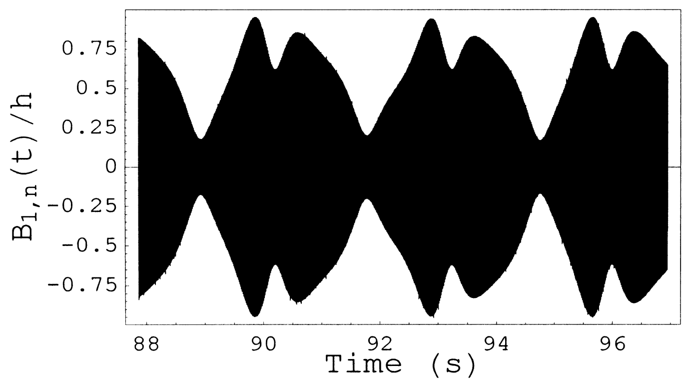
Conventional Galerkin



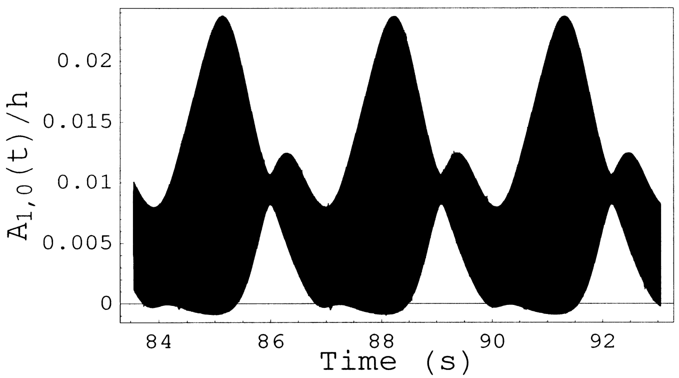


Driven mode $A_{1,n}(t)$





Companion mode $B_{1,n}(t)$

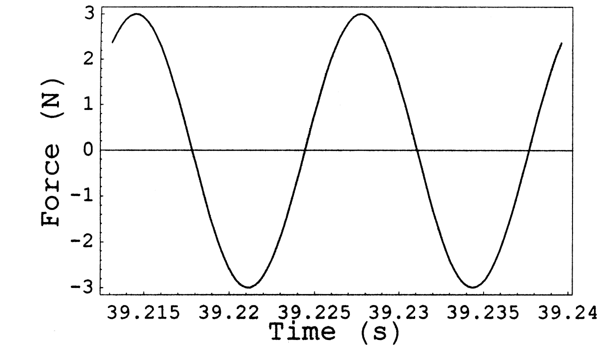


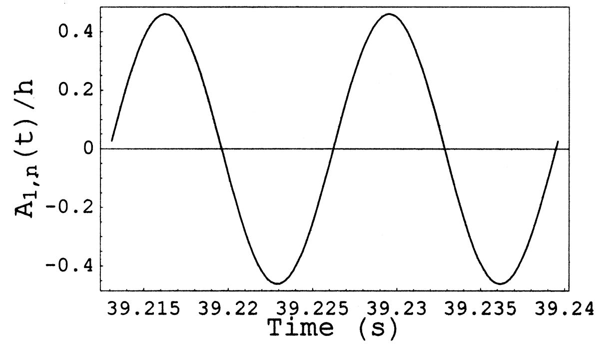
1st axisymmetric mode $A_{1,0}(t)$

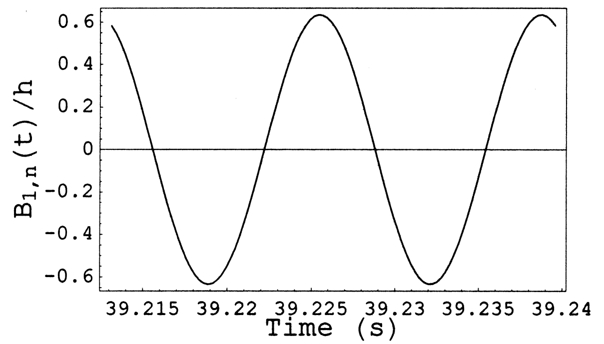
Case "c" : time response

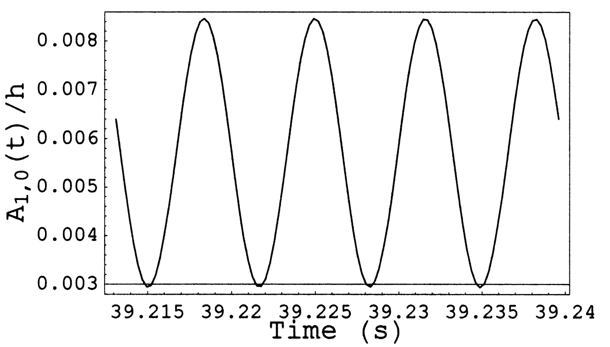
Harmonic force excitation

at $\omega/\omega_{1,n}$ = 0.995





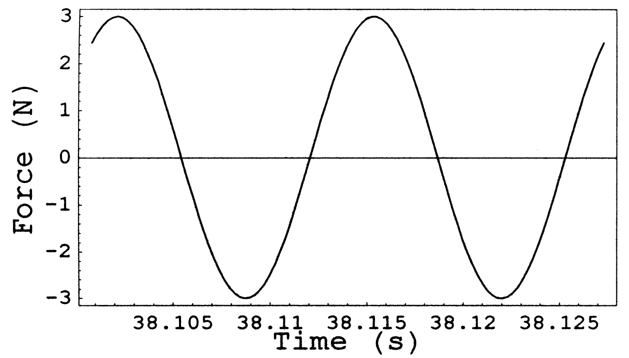


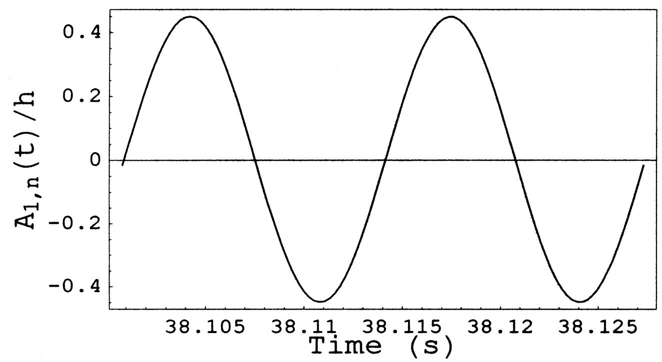


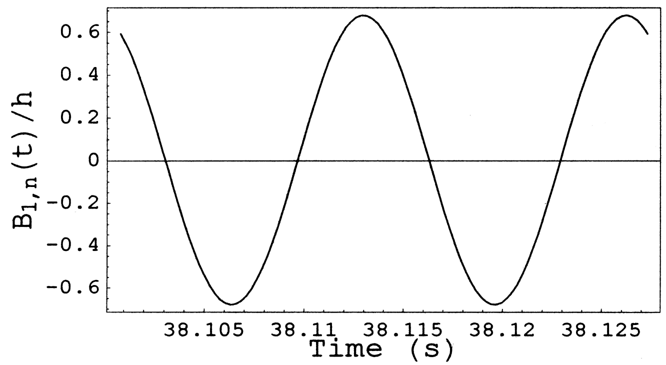
Driven mode $A_{1,n}(t)$

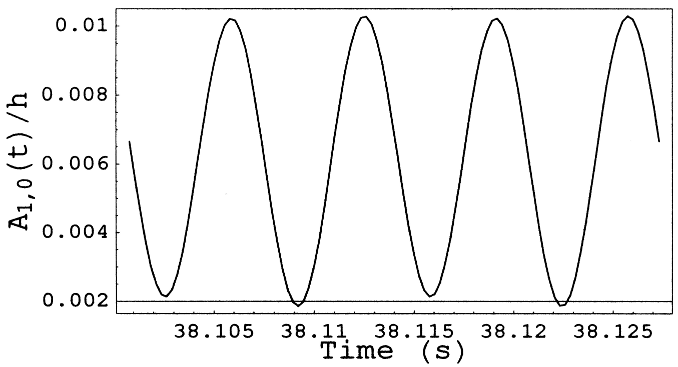
Companion mode $B_{1,n}(t)$

1st axisym. mode $A_{1,0}(t)$









Conclusions

- Dimension of model for nonlinear vibrations of shells has been reduced from 16 to 3 dofs by using the POD method

- An accurate reduced-order model has been built

- The most accurate reduced-order model has been built by using the time response with amplitude modulations

Nonlinear Vibrations of Circular Cylindrical Panels and Rectangular Plates

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Elastic Strain Energy of the Panel

$$\varepsilon_x = \varepsilon_{x,0} + z\,k_x$$

$$\varepsilon_\theta = \varepsilon_{\theta,0} + z\,k_\theta$$

$$\gamma_{x\theta} = \gamma_{x\theta,0} + z\,k_{x\theta}$$

Middle surface strain-displacement (Donnell)

$$\varepsilon_{x,0} = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 + \frac{\partial w}{\partial x}\frac{\partial w_0}{\partial x}$$

$$\varepsilon_{\theta,0} = \frac{\partial v}{R\,\partial\theta} + \frac{w}{R} + \frac{1}{2}\left(\frac{\partial w}{R\,\partial\theta}\right)^2 + \frac{\partial w}{R\,\partial\theta}\frac{\partial w_0}{R\,\partial\theta}$$

$$\gamma_{x\theta,0} = \frac{\partial u}{R\,\partial\theta} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\frac{\partial w}{R\,\partial\theta} + \frac{\partial w}{\partial x}\frac{\partial w_0}{R\,\partial\theta} + \frac{\partial w_0}{\partial x}\frac{\partial w}{R\,\partial\theta}$$

u, v, w = displacements in x, θ and r directions of the mean plane

Changes in the curvature and torsion (Donnell)

$$k_x = -\frac{\partial^2 w}{\partial x^2}$$

$$k_\theta = -\frac{\partial^2 w}{R^2 \partial \theta^2}$$

$$k_{x\theta} = -2\frac{\partial^2 w}{R \partial x \partial \theta}$$

Middle surface strain-displacement (Novozhilov)

$$\varepsilon_{x,0} = \frac{\partial u}{\partial x} + \frac{1}{2}\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial x}\right)^2\right] + \frac{\partial w}{\partial x}\frac{\partial w_0}{\partial x}$$

$$\varepsilon_{\theta,0} = \frac{\partial v}{R\partial \theta} + \frac{w}{R} + \frac{1}{2R^2}\left[\left(\frac{\partial u}{\partial \theta}\right)^2 + \left(\frac{\partial v}{\partial \theta} + w\right)^2 + \left(\frac{\partial w}{\partial \theta} - v\right)^2\right]$$
$$+ \frac{1}{R^2}\left[\frac{\partial w_0}{\partial \theta}\left(\frac{\partial w}{\partial \theta} - v\right) + w_0\left(w + \frac{\partial v}{\partial \theta}\right)\right]$$

$$\gamma_{x\theta,0} = \frac{\partial v}{\partial x} + \frac{\partial u}{R\partial \theta} + \frac{1}{R}\left[\frac{\partial u}{\partial x}\frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial x}\left(\frac{\partial v}{\partial \theta} + w\right) + \frac{\partial w}{\partial x}\left(\frac{\partial w}{\partial \theta} - v\right)\right.$$
$$\left. + \frac{\partial w_0}{\partial x}\left(\frac{\partial w}{\partial \theta} - v\right) + \frac{\partial w}{\partial x}\frac{\partial w_0}{\partial \theta} + \frac{\partial v}{\partial x}w_0\right]$$

Changes in the curvature and torsion (Novozhilov)

$$k_x = -\frac{\partial^2 w}{\partial x^2}$$

$$k_\theta = -\frac{\partial^2 w}{R^2 \partial \theta^2} - \frac{w}{R^2} + \frac{\partial u}{R \partial x} + \frac{\partial v}{R^2 \partial \theta}$$

$$k_{x\theta} = -2\frac{\partial^2 w}{R \partial x \partial \theta} + \frac{\partial v}{R \partial x} - \frac{\partial u}{R^2 \partial \theta}$$

Elastic strain energy

$$U_S = \frac{1}{2}\int_0^{\alpha}\int_0^{L}\int_{-h/2}^{h/2}\left(\sigma_x\varepsilon_x + \sigma_\theta\varepsilon_\theta + \tau_{x\theta}\gamma_{x\theta}\right)\mathrm{d}x\,R(1+z/R)\,\mathrm{d}\theta\,\mathrm{d}z$$

$$\sigma_x = \frac{E}{1-\nu^2}\left(\varepsilon_x + \nu\varepsilon_\theta\right)$$

$$\sigma_\theta = \frac{E}{1-\nu^2}\left(\varepsilon_\theta + \nu\varepsilon_x\right)$$

$$\tau_{x\theta} = \frac{E}{2\left(1+\nu\right)}\gamma_{x\theta}$$

Membrane energy

Bending energy

Membrane and Bending energy coupled

$$U_S = \frac{1}{2}\frac{Eh}{1-\nu^2}\int_0^{\alpha}\int_0^{L}\left(\varepsilon_{x,0}^2 + \varepsilon_{\theta,0}^2 + 2\nu\varepsilon_{x,0}\varepsilon_{\theta,0} + \frac{1-\nu}{2}\gamma_{x\theta,0}\right)\mathrm{d}x\,R\,\mathrm{d}\theta$$

$$+\frac{1}{2}\frac{Eh^3}{12\left(1-\nu^2\right)}\int_0^{\alpha}\int_0^{L}\left(k_x^2 + k_\theta^2 + 2\nu k_x k_\theta + \frac{1-\nu}{2}k_{x\theta}\right)\mathrm{d}x\,R\,\mathrm{d}\theta$$

$$+\frac{1}{2}\frac{Eh^3}{6R\left(1-\nu^2\right)}\int_0^{\alpha}\int_0^{L}\left(\varepsilon_{x,0}k_x + \varepsilon_{\theta,0}k_\theta + \nu\varepsilon_{x,0}k_\theta + \nu\varepsilon_{\theta,0}k_x + \frac{1-\nu}{2}\varepsilon_{x\theta,0}k_{x\theta}\right)\mathrm{d}x\,R\,\mathrm{d}\theta + O(h^4),$$

Mode Expansion, Kinetic Energy and External Loads

$$T_S = \frac{1}{2}\rho_S\, h \int_0^{\alpha}\int_0^{L} (\ddot{u}^2 + \dot{v}^2 + \dot{w}^2)\,\mathrm{d}\,x\,R\,\mathrm{d}\,\theta$$

$$W = \int_0^{\alpha}\int_0^{L} \left(q_x\, u + q_\theta\, v + q_r w\right)\mathrm{d}\,x\,R\,\mathrm{d}\,\theta$$

Single harmonic radial force

$$\begin{cases} q_x = q_\theta = 0 \\ q_r = \tilde{f}\,\delta(R\theta - R\tilde{\theta})\,\delta(x - \tilde{x})\cos(\omega\, t) \end{cases}$$

$$W = \tilde{f}\cos(\omega\, t)\left(w\right)_{x=L/2,\,\theta=\alpha/2}$$

Simply supported boundary condition with one immovable edges

$$u = v = w = w_0 = M_x = \partial^2 w_0 \big/ \partial x^2 = 0\,, \qquad \text{at } x = 0,\ L,$$

$$u = w = w_0 = N_\theta = M_\theta = \partial^2 w_0 \big/ \partial \theta^2 = 0\,, \qquad \text{at } \theta = 0,\ \alpha,$$

Basis of displacements

Panels with point excitation in the middle

only mode with odd number of circumferential and axial half-waves

$$u(x,\theta,t)=\sum_{m=1}^{M}\sum_{n=1}^{N}u_{2m,n}(t)\,\sin(n\pi\theta/\alpha)\,\sin(2m\pi x/L),$$ with odd value of n

$$v(x,\theta,t)=\sum_{m=1}^{M}\sum_{n=1}^{N}v_{m,n}(t)\,\cos(n\pi\theta/\alpha)\,\sin(m\pi x/L),$$ with odd values of m and n

$$w(x,\theta,t)=\sum_{m=1}^{M}\sum_{n=1}^{N}w_{m,n}(t)\,\sin(n\pi\theta/\alpha)\,\sin(m\pi x/L),$$ with odd values of m and n

Geometric imperfections

(Only in radial direction, associated with zero initial stress)

$$w_0(x,\theta) = \sum_{m=1}^{\tilde{M}} \sum_{n=1}^{\tilde{N}} A_{m,n} \sin(n\pi\theta/\alpha)\sin(m\pi x/L),$$

Boundary condition

$$M_x = \frac{E\,h^3}{12(1-\nu^2)}\left(k_x + \nu\, k_\theta\right) = 0$$

$$M_\theta = \frac{E\,h^3}{12(1-\nu^2)}\left(k_\theta + \nu\, k_x\right) = 0$$

$$N_\theta = \frac{E\,h}{1-\nu^2}\left(\varepsilon_{\theta,0} + \nu\,\varepsilon_{x,0}\right) = 0$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T_S}{\partial \dot{q}_j}\right)-\frac{\partial T_S}{\partial q_j}+\frac{\partial U_S}{\partial q_j}=Q_j \qquad j=1,\ldots dofs$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T_S}{\partial \dot{q}_j}\right)=\rho_S\, h\,(\alpha L/4)\, R\, \ddot{q}_j$$

$$\partial T_S \big/ \partial q_j = 0$$

$$\frac{\partial U_S}{\partial q_j}=\sum_{k=1}^{dofs} q_k\, f_k+\sum_{i,k=1}^{dofs} q_i\, q_k\, f_{i,k}+\sum_{i,k,l=1}^{dofs} q_i\, q_k\, q_l\, f_{i,k,l}$$

Numerical and Experimental Results for the limit case $R \rightarrow \infty$: Rectangular plate

Software: AUTO for bifurcation analysis and continuation of nonlinear ODE

Benchmark case

(Chu & Herrmann , Ribeiro, Kadiri & Benamar)

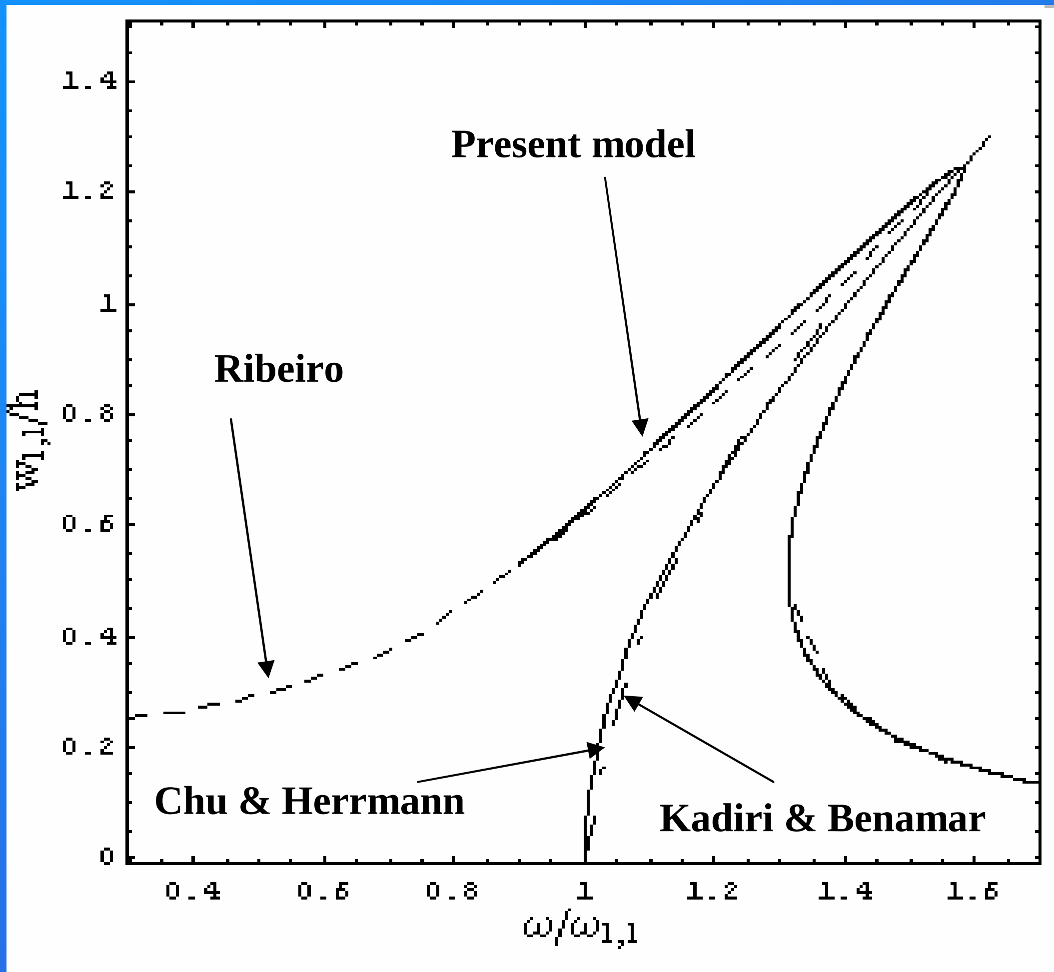
h = 1 mm
L = 300 mm
l = 300 mm
Density = 2778 Kg/m^3
Young Module = 70 Gpa
Poisson Ratio = 0.3

Boundary Conditions:

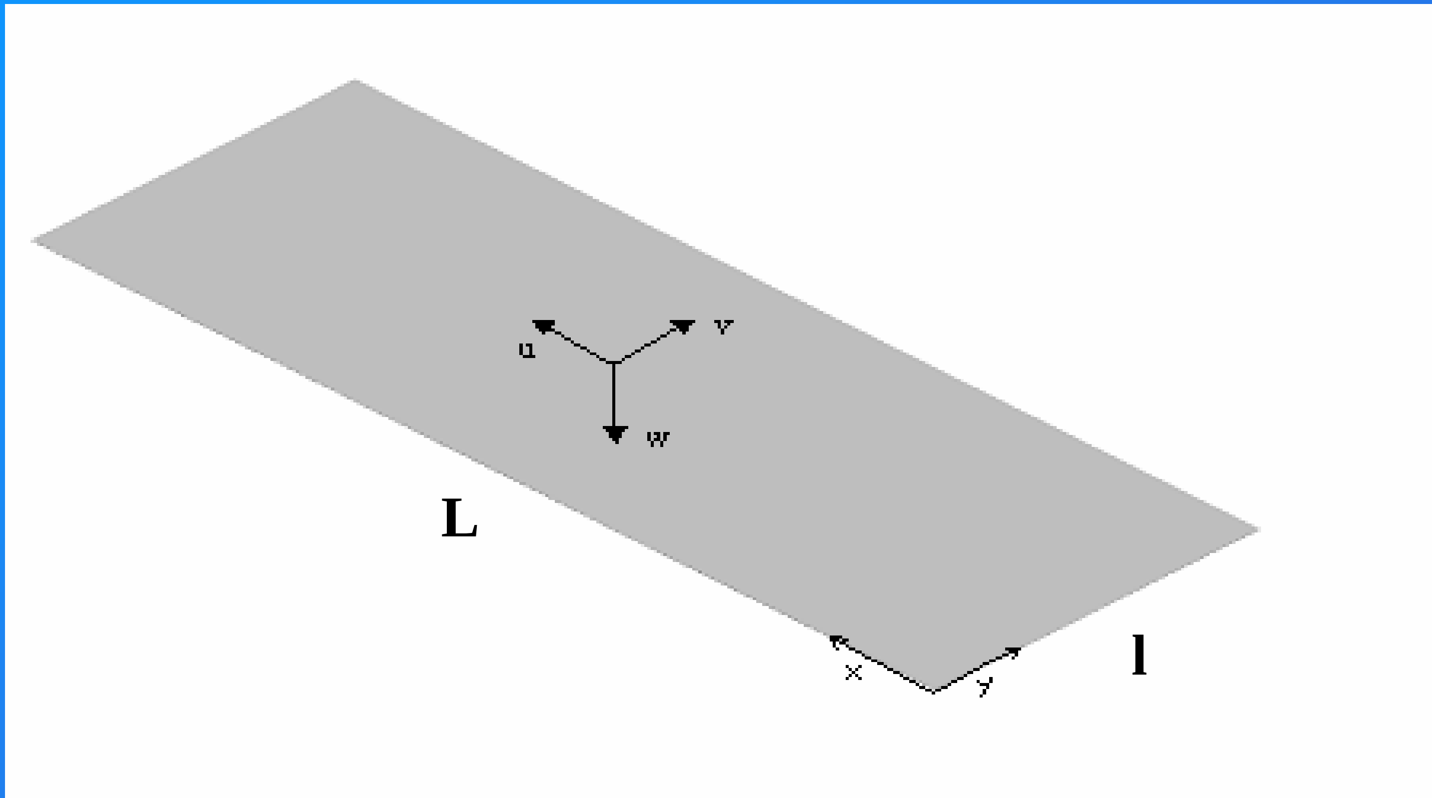
u = v = w = 0 at x = 0, L and y = 0, l

(zero displacements at the edges)

Validation of the present model (16 dofs)

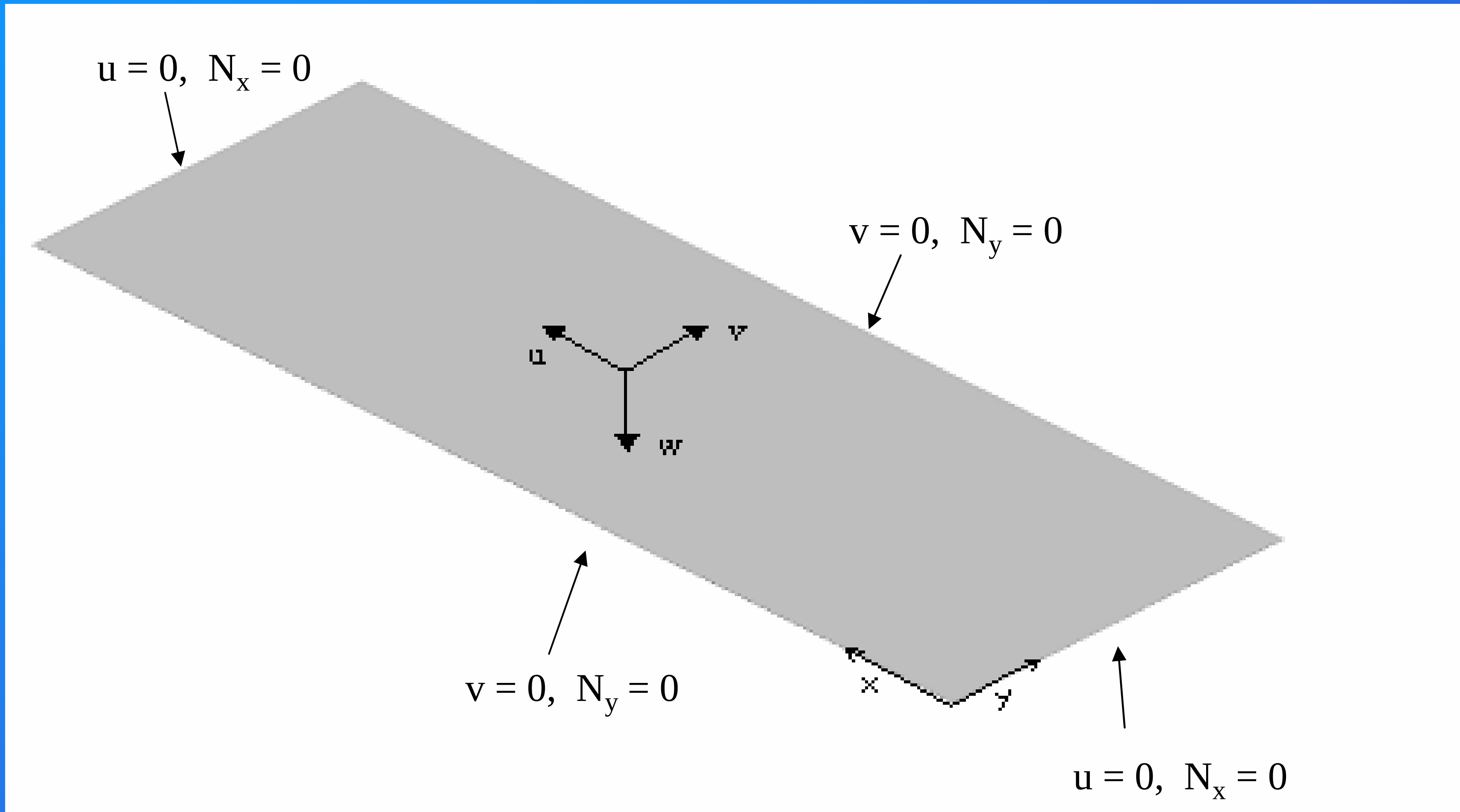


The experimental aluminum plate



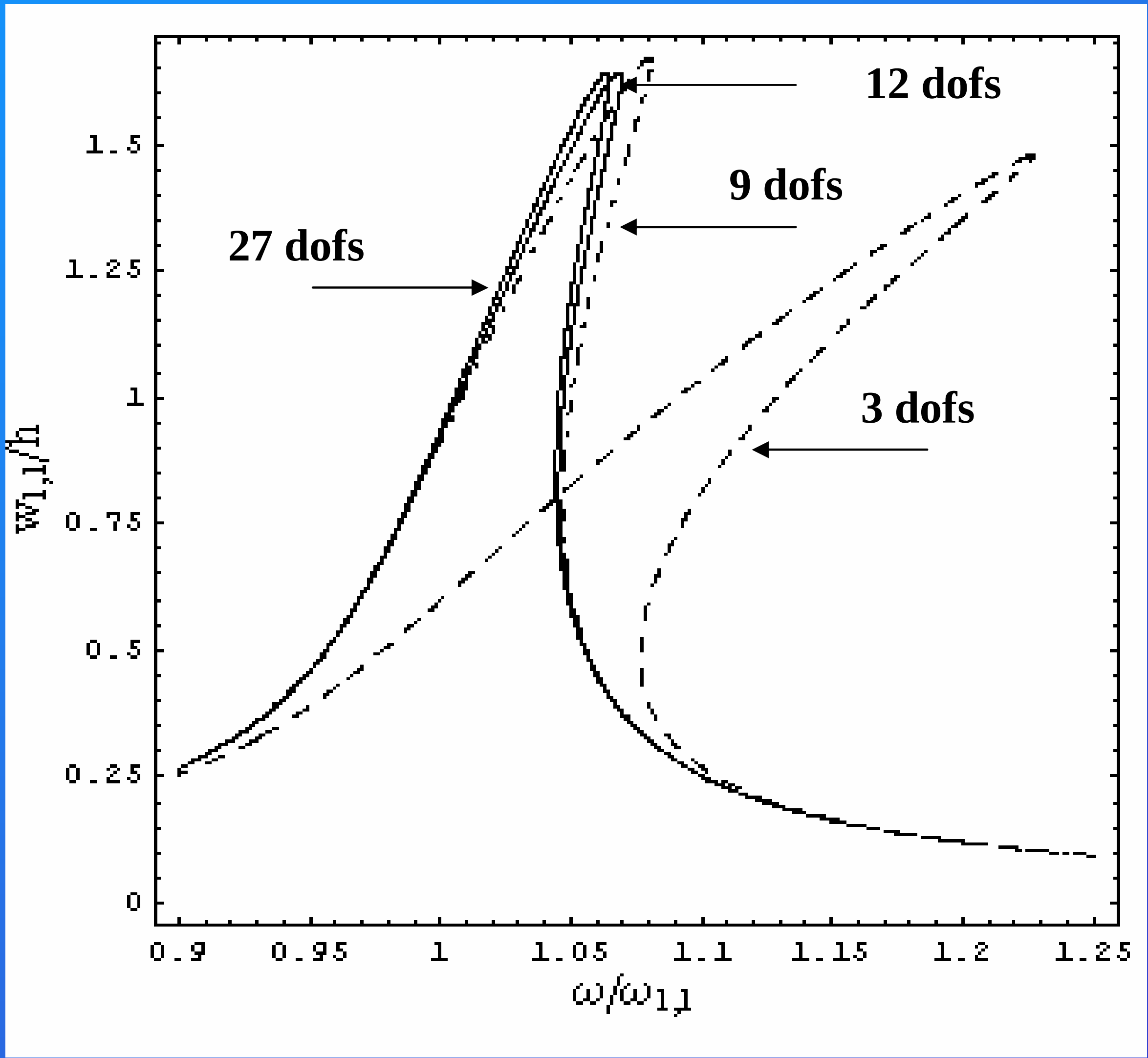
h (thickness) = 0.3 mm
L (x lenght) = 515 mm
l (y lenght) = 184 mm
Density = 2700 Kg/m^3
Young module = 69 GPa
Poisson ratio = 0,33

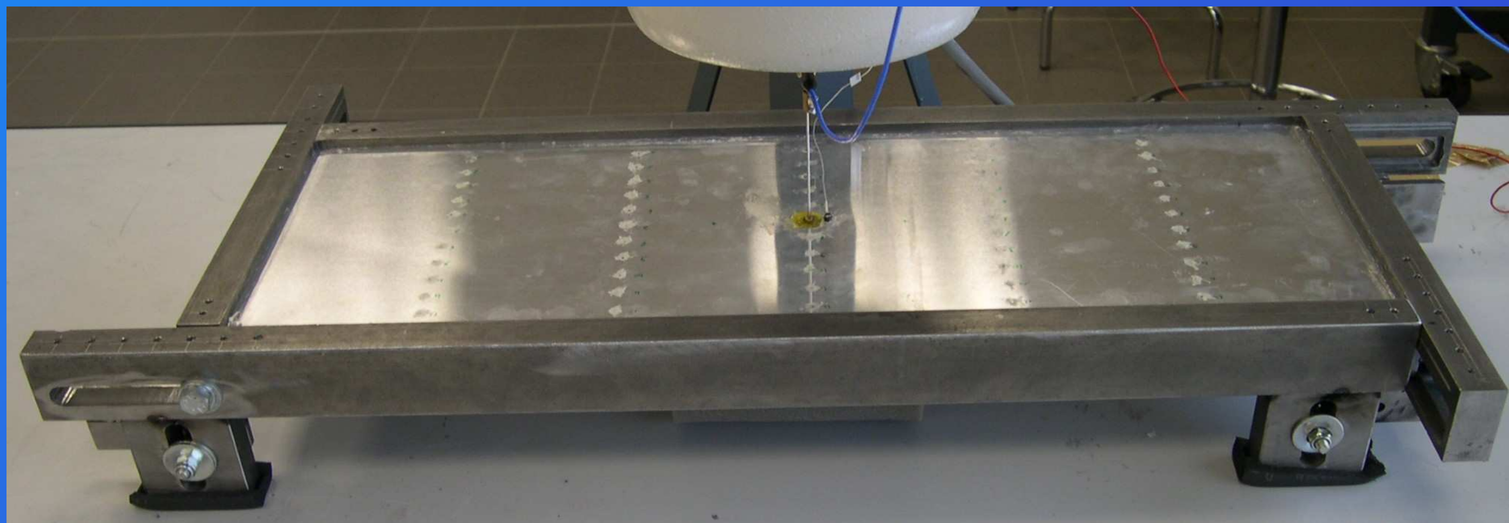
Boundary conditions used to simulate the experimental plate



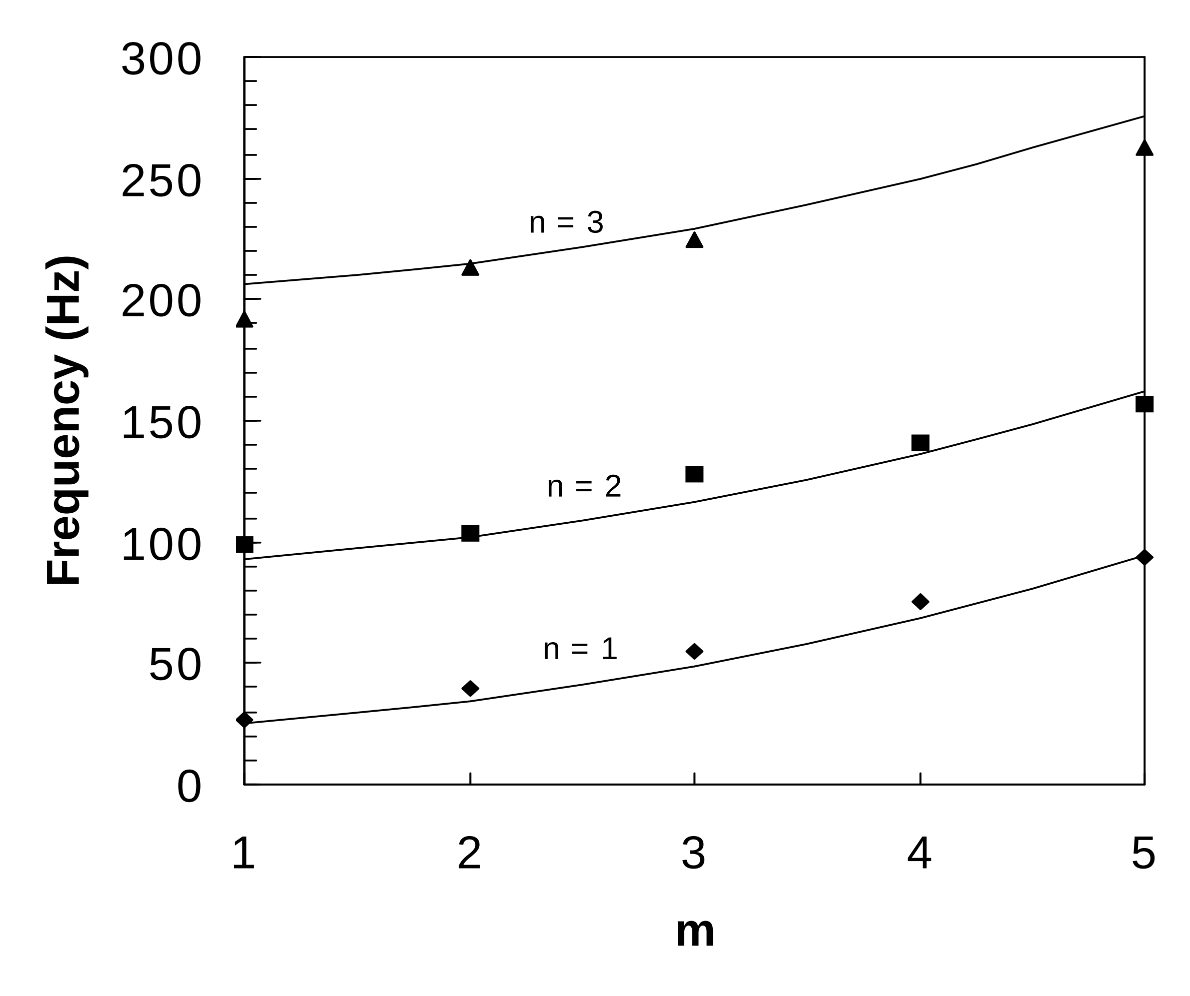
Convergence of foundamental mode

m = n =1 (half-waves in x and y directions)





Comparison between theory and experiments (linear results)

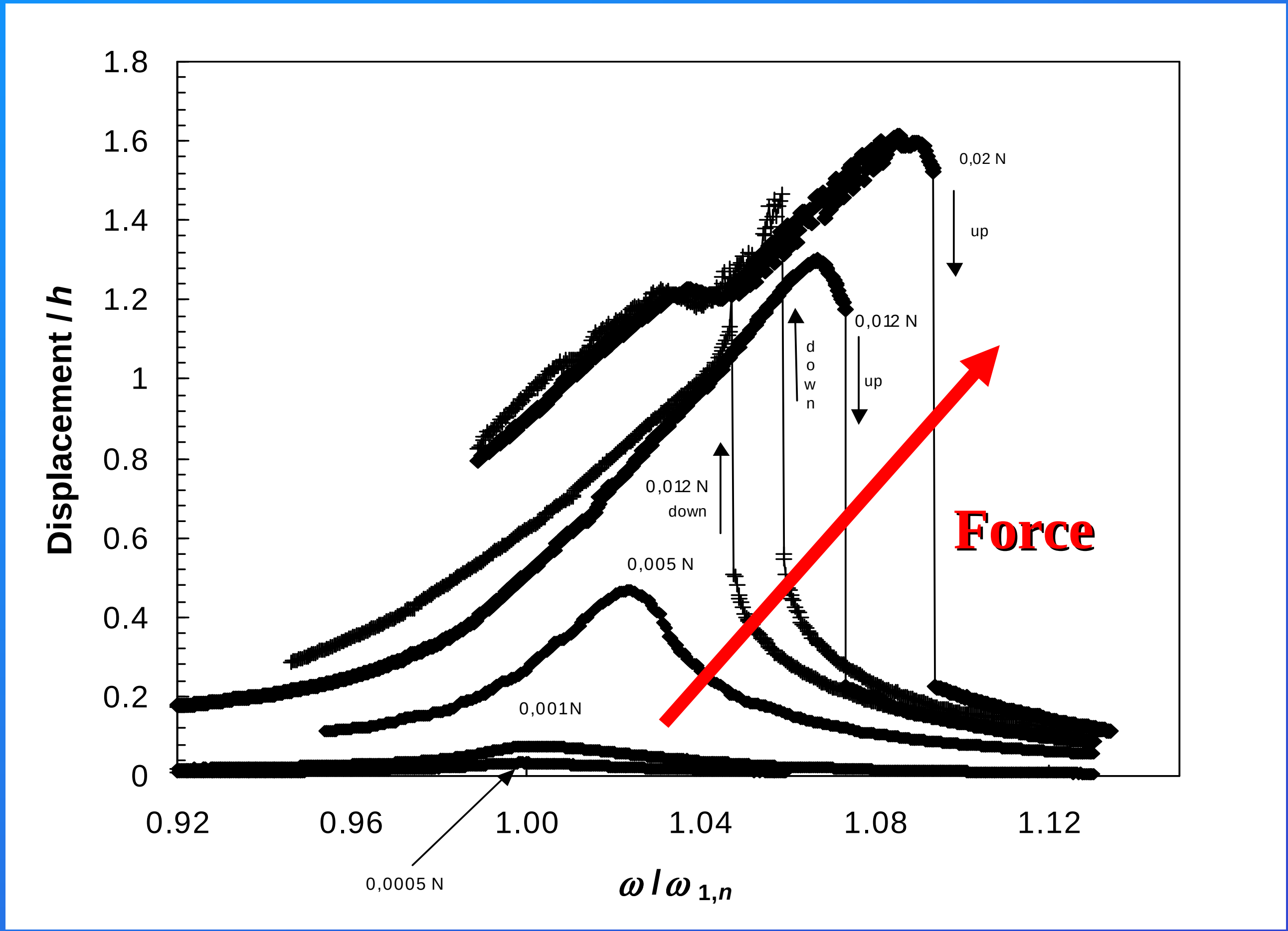


n = half-waves in y direction

m = half-waves in x direction

NONLINEAR RESPONSE – Step-sine test

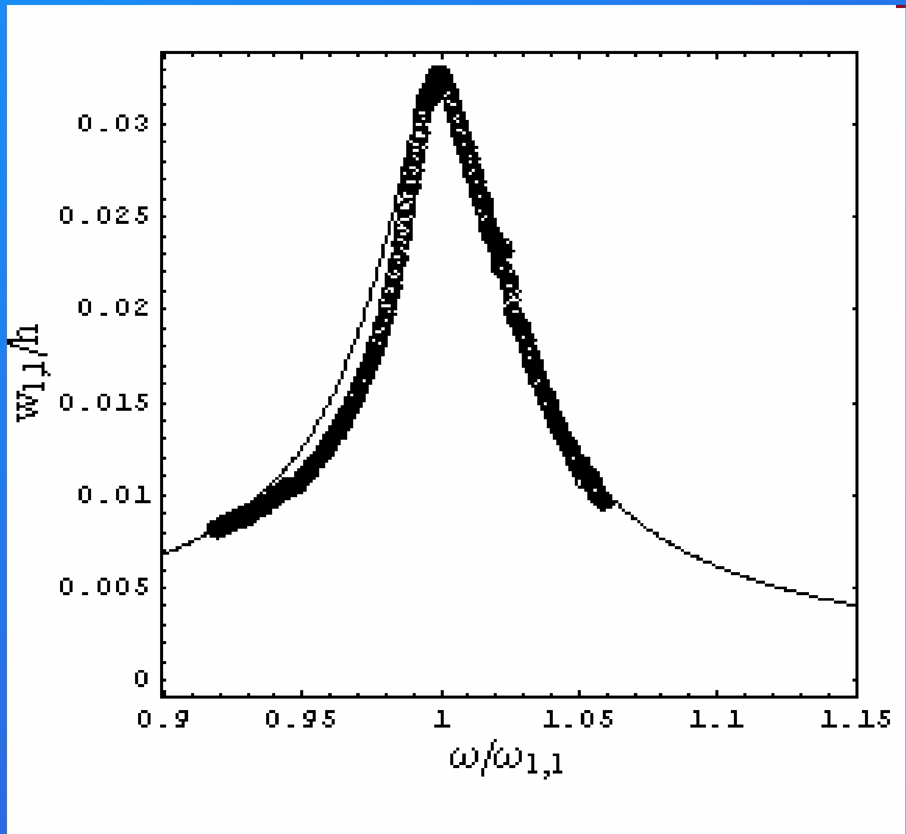
(foundamental mode n=1, m=1)



HARDENING TYPE NONLINEARITY

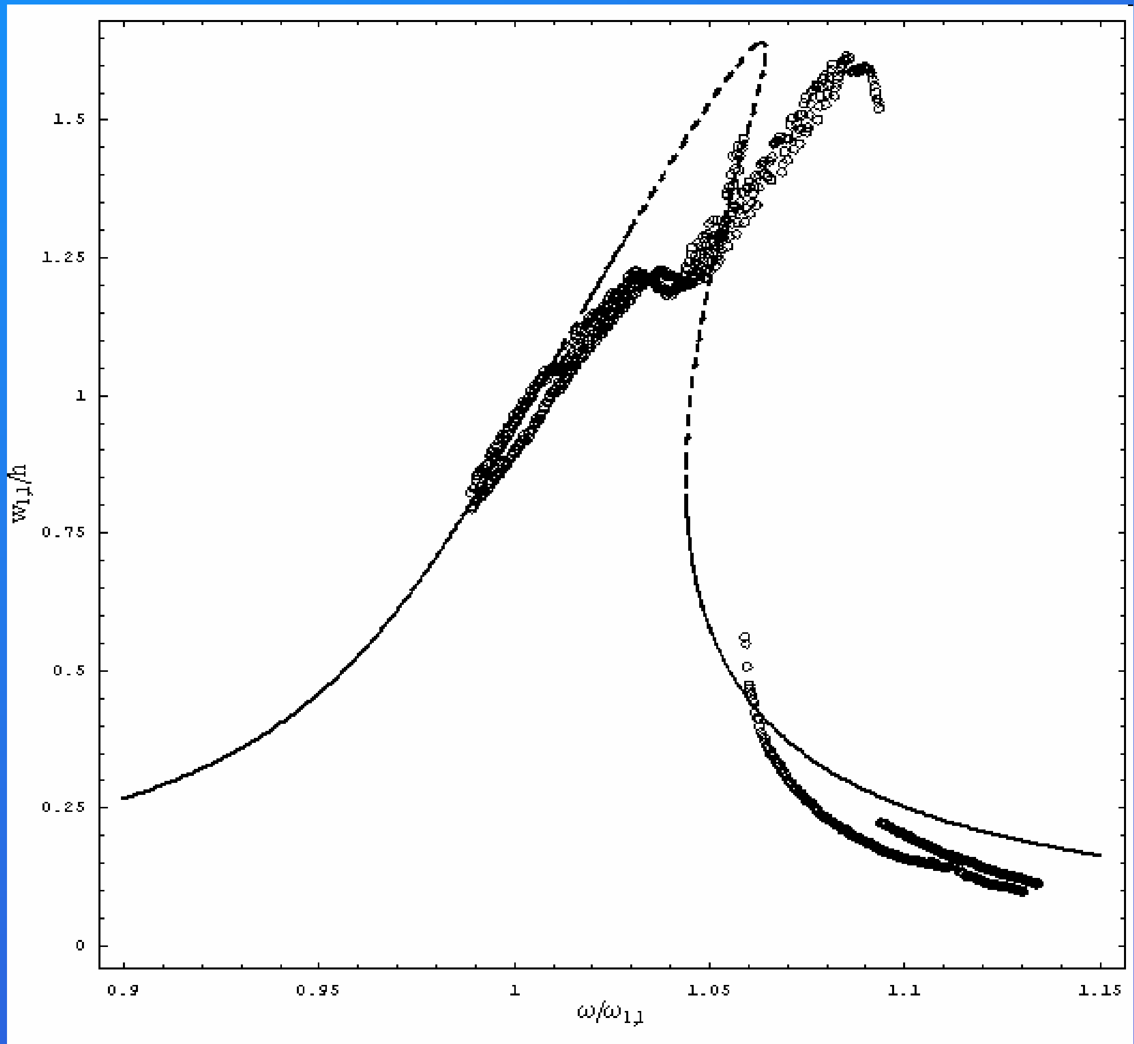
Comparison between experimental and numerical results

(Excitation force = 0.0005 N, linear case)



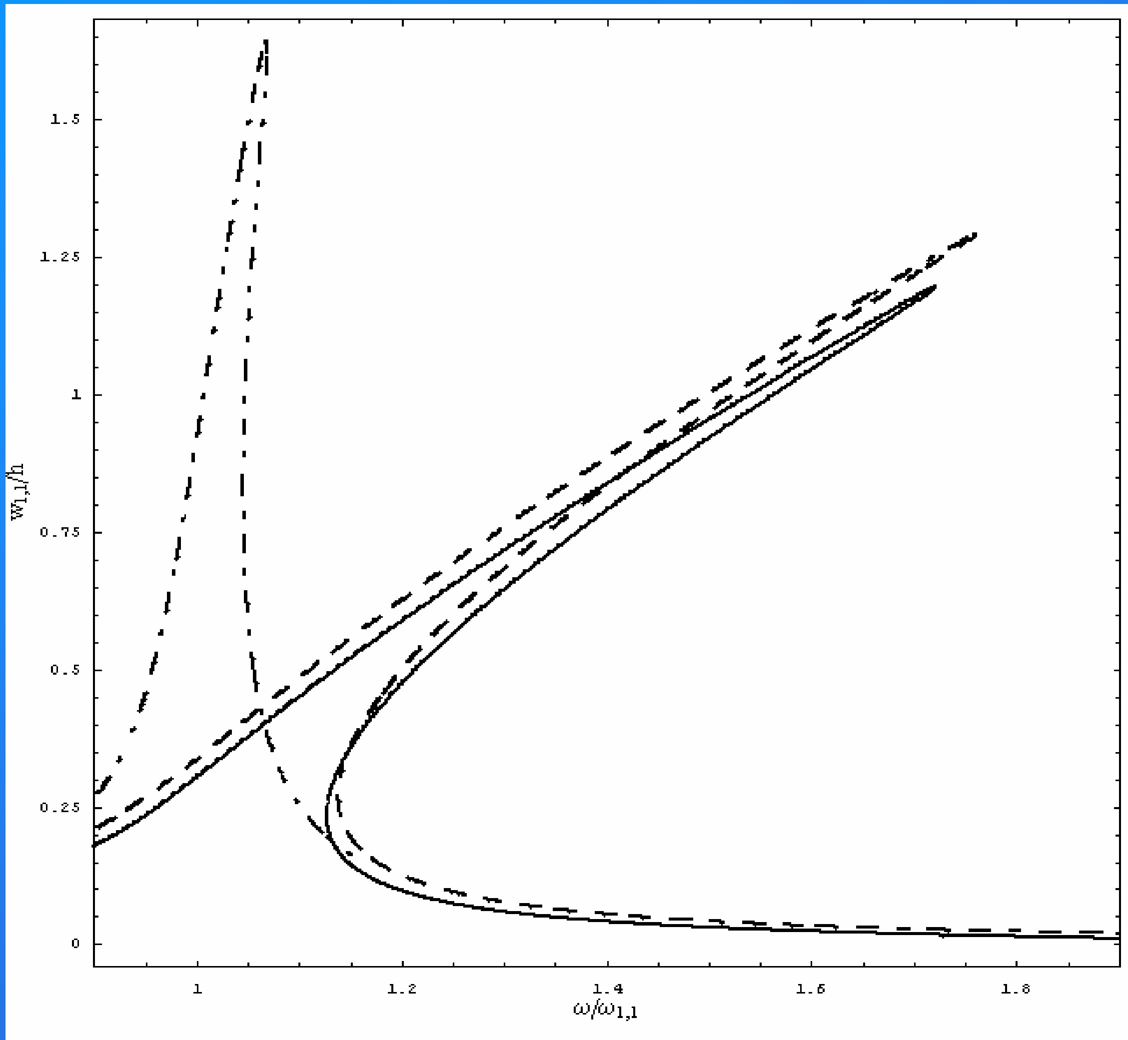
Comparison of theory and experiments

(Excitation force = 0.02 N, nonlinear case)



Hardening results

Different boundary conditions



— · — ·— : free displacements and rot. (experimental case)

— — — : zero displacements

—— : zero displacements and rotations

Conclusions

- Refined model for nonlinear vibrations of circular cylindrical panels and flat plates
- Convergence of solution
- Good agreement with experimental results
- Model suitable for different boundary conditions

Nonlinear Supersonic Flutter of Imperfect Circular Cylindrical Shells

LINEARIZED PISTON THEORY

$$p_1 = -\frac{\gamma\, p_\infty\, M^2}{(M^2-1)^{1/2}}\left[\frac{\partial (w+w_0)}{\partial x} + \frac{1}{M\, a_\infty}\left(\frac{M^2-2}{M^2-1}\right)\frac{\partial w}{\partial t} - \frac{w+w_0}{2\,(M^2-1)^{1/2}\, R}\right]$$

THIRD ORDER PISTON THEORY

$$p_3 = p_1 - \gamma\, p_\infty \left\{ \frac{\gamma+1}{4}\left[M\,\frac{\partial (w+w_0)}{\partial x} + \frac{1}{a_\infty}\frac{\partial w}{\partial t}\right]^2 + \frac{\gamma+1}{12}\left[M\,\frac{\partial (w+w_0)}{\partial x} + \frac{1}{a_\infty}\frac{\partial w}{\partial t}\right]^3 \right\}$$

M = Mach number

γ= adiabatic esponent

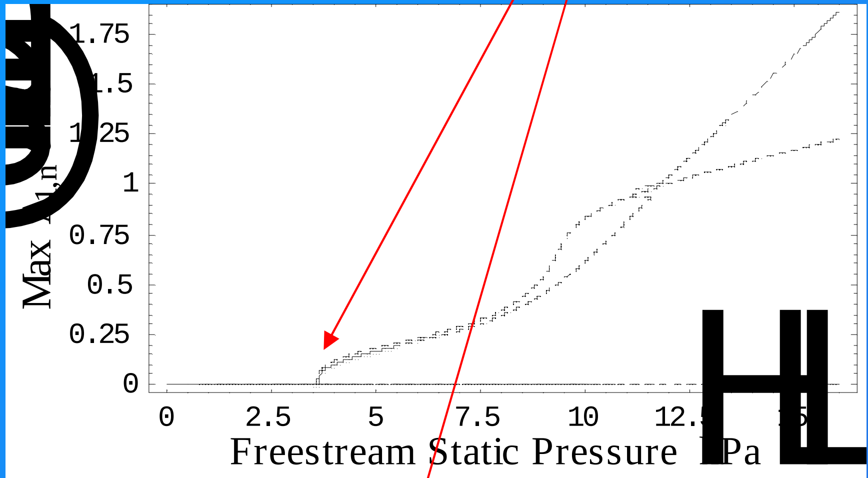
$M > 1.6$

a_∞ = free-stream speed of sound

p_∞ = free-stream static pressure

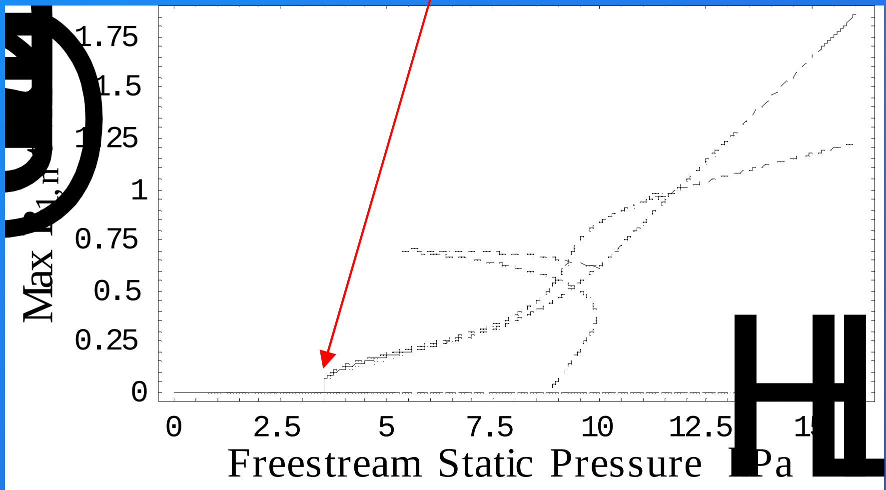
22 DOF Model with Geometric Imperfections

Onset of Flutter (Hopf bifurcation)



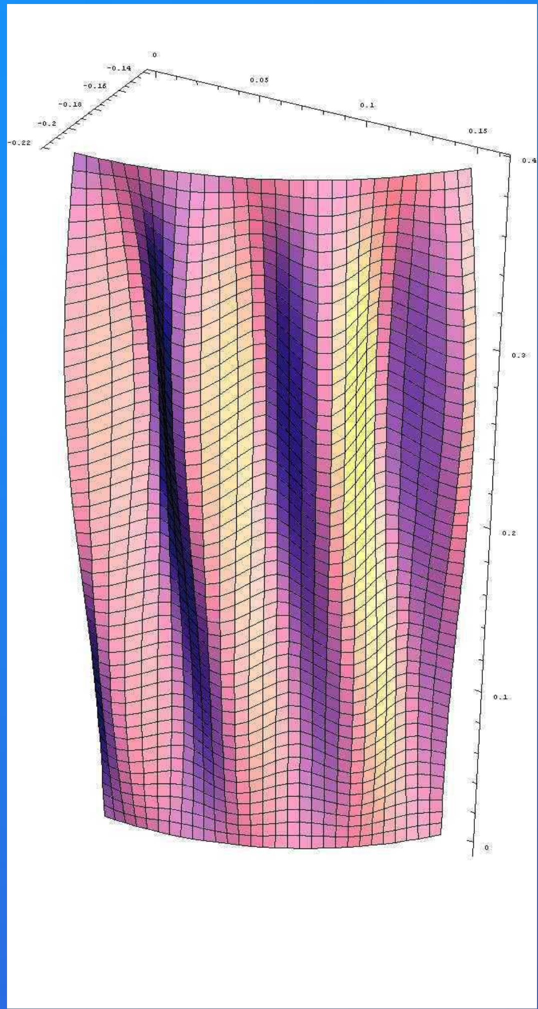
Maximum Amplitude of $A_{1,n}(t)$

Travelling Wave Flutter



Maximum Amplitude of $B_{1,n}(t)$

PARTIAL REPRESENTATION OF THE FLUTTERING SHELL

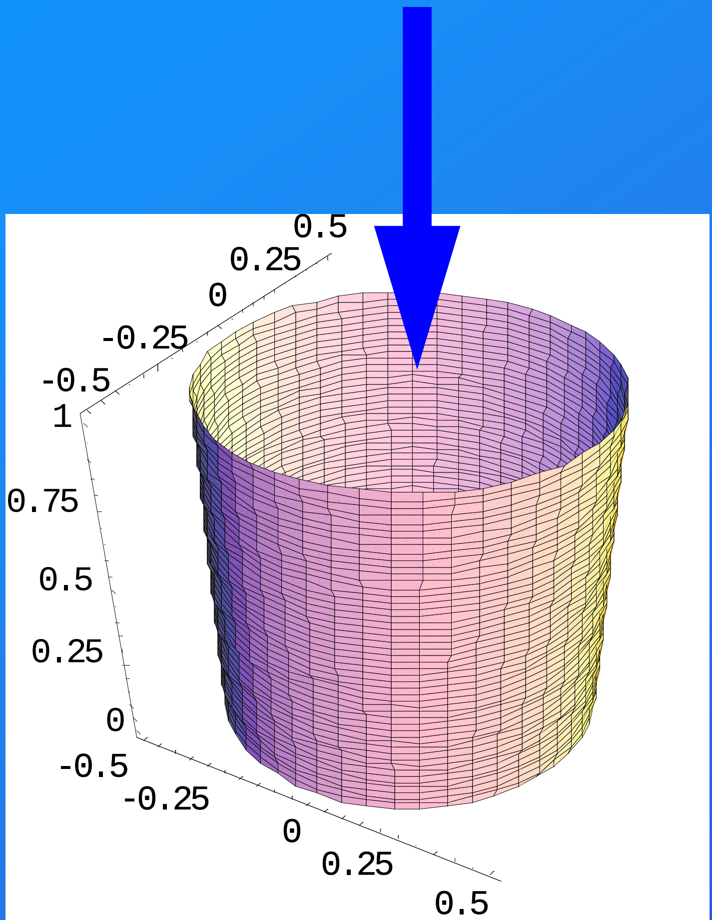


$n = 23$

$p_\infty = 4500 \text{ Pa}$

displacement augmented 1000 times

Nonlinear stability of circular cylindrical shells conveying flowing liquid



Fluid-Structure Interaction: Potential Flow

mean flow velocity

$$\mathbf{v} = -\nabla \Psi \qquad \Psi = -U\,x + \Phi$$

Unsteady potential

Fluid velocity

Steady potential

$$\nabla^2 \Phi = \frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial r^2} + \frac{1}{r}\frac{\partial \Phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \Phi}{\partial \theta^2} = 0$$

Laplace equation

$$p = \rho_F \left(\frac{\partial \Phi}{\partial t} + U \frac{\partial \Phi}{\partial x} \right)$$

Fluid pressure

$$\left(\frac{\partial \Phi}{\partial r} \right)_{r=R} = \left(\frac{\partial w}{\partial t} + U \frac{\partial w}{\partial x} \right)$$

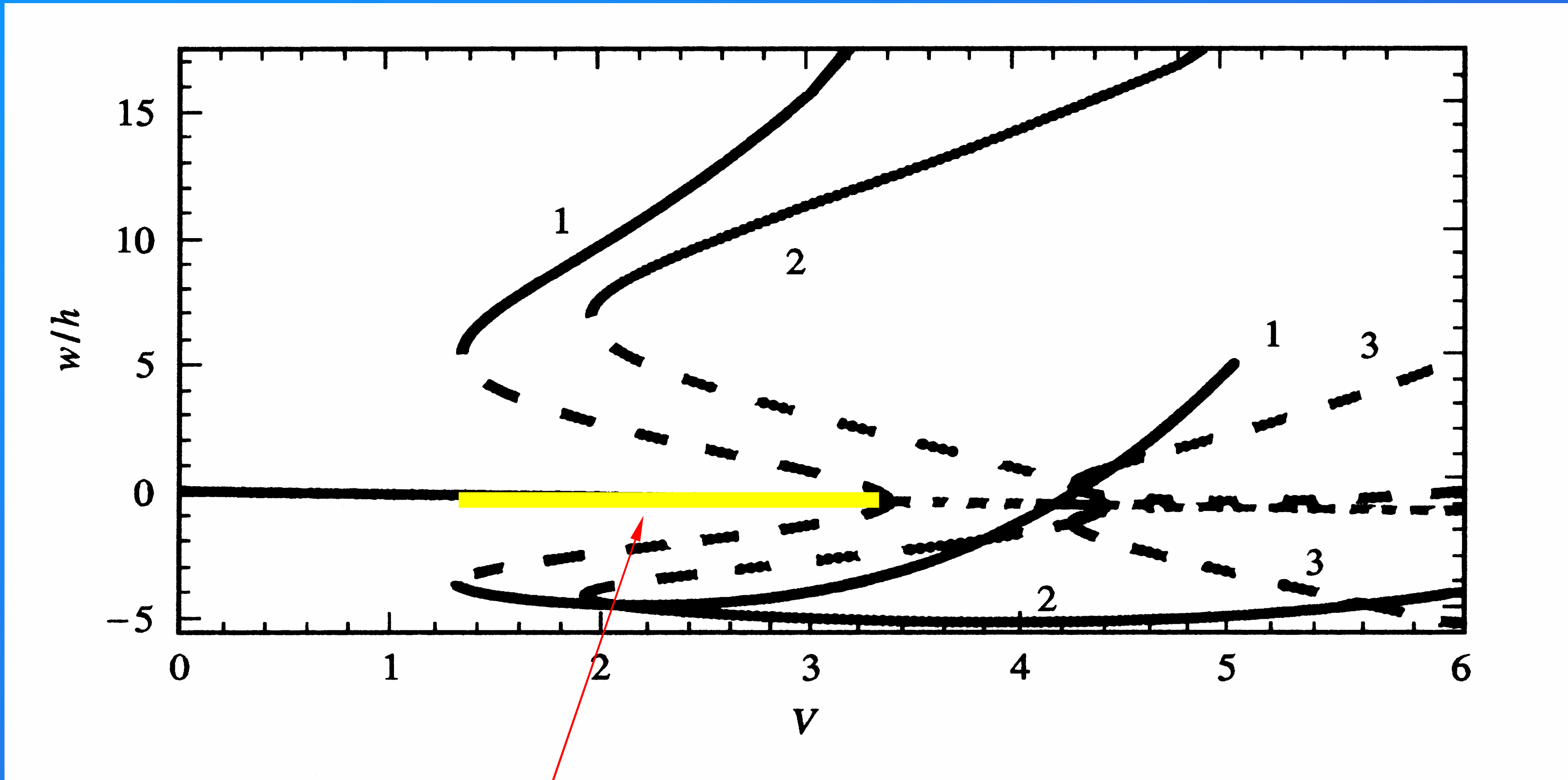
Boundary condition at shell-fluid interface

(shell periodically supported)

$$p_{r=R} = \rho_F \frac{L}{m\pi} \frac{\mathrm{I}_n(m\pi R / L)}{\mathrm{I}'_n(m\pi R / L)} \left(\frac{\partial}{\partial t} + U \frac{\partial}{\partial x} \right)^2 w$$

Pressure at interface

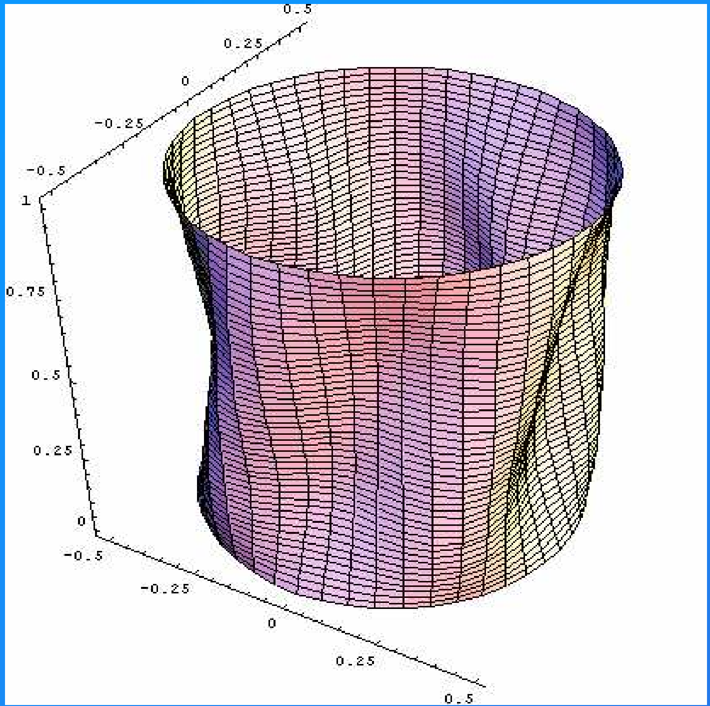
Divergence (Buckling) of Shell by Flow

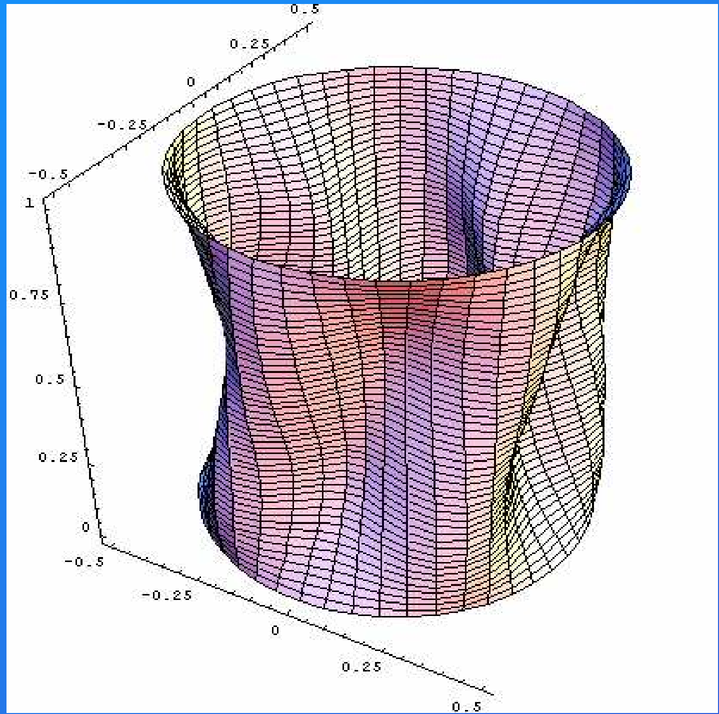


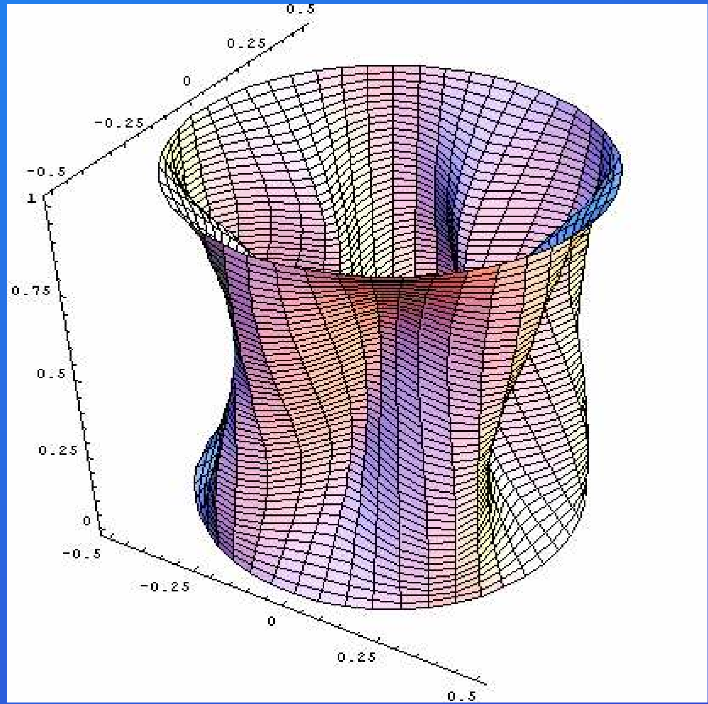
Jumps under perturbation

Buckled Shell (Divergence Instability due to flow)

Coupled-mode bifurcation



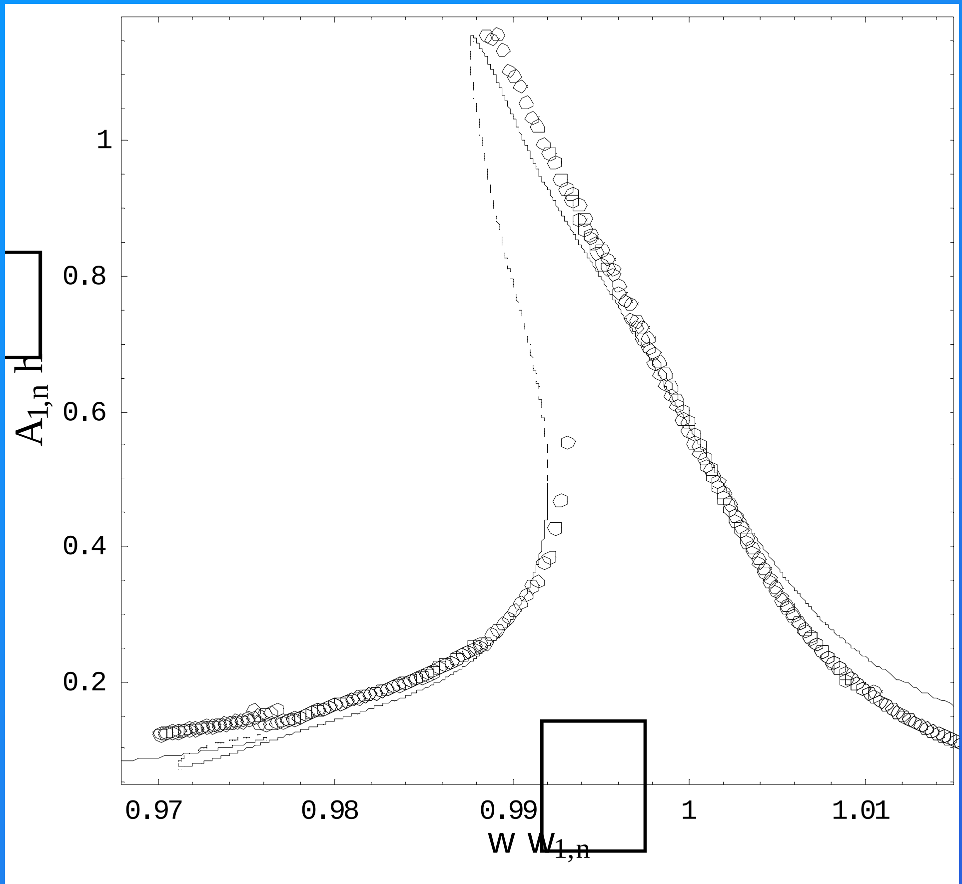




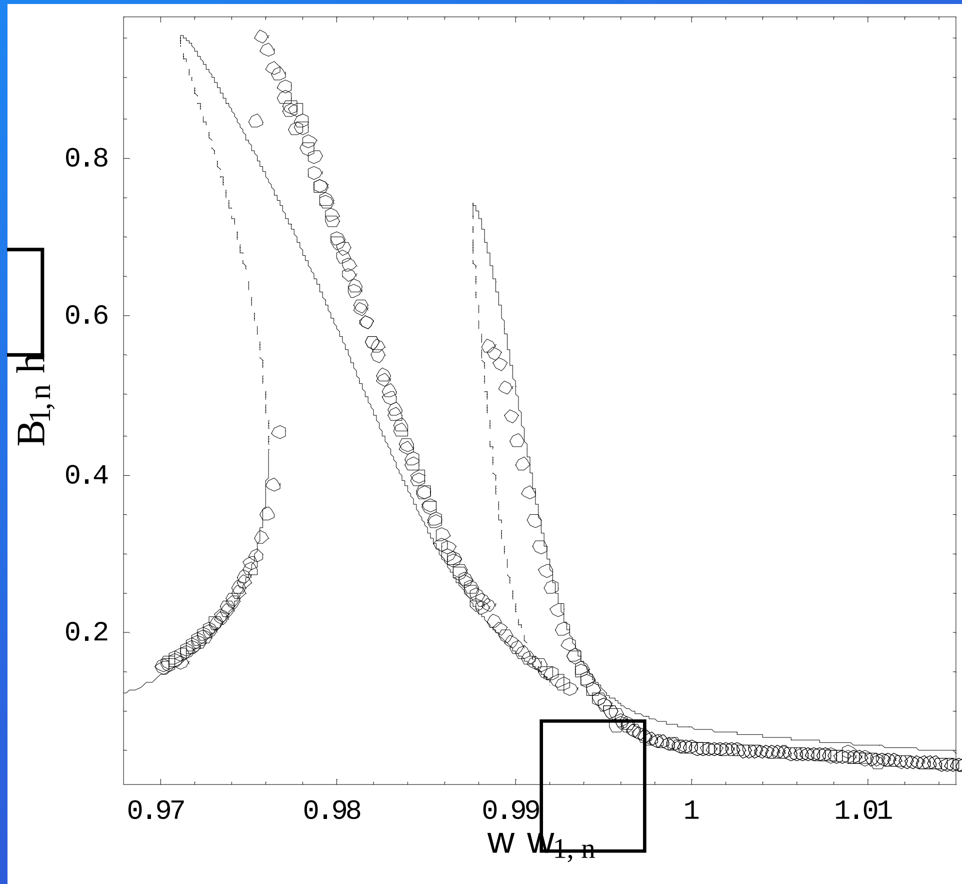
$V = 2$ ($< V_{CRIT.\ LIN.}$) **$V = 3$ ($< V_{CRIT.\ LIN.}$)** **$V = 4$**

Mode $n = 5$

LARGE-AMPLITUDE FORCED VIBRATIONS OF A WATER-FILLED CIRCULAR CYLINDRICAL SHELL WITH IMPERFECTIONS: COMPARISON OF THEORY AND EXPERIMENTS

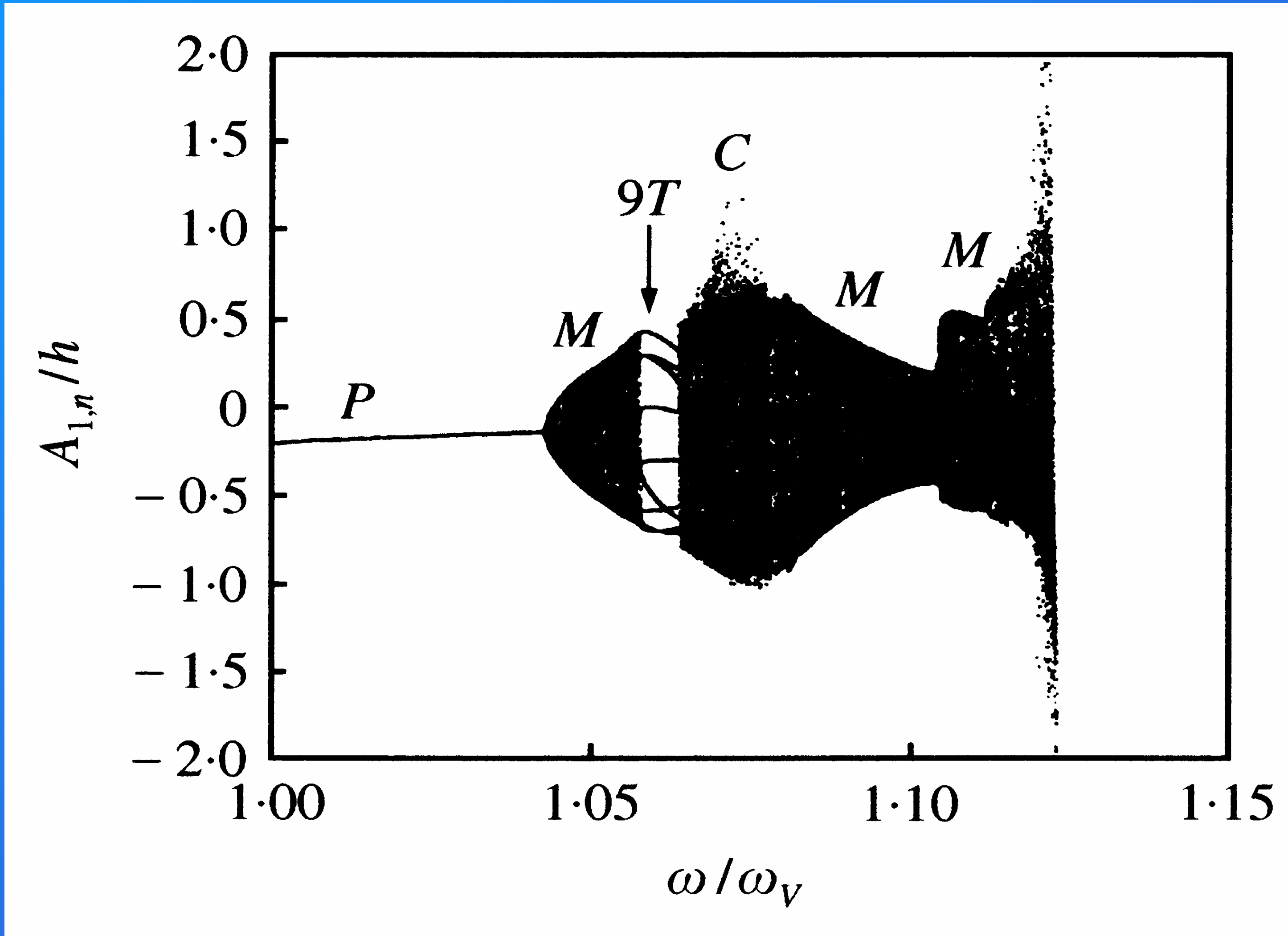


Maximum Amplitude of $A_{1,n}(t)$



Maximum Amplitude of $B_{1,n}(t)$

BIFURCATION DIAGRAM: SHELL WITH FLOWING FLOW UNDER HARMONIC EXCITATION (CHANGING FREQUENCY)



Conclusions

- It is necessary to study complex nonlinear dynamics and fluid-structure interaction by using small dimension models

- Model are based on global discretization (hyper-elements)

- Fluid-structure interaction can be described with simplified but accurate models in several applications without CFD

- Reduced-order models (POD) can be useful and accurate