

Approches sans maillage en mise en forme de matériaux

F. Chinesta



<http://lms-web.paris.ensam.fr/lms/>

Les besoins identifiés pour le domaine des procédés de production

Quels sont les blocages ?

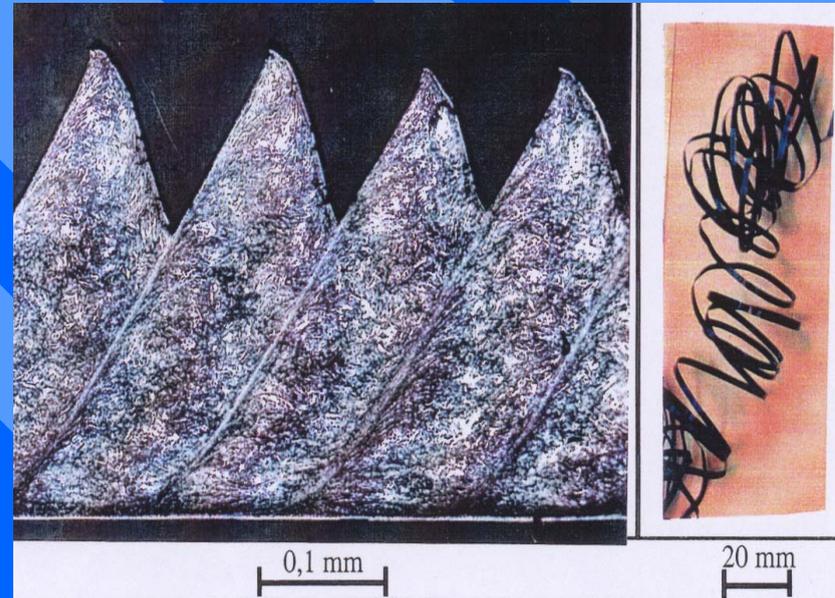
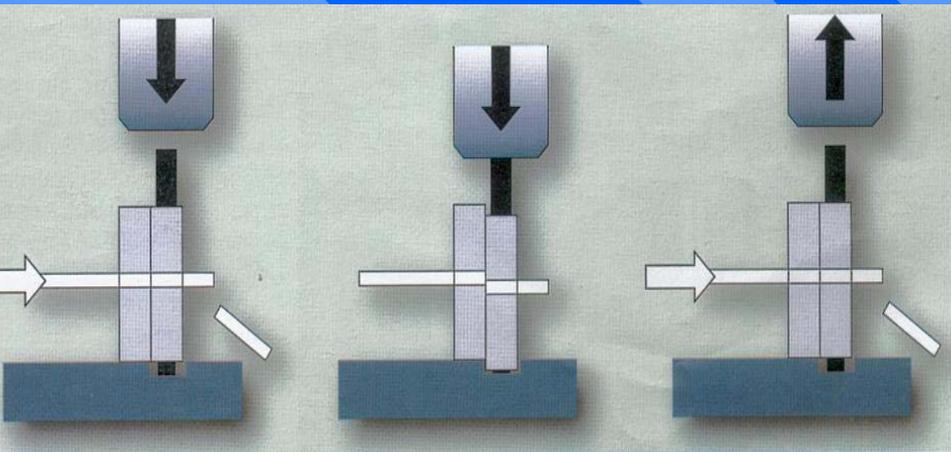
Scientifiques :

- **Données physiques (accès, identification en situation extrême),**
- **Interdisciplinarité,**
- **Algorithmique pour les situations extrêmes des simulations.**

Modélisation :

- Méthodes numériques de représentation des grandes évolutions spatio-temporelles, et de traitement des problèmes couplés de grande taille, et fortement non linéaires,
- Dialogue calcul – essais,
- Modélisation multiéchelle et multiphysique,
- Interfaces, ...

Modélisation numérique...



Choix d'un **outil numérique** adapté au traitement des **grandes déformations** et des **discontinuités**

Méthodes des éléments finis, X-FEM ?

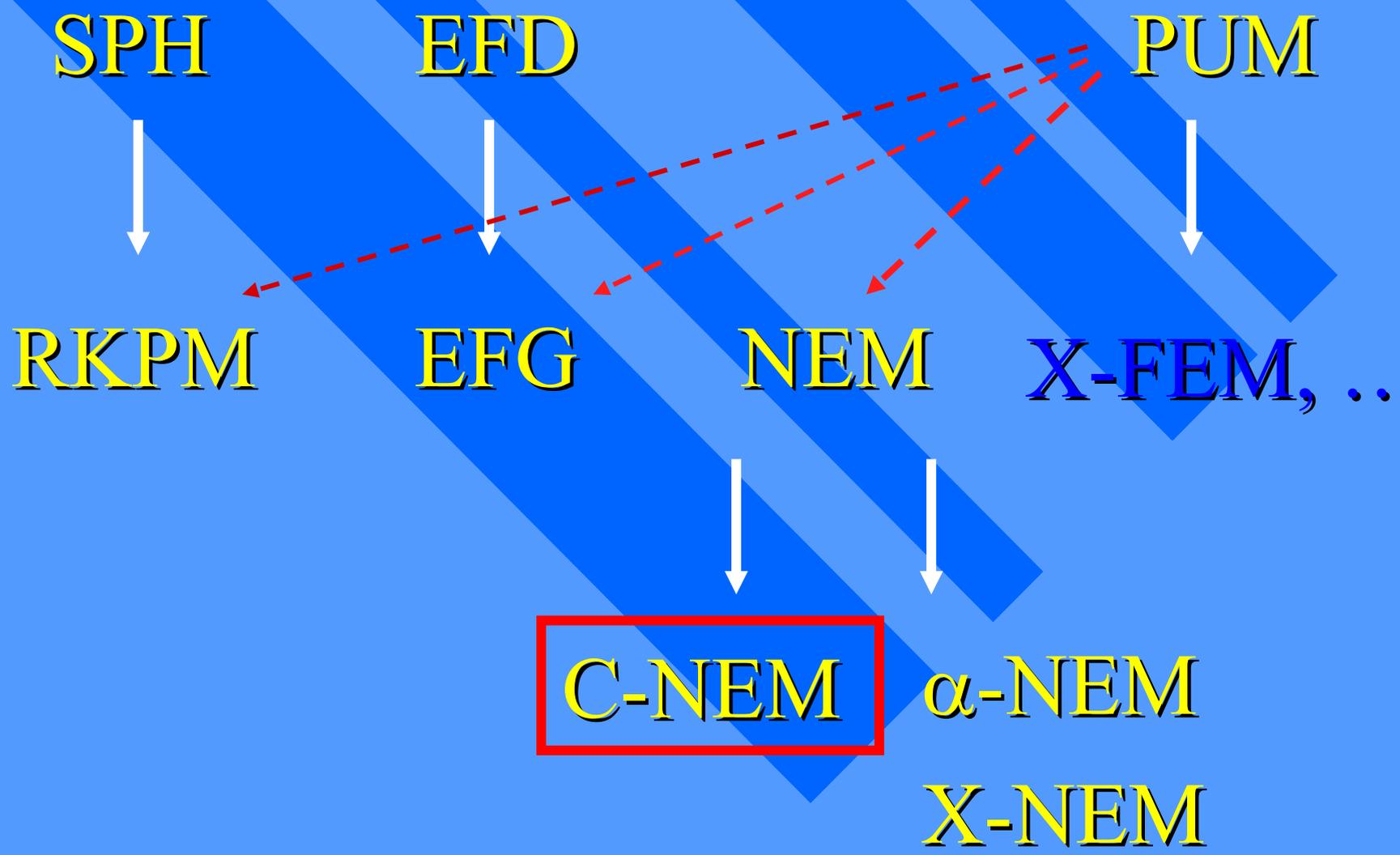
Remaillage du aux grandes déformations

Méthodes meshless ?

RKPM, EFG ? Imposition des conditions aux limites délicate

NEM ? Problèmes pour les géométries complexes (non convexes)

Un peu d'histoire ... la mode meshless !!



S.P.H. (I)

$$u^h(\underline{x}) = \int_{\Omega} w(\underline{x} - \underline{y}, h) u(\underline{y}) d\Omega$$

$$w(\underline{x} - \underline{y}, h) > 0 \quad \Omega_s \subset \Omega$$

$$w(\underline{x} - \underline{y}, h) = 0 \quad \Omega - \Omega_s$$

$$\int_{\Omega} w(\underline{x} - \underline{y}, h) d\Omega = 1$$

$$\frac{dw(s, h)}{ds} < 0 \quad s = \|\underline{x} - \underline{y}\|, \quad s > 0$$

$$w(s, h) \xrightarrow{h \rightarrow 0} \delta(s)$$

$$u^h(\underline{x}) = \sum_l w(\underline{x} - \underline{x}_l) u_l \Omega_l = \sum_l N_l u_l$$

$$u_l \neq u^h(\underline{x}_l)$$

Technique bien adaptée aux simulations de type dynamique moléculaire

S.P.H. (II)

Consistance :

$$\int_{\Omega} w(x-y, h) 1 d\Omega = 1$$

$$\int_{\Omega} w(x-y, h) y d\Omega = x$$



$$\int_{\Omega} w(x-y, h) x d\Omega = x$$



$$\int_{\Omega} w(x-y, h) (x-y) d\Omega = 0$$

La formulation discrète ne vérifie pas la consistance linéaire

R.K.P.M. (I)

$$\left\{ \begin{array}{l} \int_{\Omega} \Phi_a(x-s) ds = 1 \\ \int_{\Omega} \Phi_a(x-s) s ds = x \\ \vdots \\ \int_{\Omega} \Phi_a(x-s) s^N ds = x^N \\ \int_{\Omega} \Phi_a(x-s) f^e(s) ds = f^e(x) \end{array} \right.$$

$$\left\{ \begin{array}{l} \int_{\Omega} C(x;x-s) \Phi_a(x-s) ds = 1 \\ \int_{\Omega} C(x;x-s) \Phi_a(x-s) s ds = x \\ \vdots \\ \int_{\Omega} C(x;x-s) \Phi_a(x-s) s^N ds = x^N \\ \int_{\Omega} C(x;x-s) \Phi_a(x-s) f^e(s) ds = f^e(x) \end{array} \right.$$

$$C(x;x-s) = H^T(x-s)b(x)$$

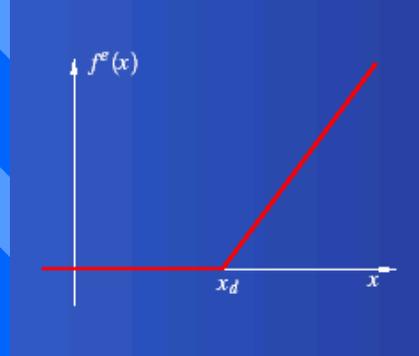
$$H^T(x-s) = [1, x-s, \dots, (x-s)^N, f^e(x-s)]$$

$$\underbrace{\left[\int_{\Omega} R(s) H^T(x-s) \Phi_a(x-s) ds \right]}_{M(x) \text{ Moment Matrix}} b(x) = \underbrace{R(x)}_{\text{Reproducing Vector}}$$

$$f^R(x) \cong \sum_{I=1}^{NP} \Psi_I(x) f_I$$

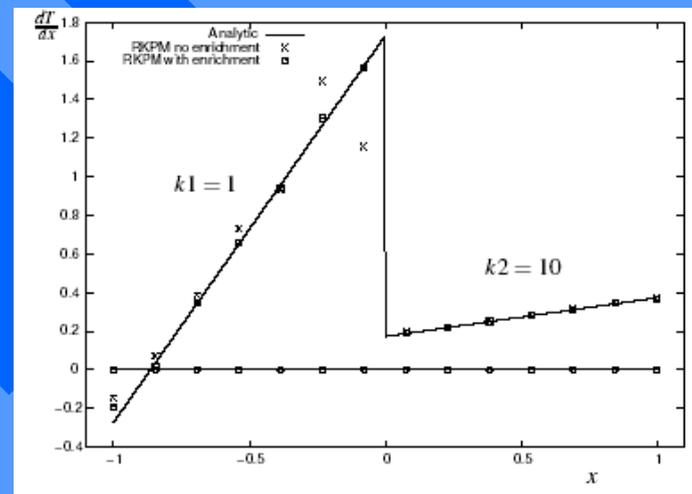
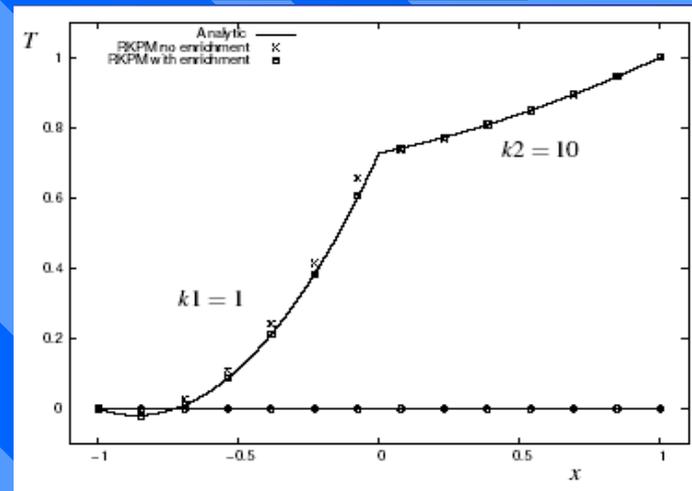
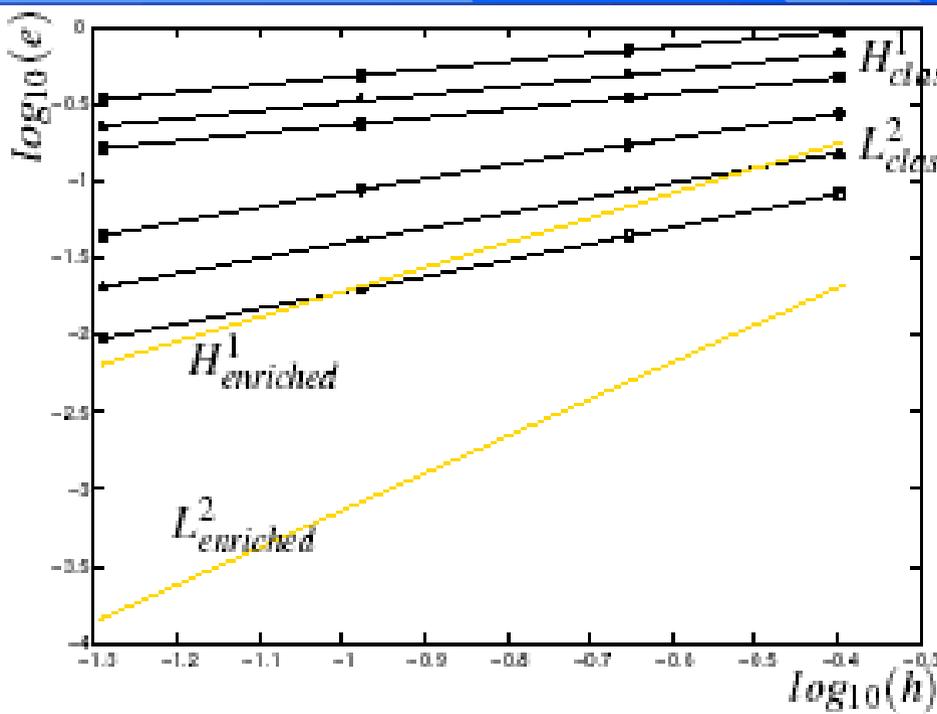
R.K.P.M. (II)

$$f^e(x) = \Theta(x, y) H_0(\Theta(x, y))$$



$$T_1 \quad \boxed{k_1} \quad \boxed{k_2} \quad T_2$$

$$\frac{d}{dx} \left(k(x) \frac{dT}{dx} \right) = g \quad \text{for } -1 \leq x \leq 1$$



Moindres Carrés Mobiles

$$u^h(\underline{x}) = \sum_{j=1}^{m} p_j(\underline{x}) a_j(\underline{x}) = \underline{p}^T(\underline{x}) \underline{a}(\underline{x})$$

p.e. $\underline{p}^T(\underline{x}) = (1, x, y)$
 $m=3$

E.F.D. / E.F.G.

$$\underline{\underline{A}}(\underline{x}) \underline{a}(\underline{x}) = \underline{\underline{B}} \underline{u}$$

$$\underline{\underline{A}}(\underline{x}) = \sum_I^n w(\underline{x} - \underline{x}_I) \underline{p}(\underline{x}_I) \underline{p}^T(\underline{x}_I)$$

$$\underline{\underline{B}}(\underline{x}) = \left[w(\underline{x} - \underline{x}_1) \underline{p}(\underline{x}_1) \cdots w(\underline{x} - \underline{x}_n) \underline{p}(\underline{x}_n) \right]$$

$$\underline{\underline{u}}^T = [u_1 \cdots u_n]$$

$$J(\underline{a}) = \sum_{I=1}^{I=n} w(\underline{x} - \underline{x}_I) \left[\underline{p}^T(\underline{x}_I) \underline{a}(\underline{x}) - u_I \right]$$

$$u_l \neq u^h(\underline{x}_l)$$

$$u^h(\underline{x}) = \sum_I^n \sum_{j=1}^m p_j(\underline{x}) \left(\underline{\underline{A}}^{-1}(\underline{x}) \underline{\underline{B}}(\underline{x}) \right)_{jI} u_I$$

$$u^h(\underline{x}) = \sum_l N_l u_l$$

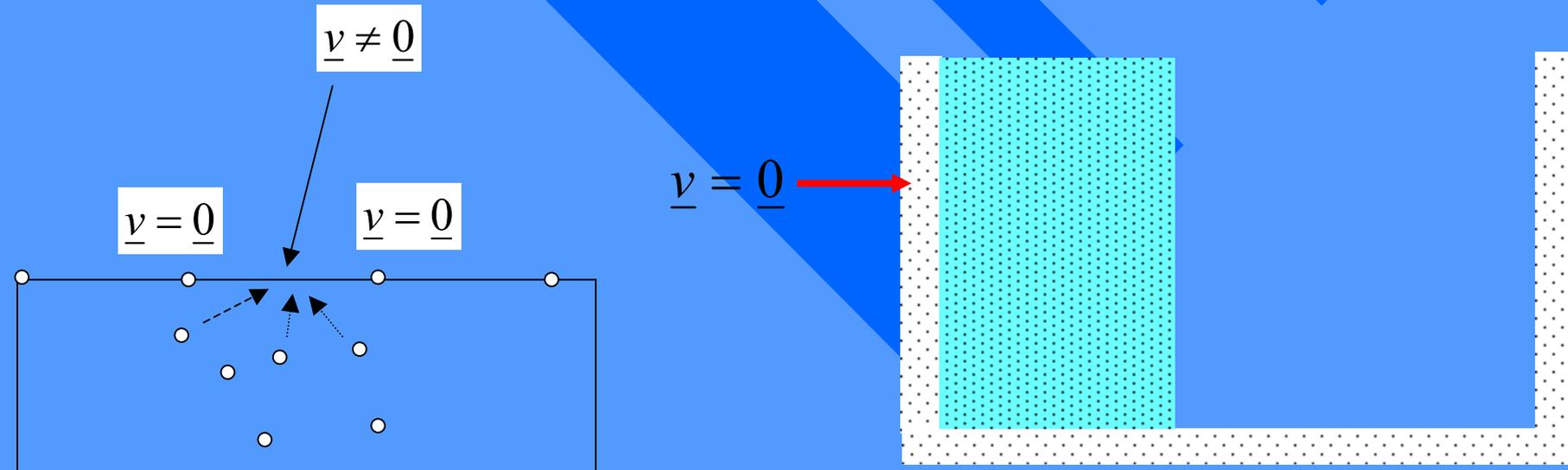
Imposition des conditions Dirichlet :

Multiplicateur de Lagrange :

$$\int_{\Omega} \underline{\underline{\sigma}} : \underline{\underline{\varepsilon}}^* d\Omega - \int_{\Omega} \underline{b} \cdot \underline{u}^* d\Omega - \int_{\Gamma_t} \underline{T} \cdot \underline{u}^* d\Gamma - \int_{\Gamma_u} \lambda^* (\underline{u} - \underline{u}_d) d\Gamma - \int_{\Gamma_u} \lambda \underline{u}^* d\Gamma = 0$$

$$\forall \underline{u}^* \in H^1, \forall \lambda^* \in H^0$$

Pénalisation :



Notre choix :

La Méthode des Eléments Naturels

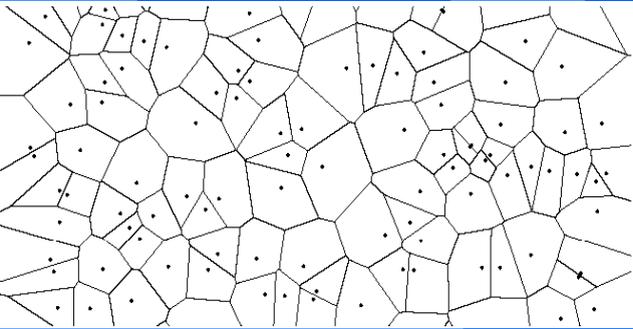
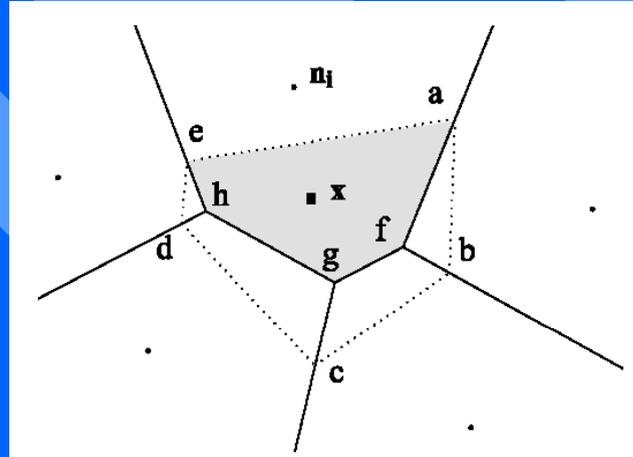


Diagramme de Voronoï

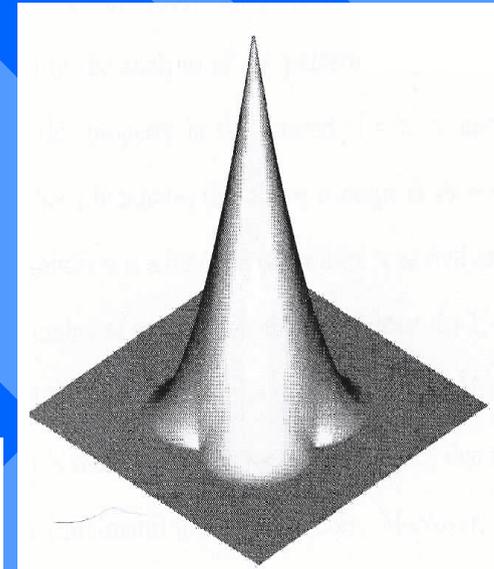
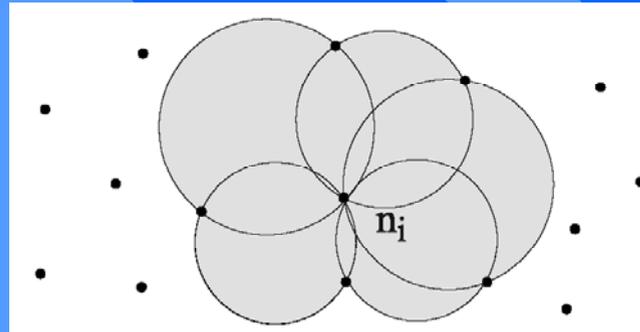


Calcul des fonctions de forme

$$\mathbf{u}^h(\mathbf{x}) = \sum_{i=1}^n \phi_i(\mathbf{x}) \mathbf{u}_i$$

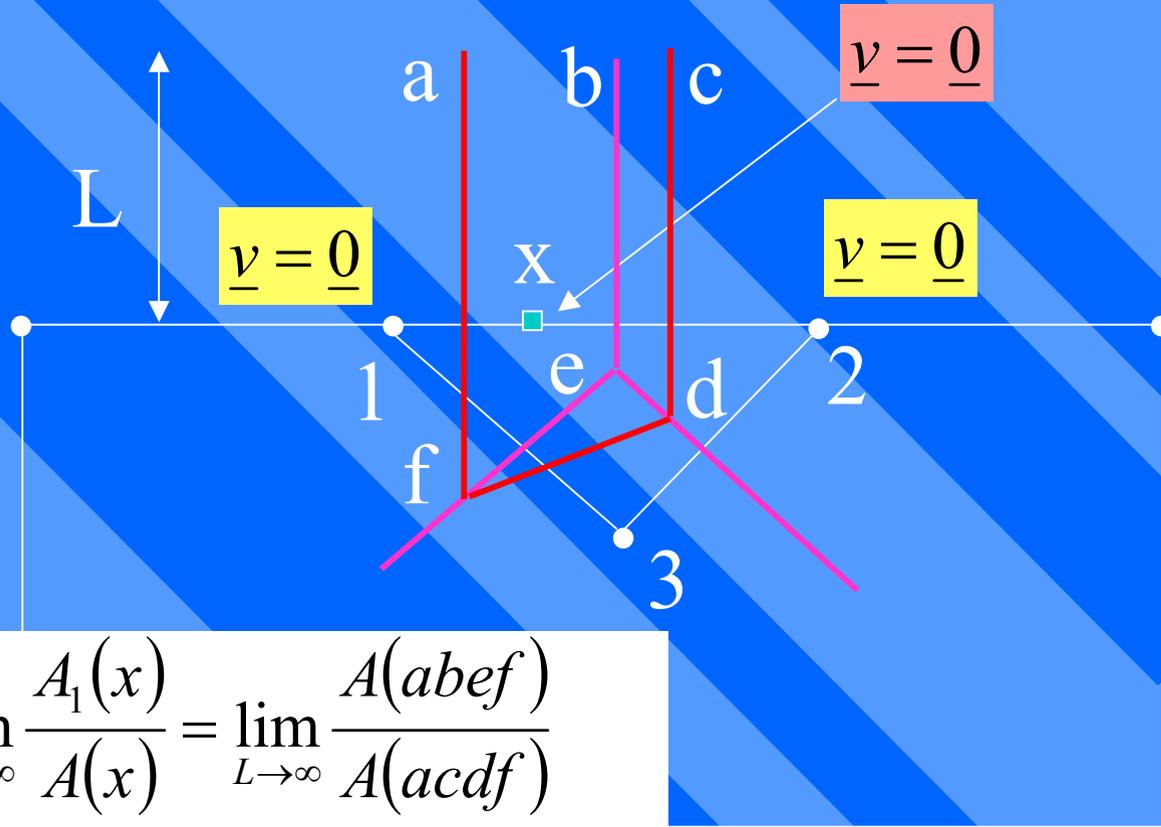
Interpolation

$$\phi(\mathbf{x}) = \frac{\text{Area}(afghe)}{\text{Area}(abcde)}$$



- **Propriété du delta de Kronecker**
- **Partition de l'unité**
- **Consistance linéaire**
- **Variation linéaire sur les bords (convexes)**
 - **La méthode C-NEM: linéarité sur tout bord**
- **Dérivabilité**
 - **Grandes transformation**
 - **Gestion des discontinuités fixes ou mobiles**
 - **Adaptation, ...**

Imposition des conditions Dirichlet

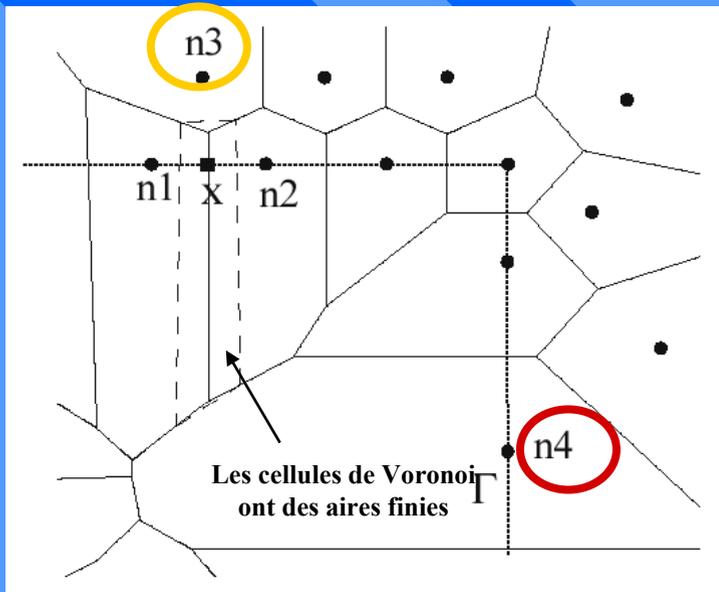


$$\phi_1(x) = \lim_{L \rightarrow \infty} \frac{A_1(x)}{A(x)} = \lim_{L \rightarrow \infty} \frac{A(abef)}{A(acdf)}$$

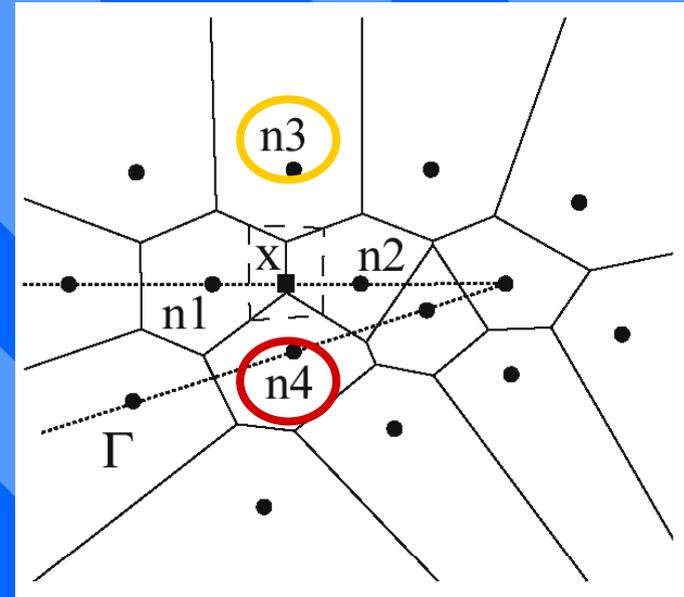
$$\phi_2(x) = \lim_{L \rightarrow \infty} \frac{A_2(x)}{A(x)} = \lim_{L \rightarrow \infty} \frac{A(bcde)}{A(acdf)}$$

$$\phi_3(x) = \lim_{L \rightarrow \infty} \frac{A_3(x)}{A(x)} = \lim_{L \rightarrow \infty} \frac{A(def)}{A(acdf)} = 0$$

Problèmes de la méthode des éléments naturels sur les domaines non convexes



Perte de linéarité sur les bords non convexes

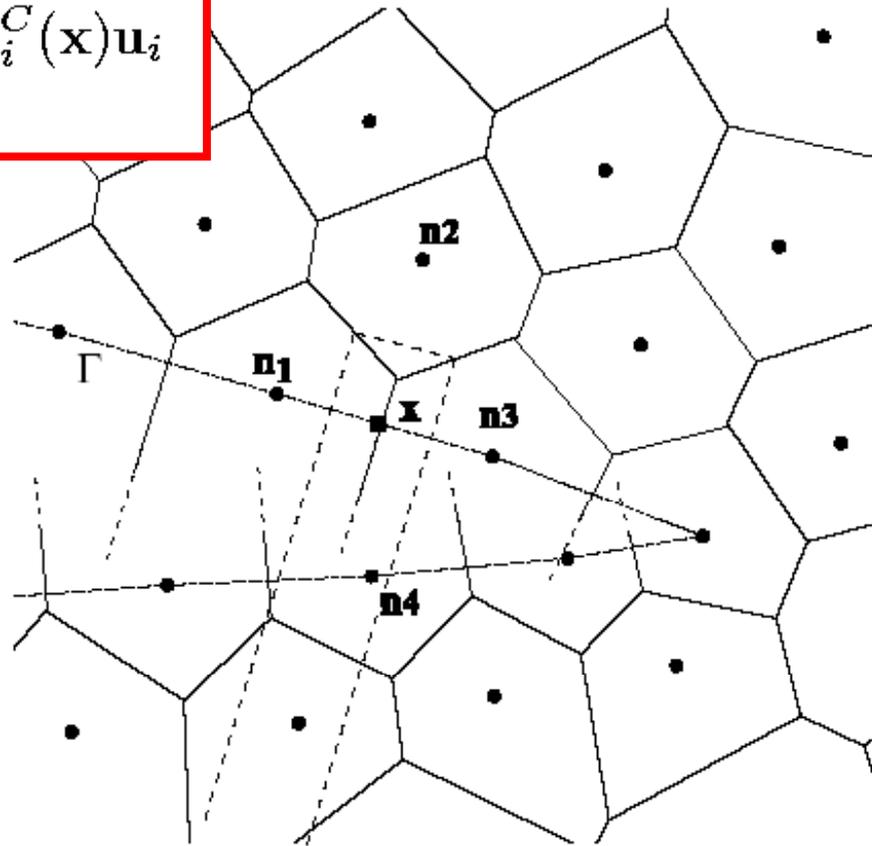
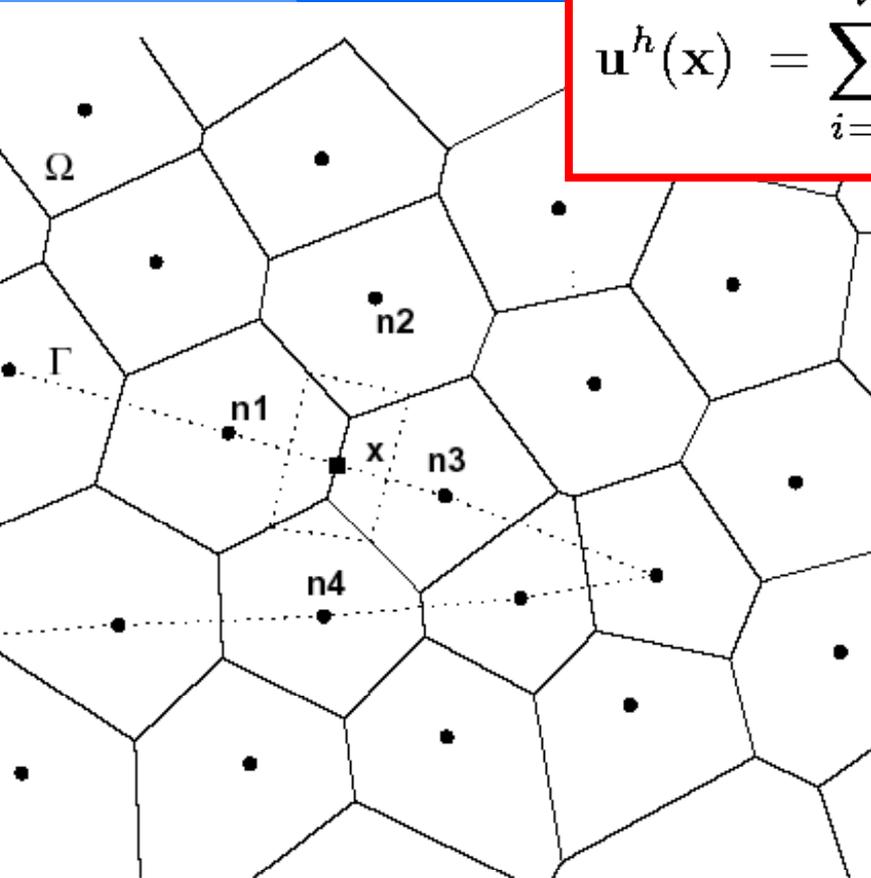


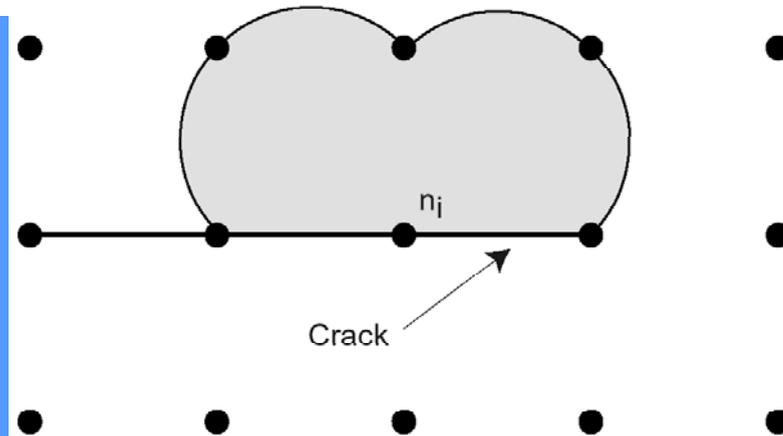
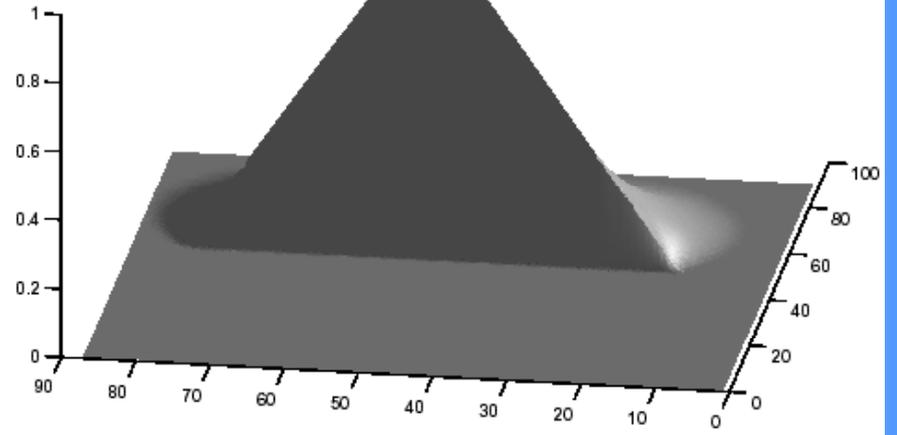
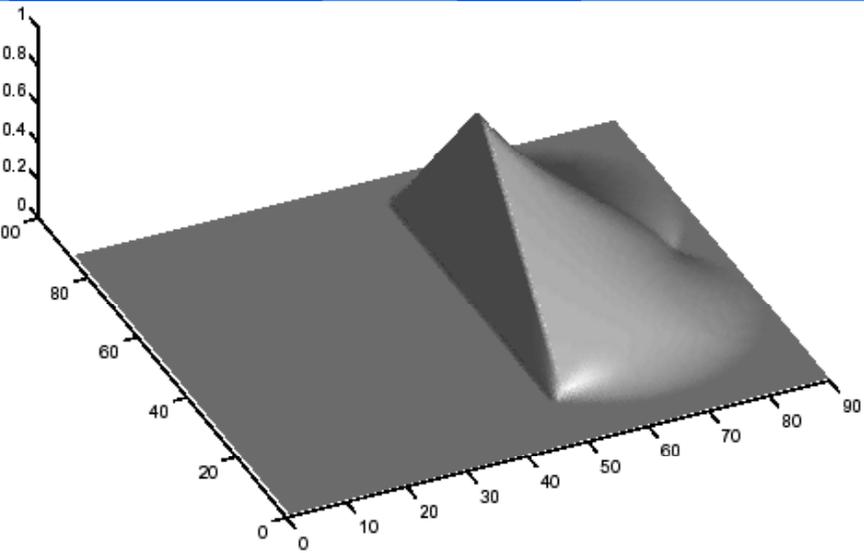
Influences parasites entre nœuds sur les bords non convexes

Introduction d'un critère de visibilité:

Triangulation de Delaunay contrainte et diagramme de Voronoï contraint

$$\mathbf{u}^h(\mathbf{x}) = \sum_{i=1}^V \phi_i^C(\mathbf{x}) \mathbf{u}_i$$





Dans la limite de l'incompressibilité

- Formulation variationnelle :

$$\int_{\Omega(t)} \underline{\underline{\sigma}} : \underline{\underline{D}}^* d\Omega = \int_{\Gamma_2} (\underline{\underline{\sigma}} \underline{\underline{v}}^*) \underline{\underline{n}} d\Gamma \quad \forall \underline{\underline{v}}^* \in U^*$$

$$\int_{\Omega(t)} -\text{Div} \underline{\underline{v}} d\Omega = 0 \quad \forall p^* \in L^2(\Omega(t))$$

$$\begin{cases} \text{Div} \underline{\underline{\sigma}} = \underline{\underline{0}} \\ \text{Div} \underline{\underline{v}} = 0 \\ \underline{\underline{\sigma}} = -p \underline{\underline{Id}} + 2\eta(\underline{\underline{D}}) \underline{\underline{D}} \\ \eta(\underline{\underline{D}}) = k \left(\sqrt{2 \underline{\underline{D}} : \underline{\underline{D}}} \right)^{n-1} \end{cases}$$

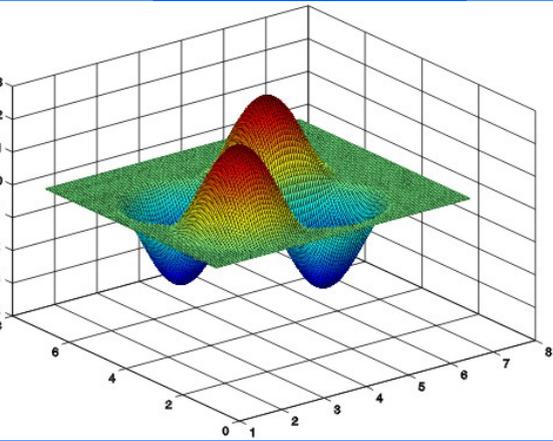
- Interpolation des vitesses C-NEM : C^0
- Interpolation des pressions : C^{-1}

$$p^h = \sum_{I=1}^n \frac{1}{n} \bar{p}_I$$

~~LBB~~

LBB via PUM-NEM ...

$$u = \sum N_i a_i + \sum N_i \phi_i b_i$$



$$\phi = xy$$

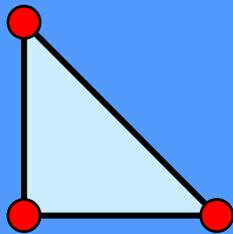
$$\inf_{w_h \in \mathcal{U}_h} \sup_{u_h \in \mathcal{U}_h} \frac{W_h^T G_h U_h}{\sqrt{W_h^T G_h W_h} \cdot \sqrt{U_h^T S_h U_h}} = \gamma_h \geq \gamma >$$

$$\|p_h\|_0^2 = W_h^T G_h W_h$$

$$\|u_h\|_1^2 = U_h^T S_h U_h$$

$$G_h \phi_h = \lambda S_h \phi_h \longrightarrow \gamma_h = \sqrt{\lambda_k}$$

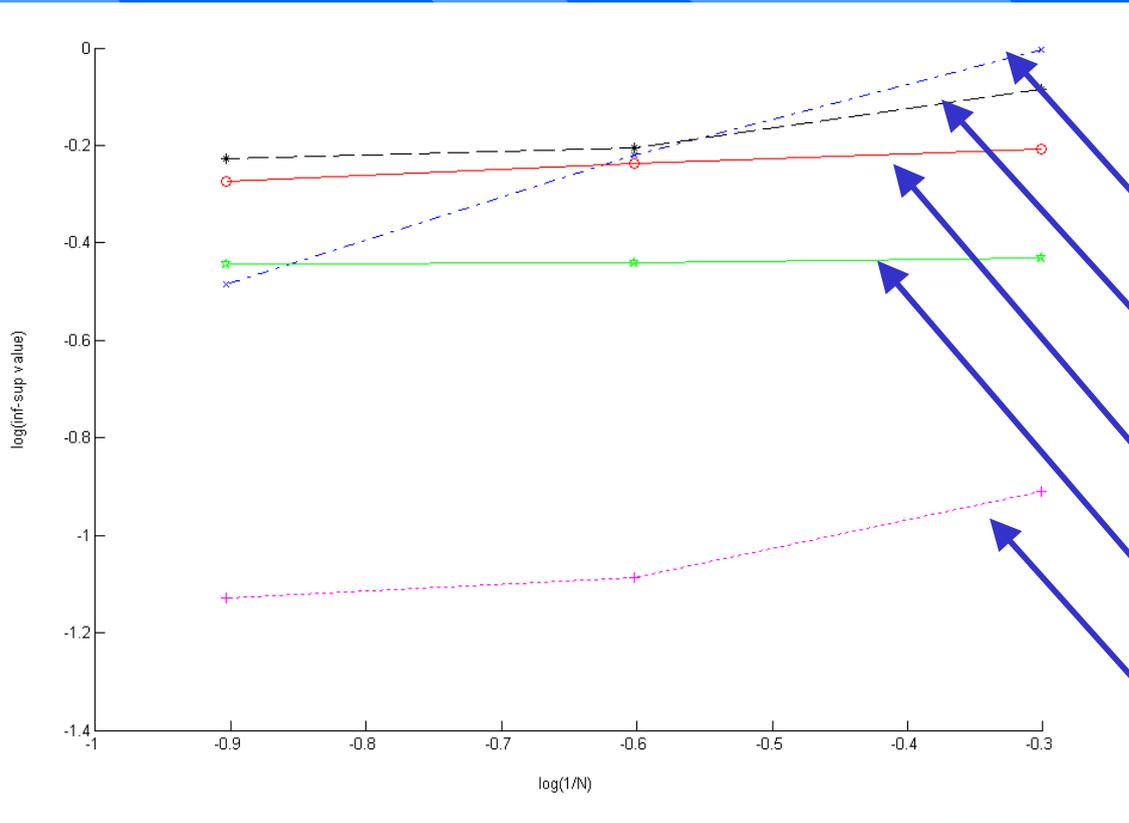
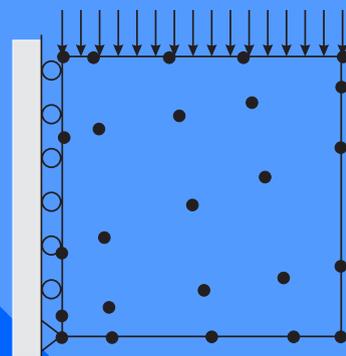
$$\begin{cases} N_1 = 1 - x - y \\ N_2 = x \\ N_3 = y \\ N_4 = N_1 \times N_2 \times N_3 = xy + x^2 y + xy^2 \end{cases}$$



$$C - NEM \rightarrow \{1, x, y\}$$

$$p^h : C - NEM$$

$$\underline{v}^h : \{1, x, y\} + \underbrace{\{1, x, y\}}_{\{xy\}} \rightarrow \{1, x, y, xy, x^2 y, xy^2\}$$



FEM, C0-C-1, spurious mode

NEM ENRICHED $\{1, x^2, y^2\}$
spurious modes

NEM ENRICHED $\{1, x, y, xy\}$
discontinuous pressures

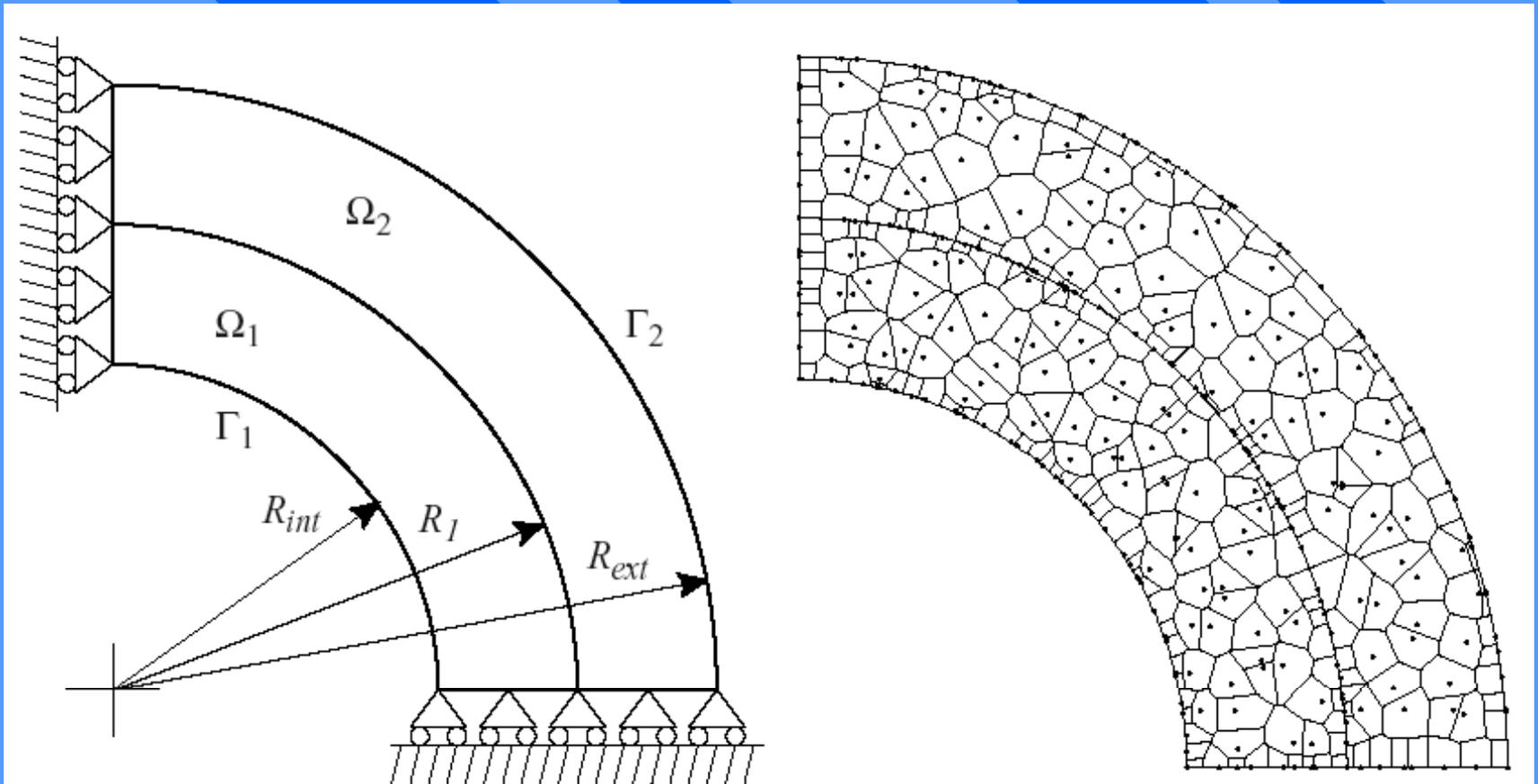
NEM ENRICHED $\{1, x, y, xy\}$
continuous pressures

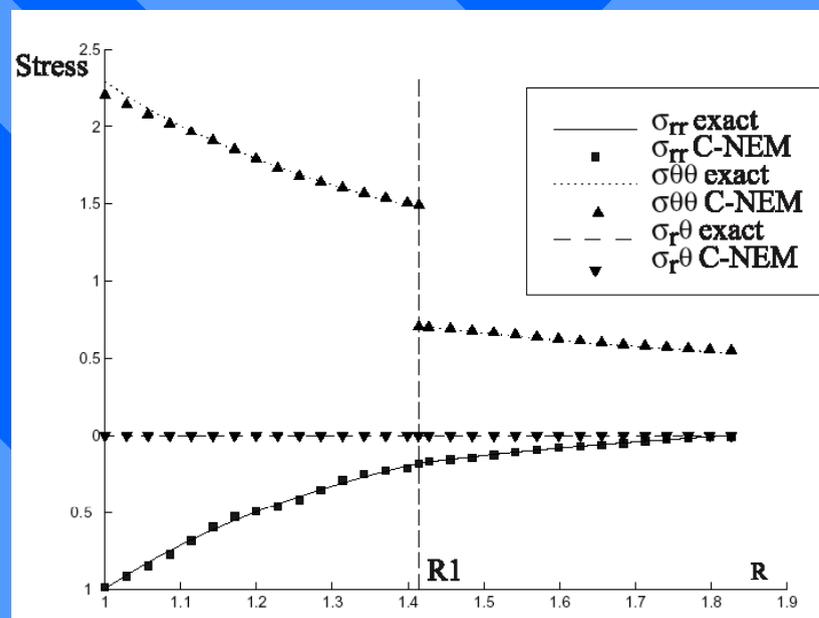
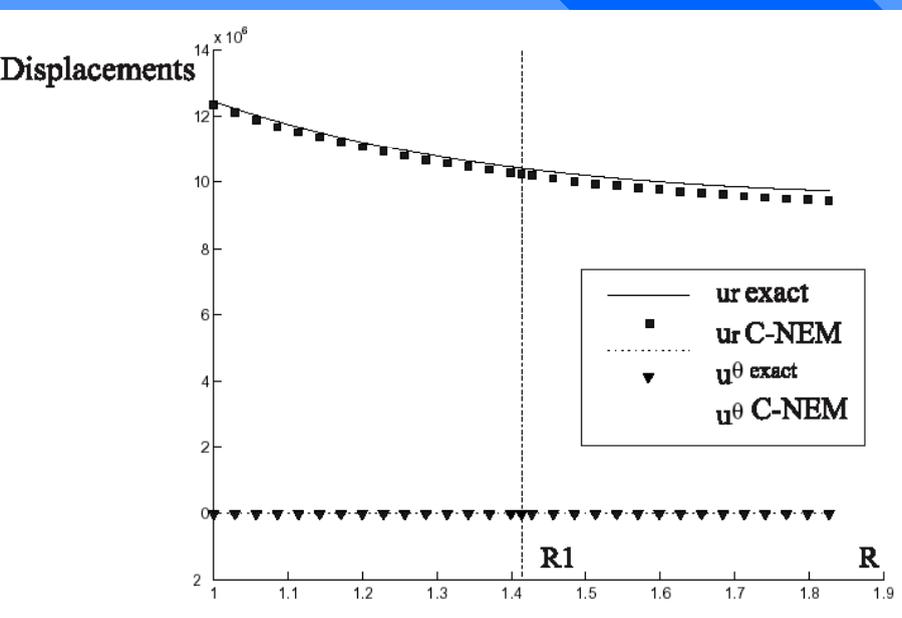
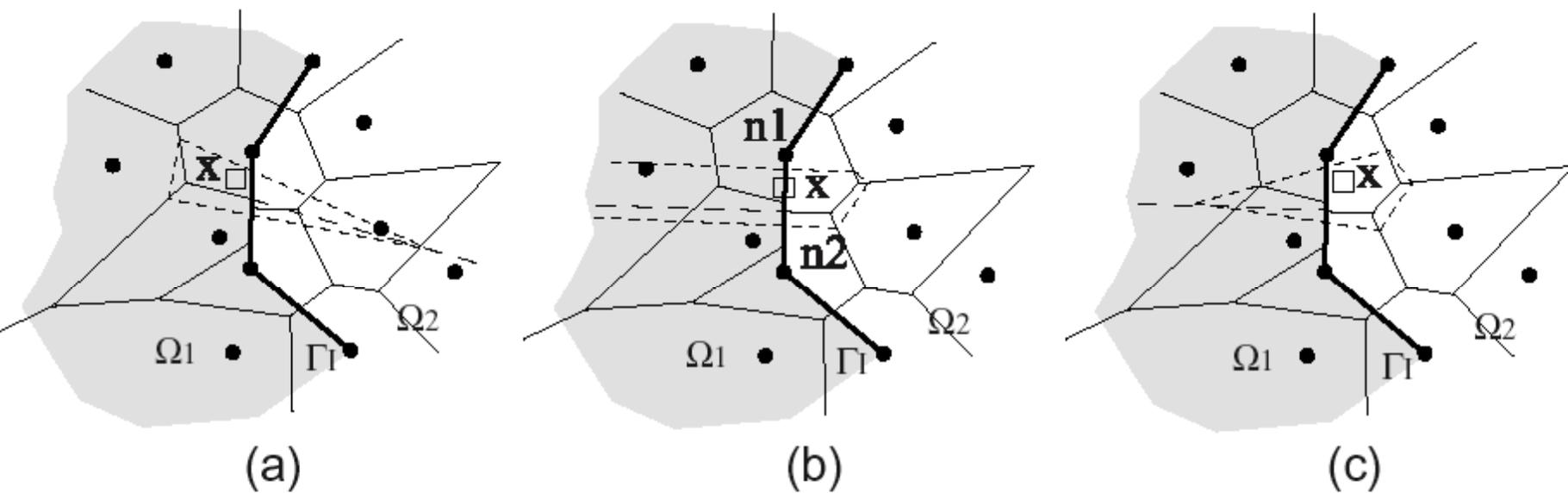
UNENRICHED NEM

Exemples numériques :

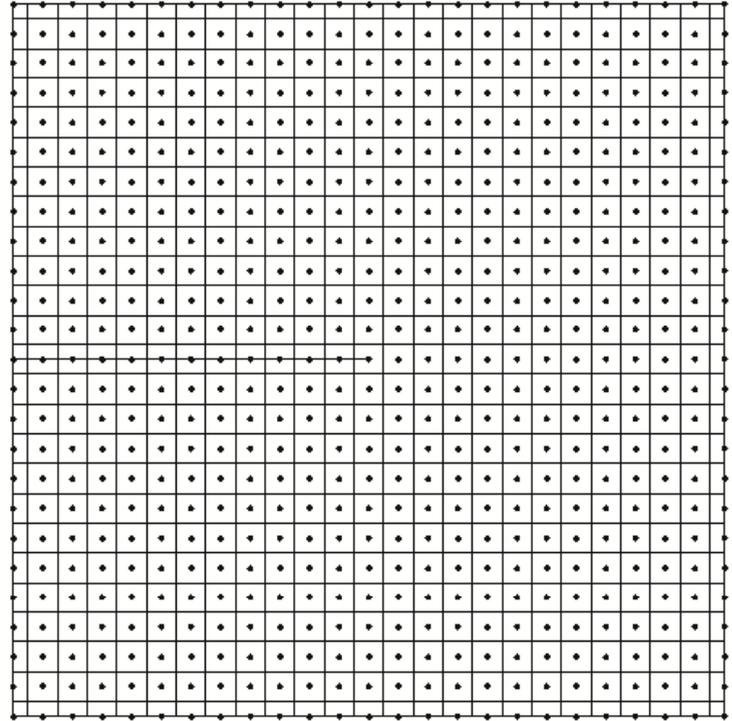
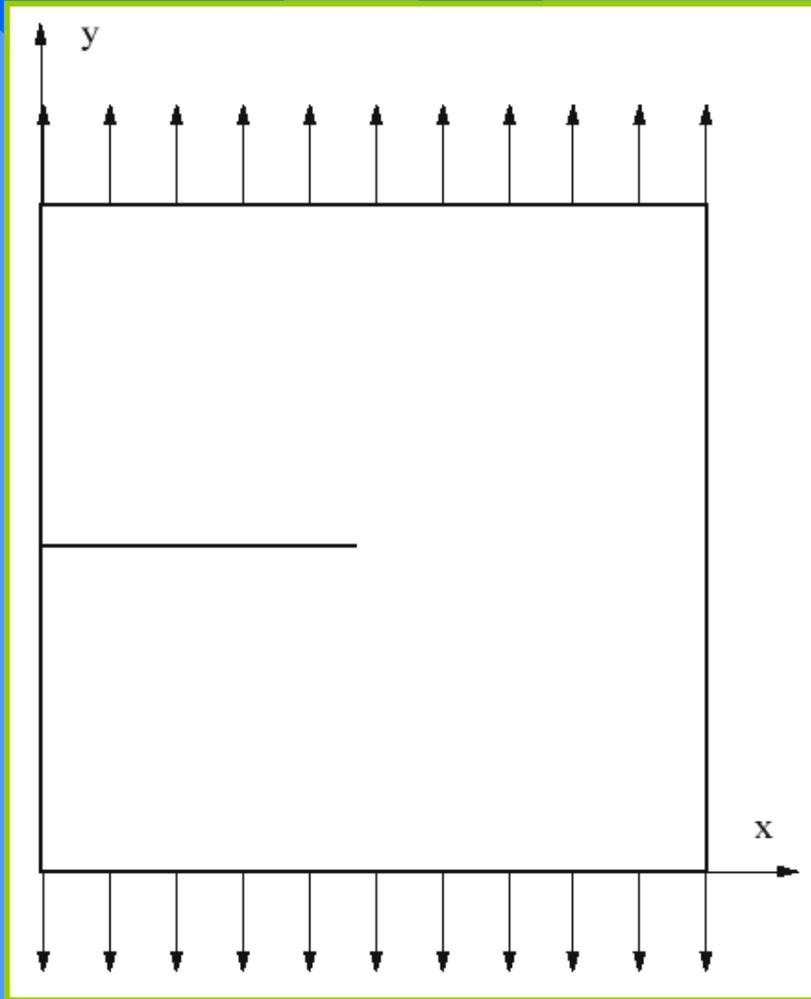
(I) Elasticité linéaire

1. Cylindre bi-matériau soumis à une pression interne

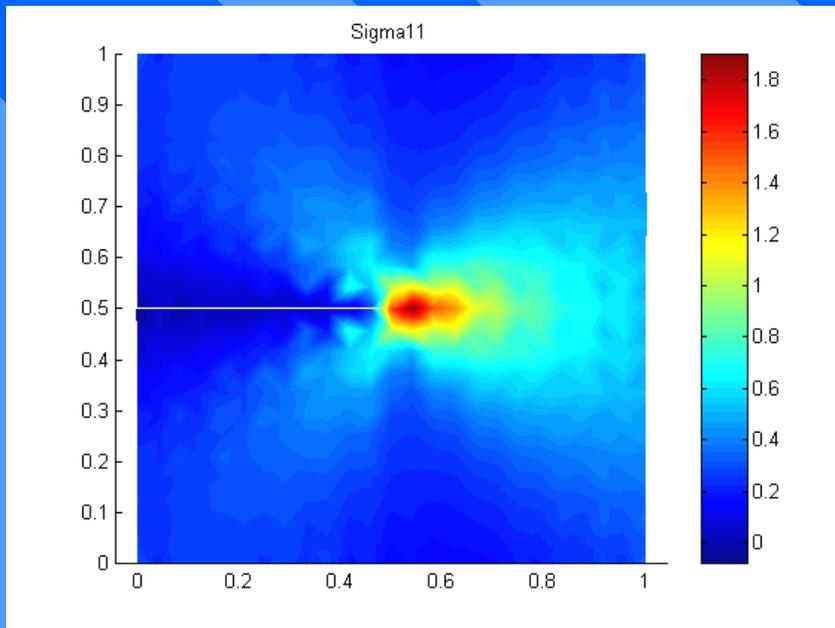




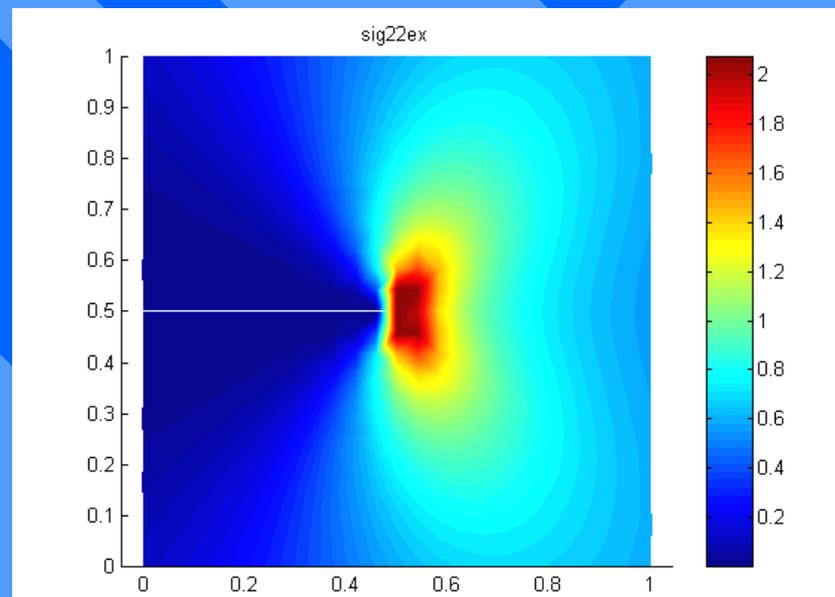
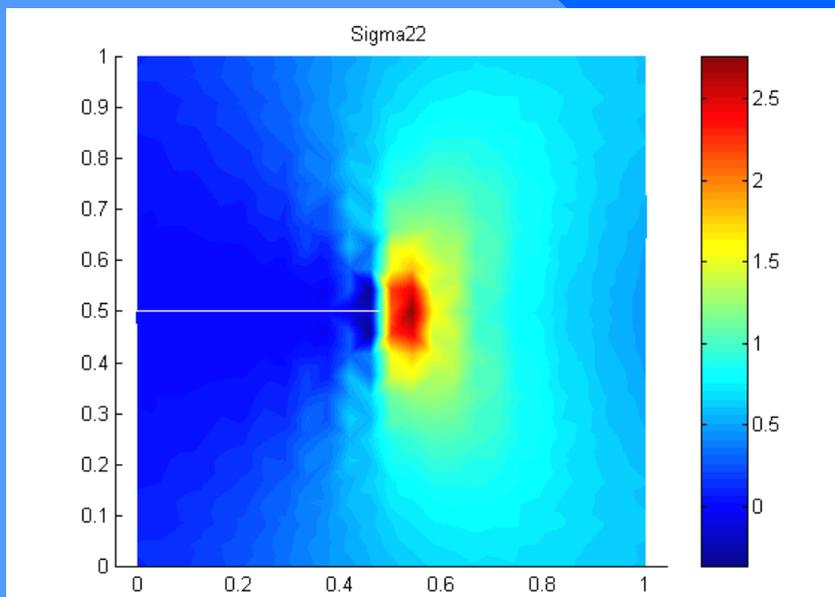
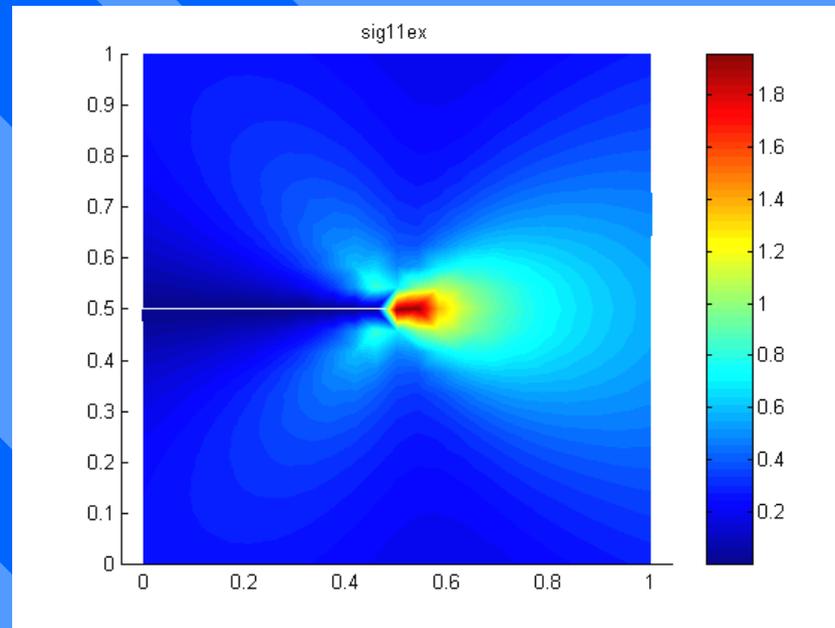
2. Fissure

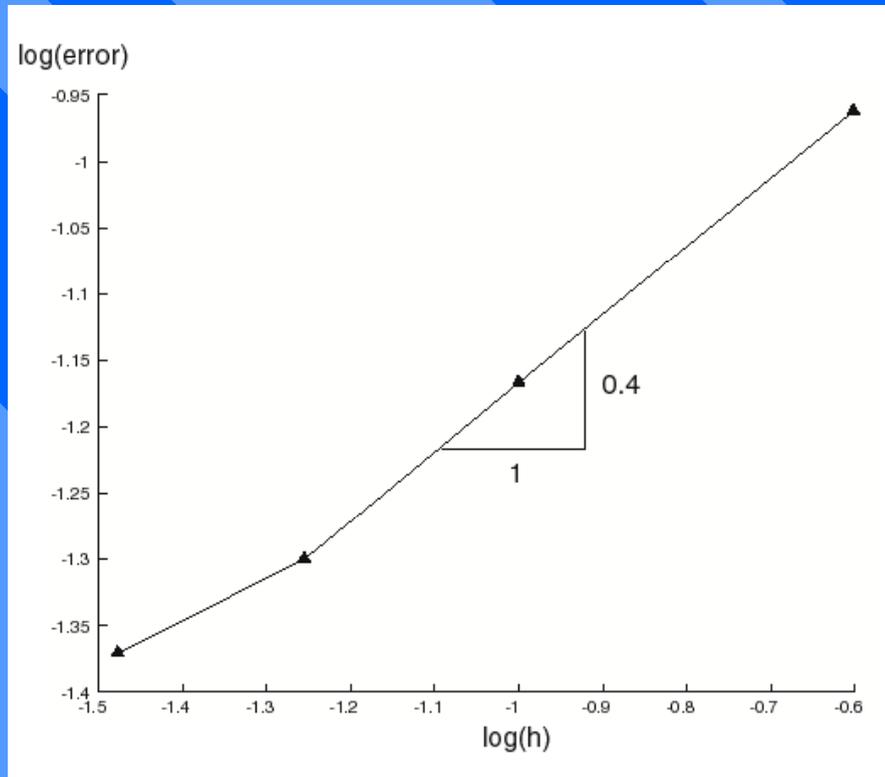


σ_{11} : solution C-NEM

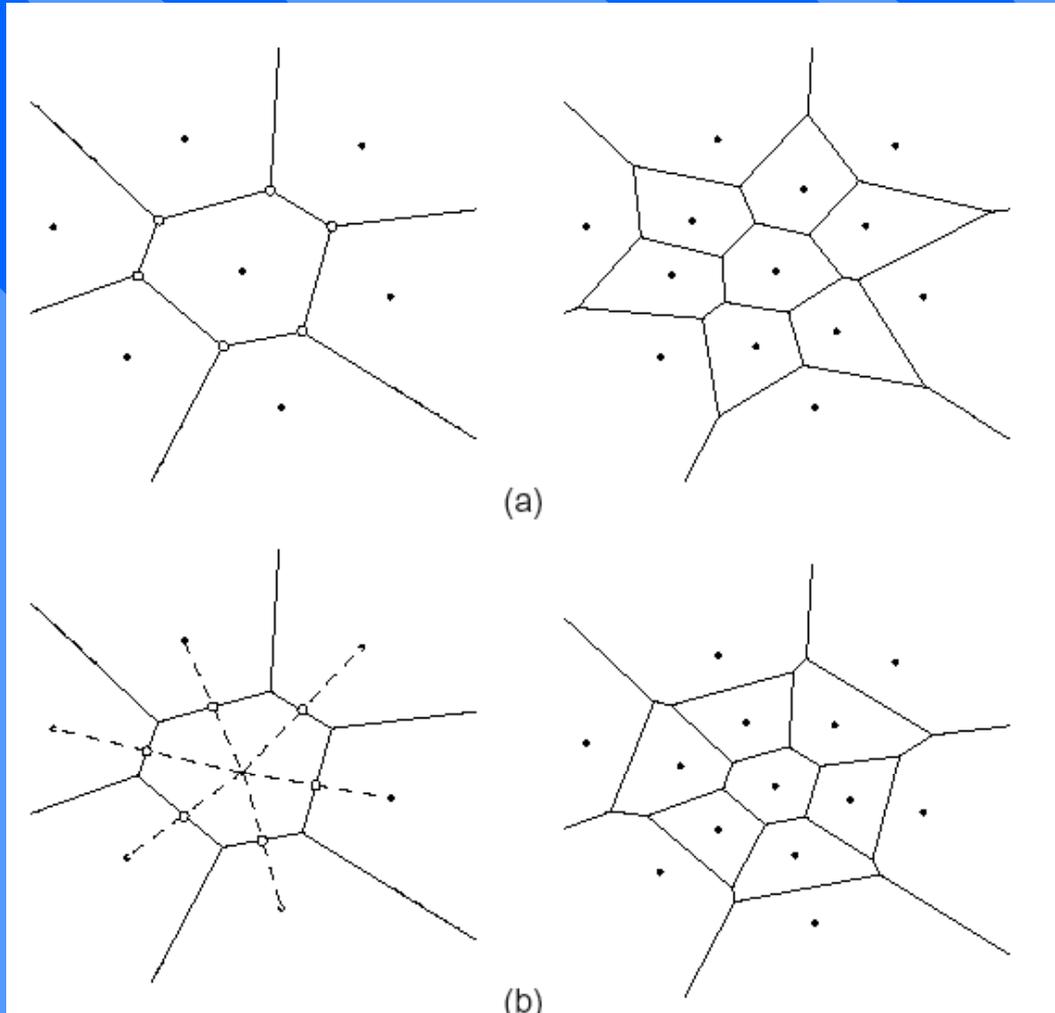


σ_{11} : solution exacte

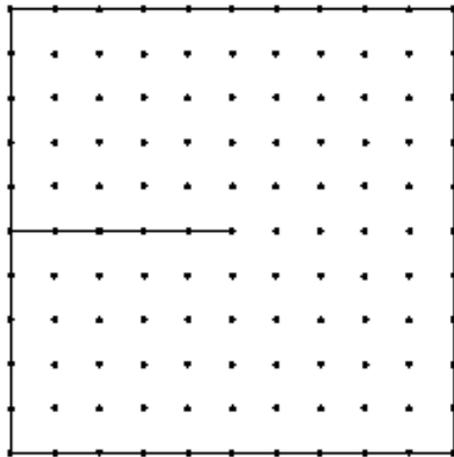




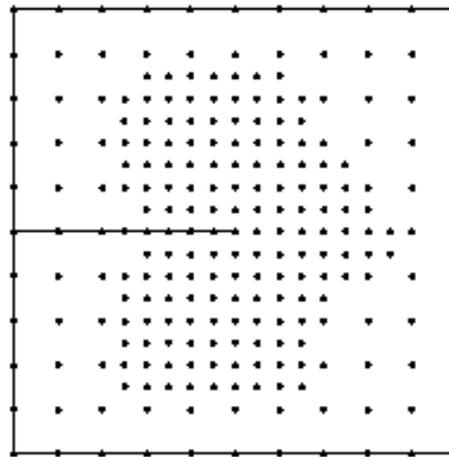
Adaptation ...



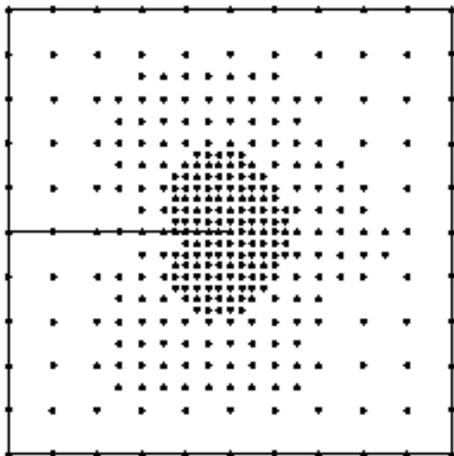
Raffinage adaptatif



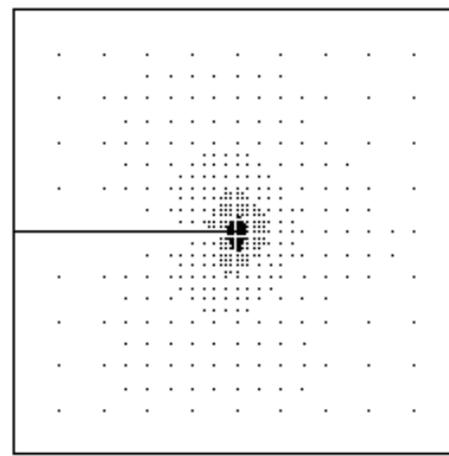
error : 11.4 %



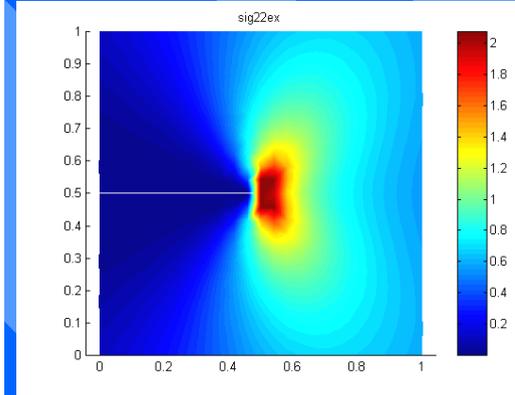
error : 8.0 %

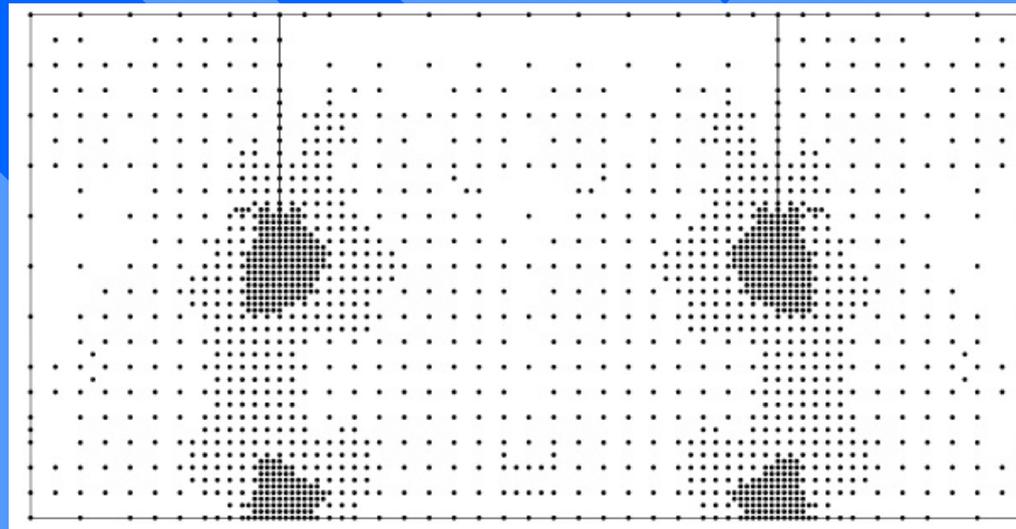
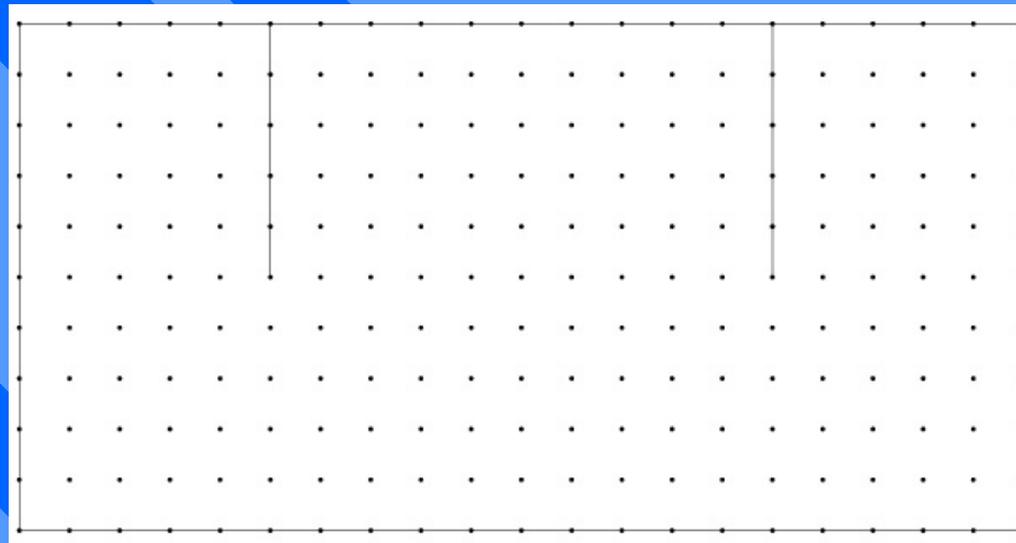
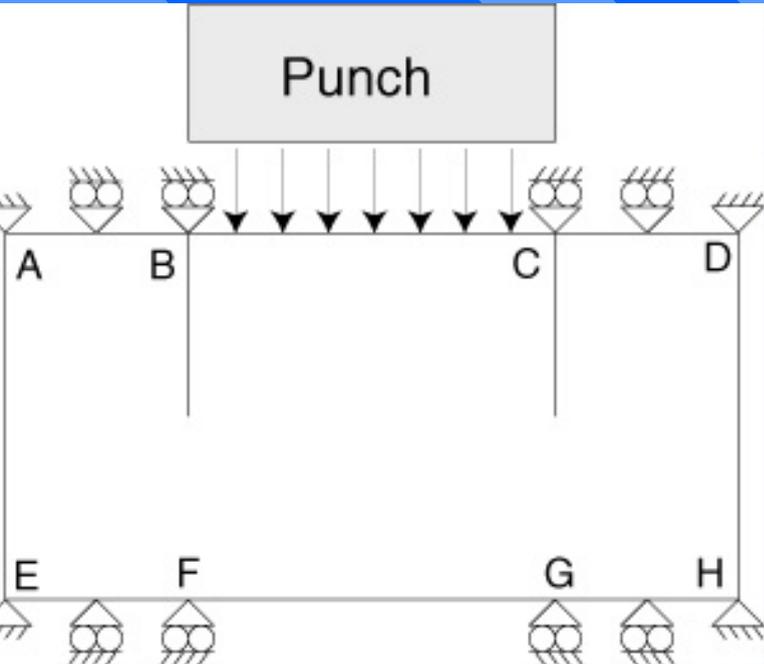


error : 6.1 %

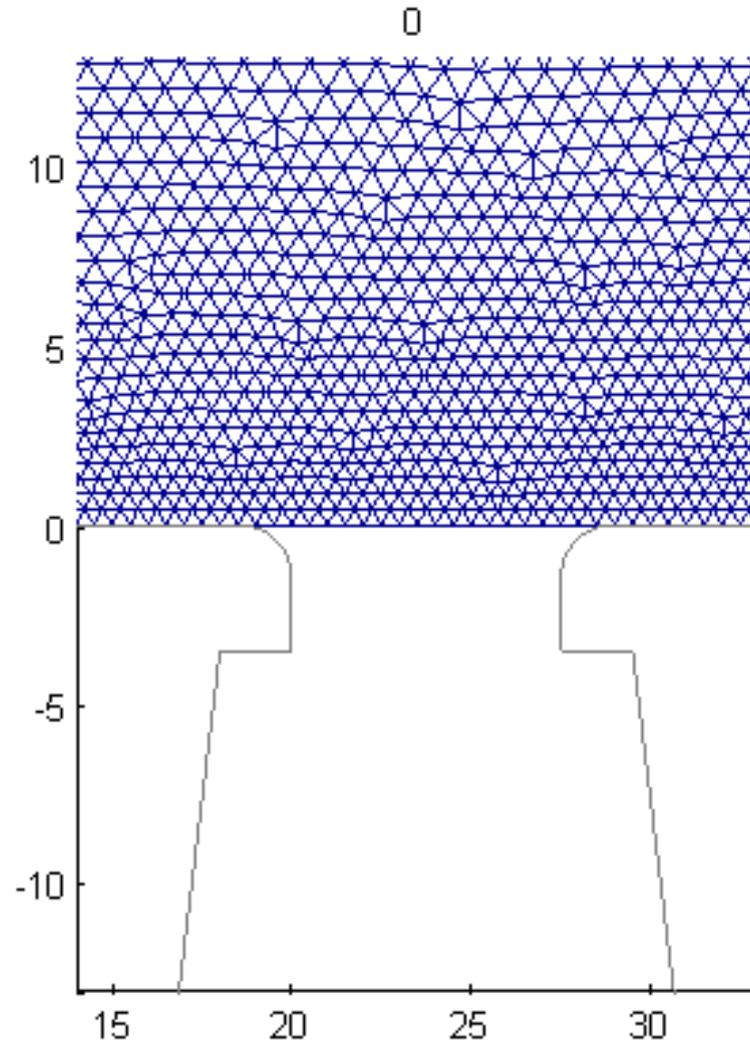


error : 4.6 %

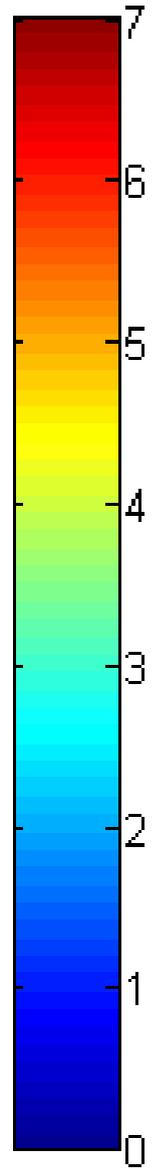
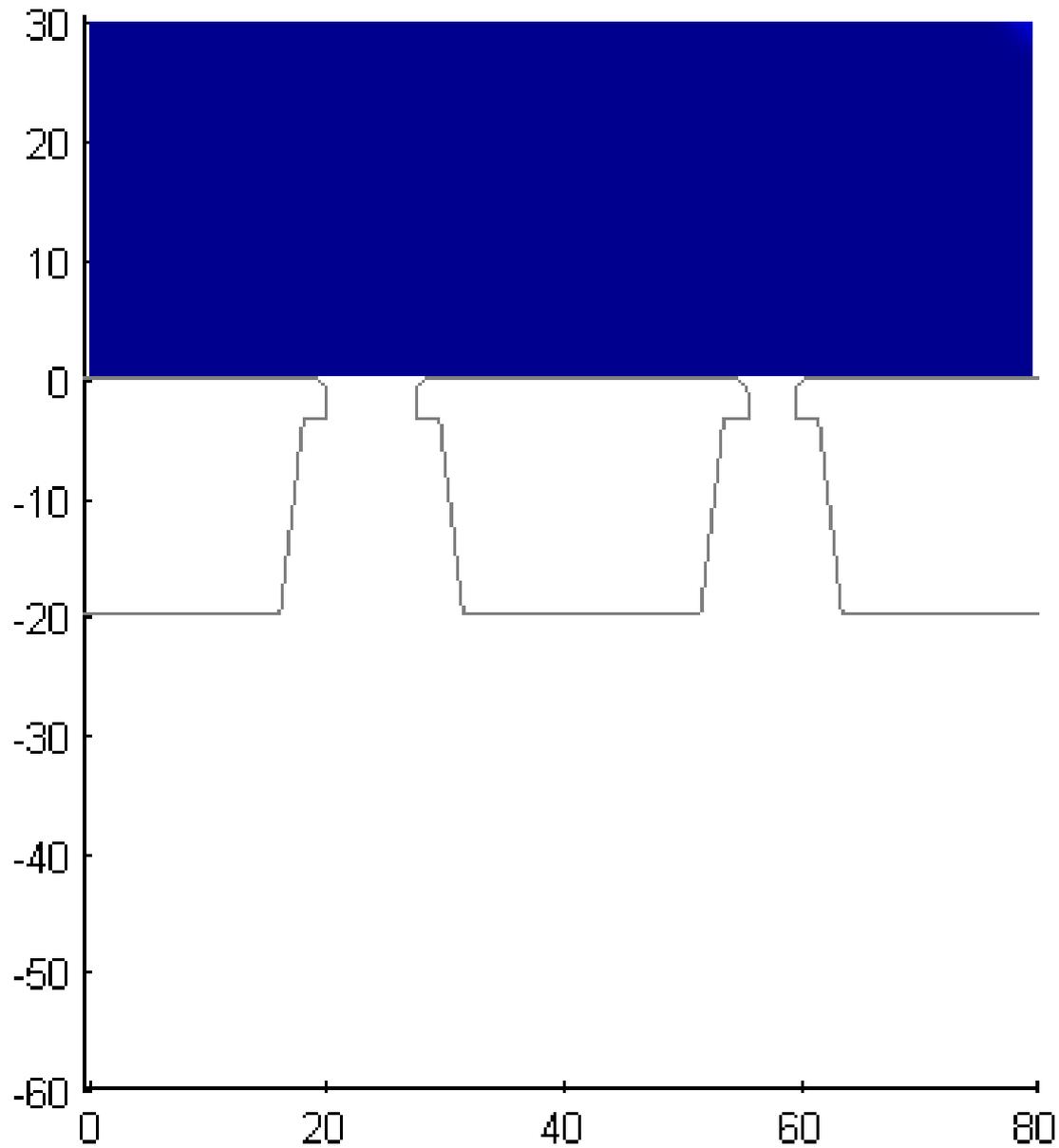




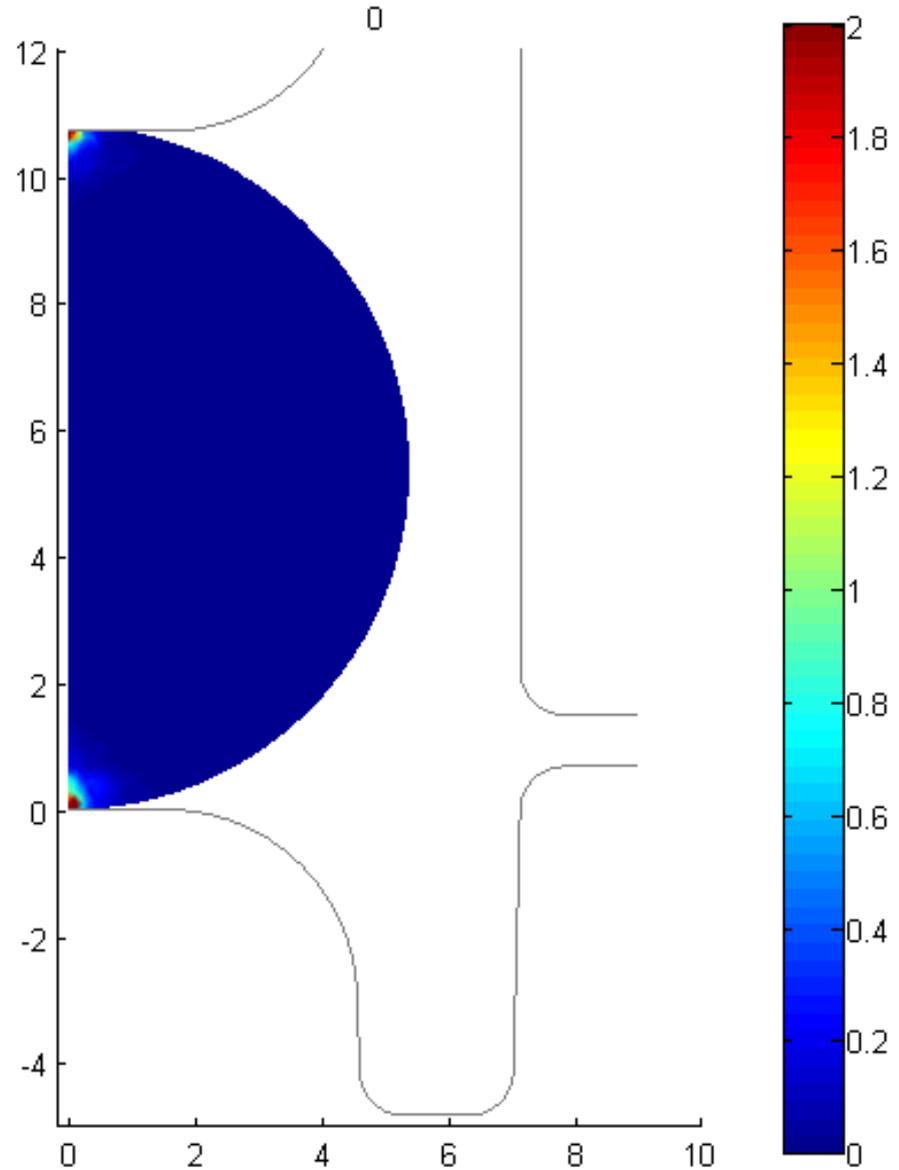
Exemples numériques : (II) Extrusion



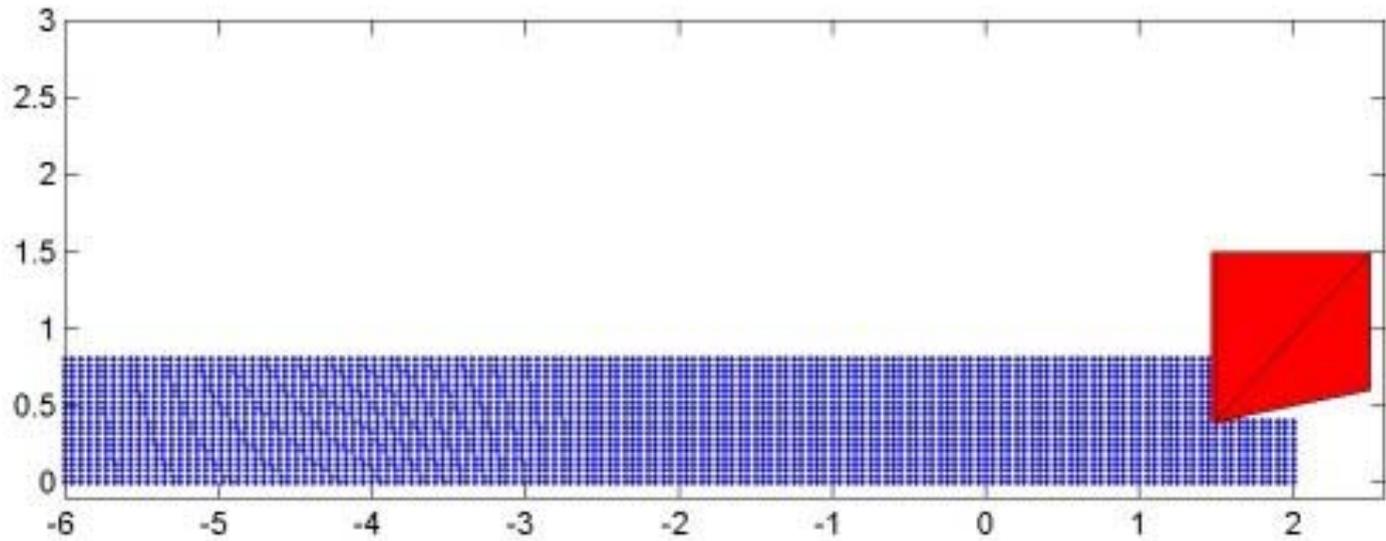
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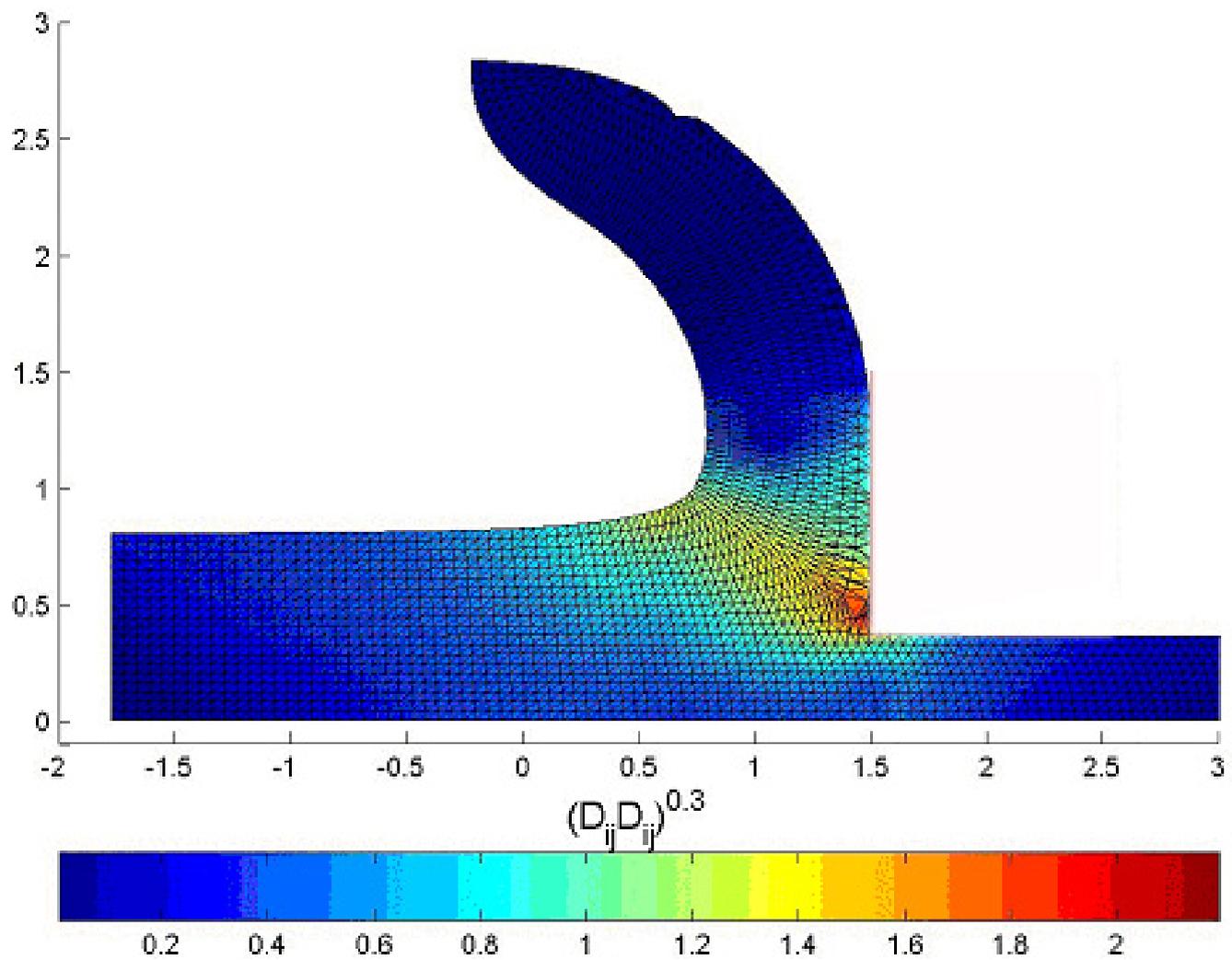


Exemples Numériques: (III) Forge

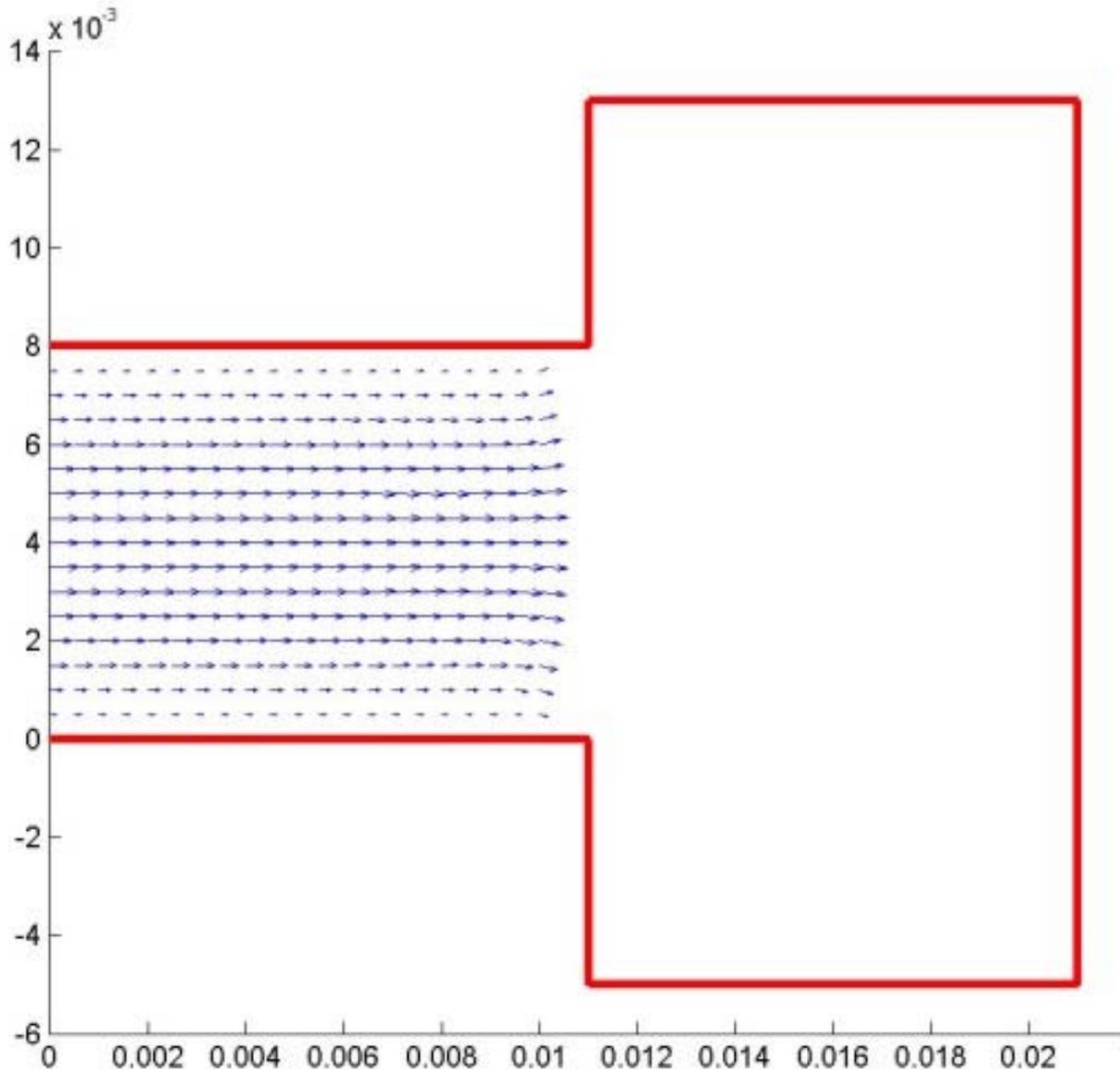


Exemples numériques : (IV) Usinage

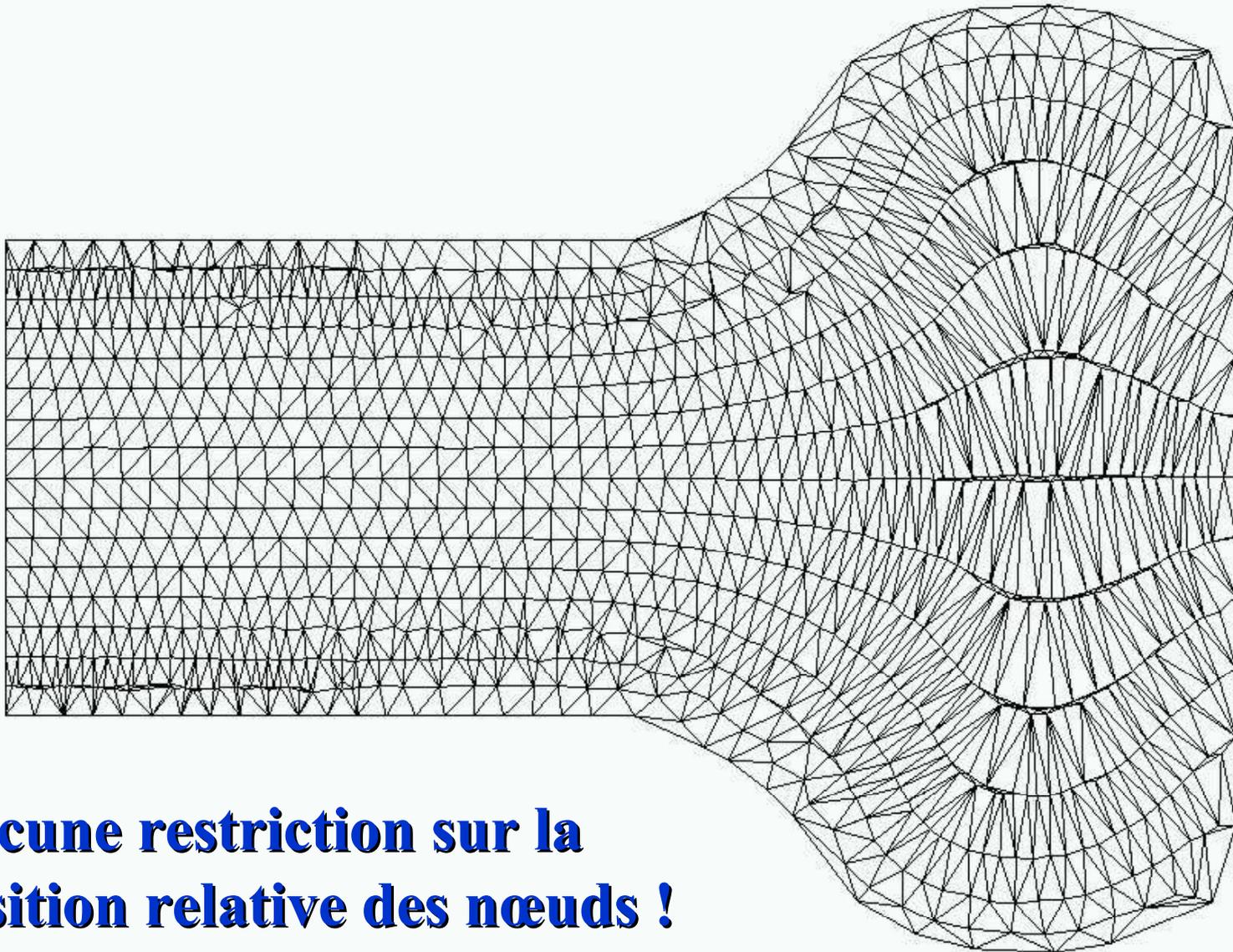




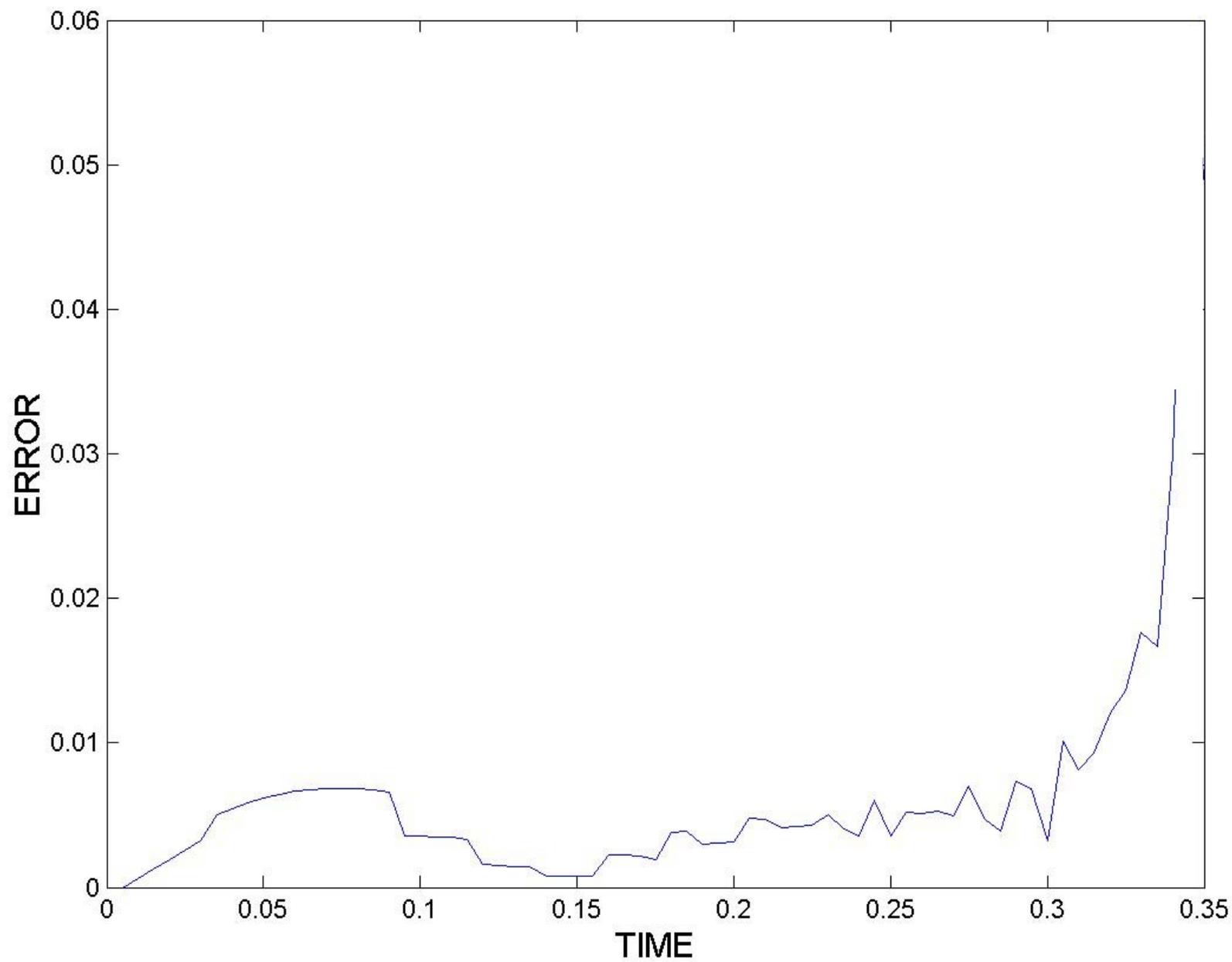
Exemples numériques : (V) Injection



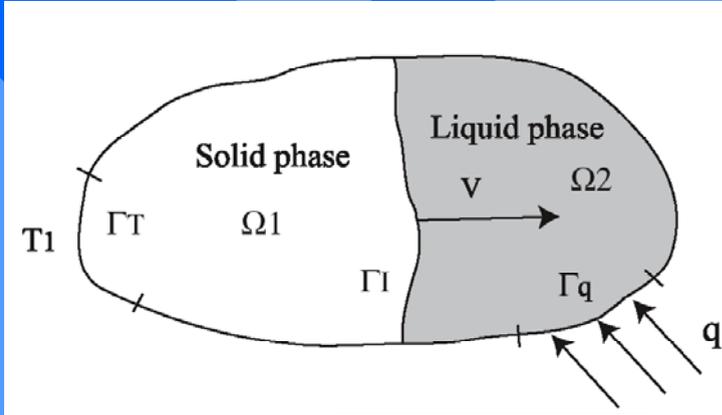
Triangulation de Delaunay ...



Aucune restriction sur la position relative des nœuds !

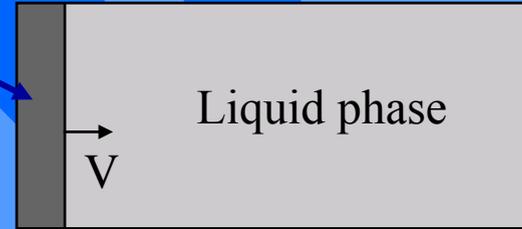


Exemples numériques : (VI) Interfaces



Solid phase

$$T_1 < T_m$$

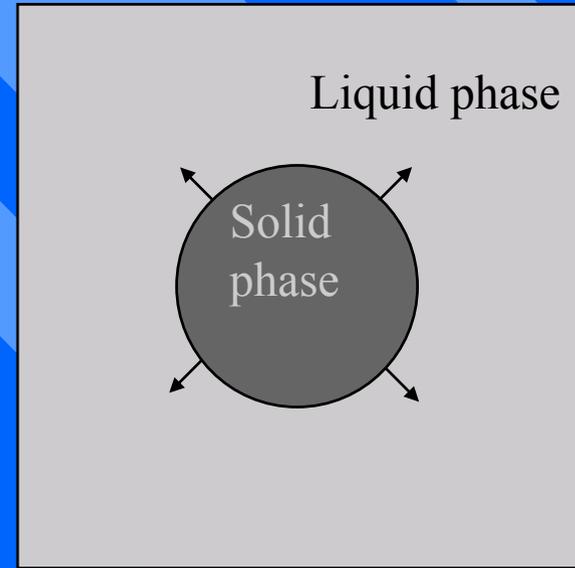


$$T_2 > T_m$$

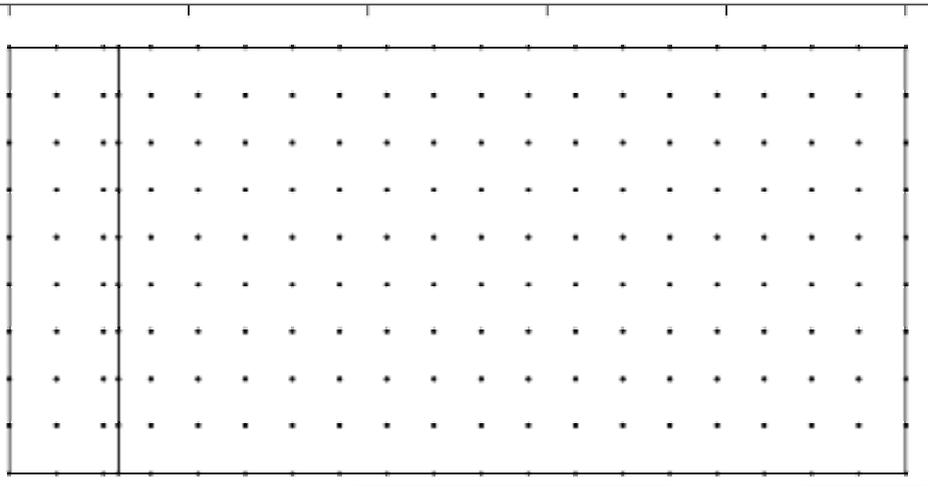
$$\mathbf{V}(\mathbf{x} \in \Gamma_I(t)) = \frac{||[q]||}{L} \mathbf{n}_I(\mathbf{x})$$

$$||[q]|| = (k_1 \nabla T|_{\Gamma_I^-(t)} - k_2 \nabla T|_{\Gamma_I^+(t)}) \cdot \mathbf{n}_I$$

$$T(\mathbf{x}, t) = T_m \quad \mathbf{x} \in \Gamma_I(t)$$



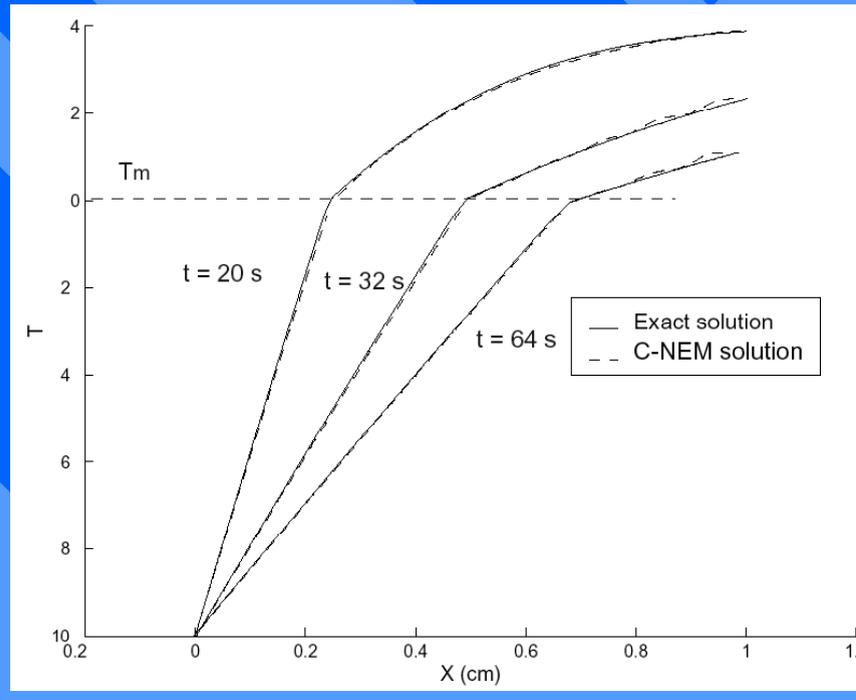
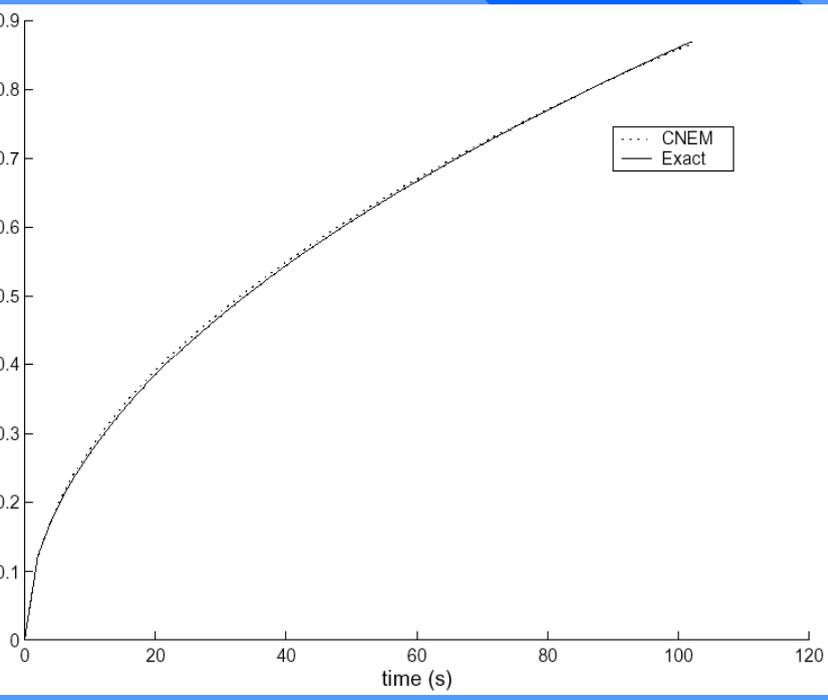
$t = 2\text{ s}$



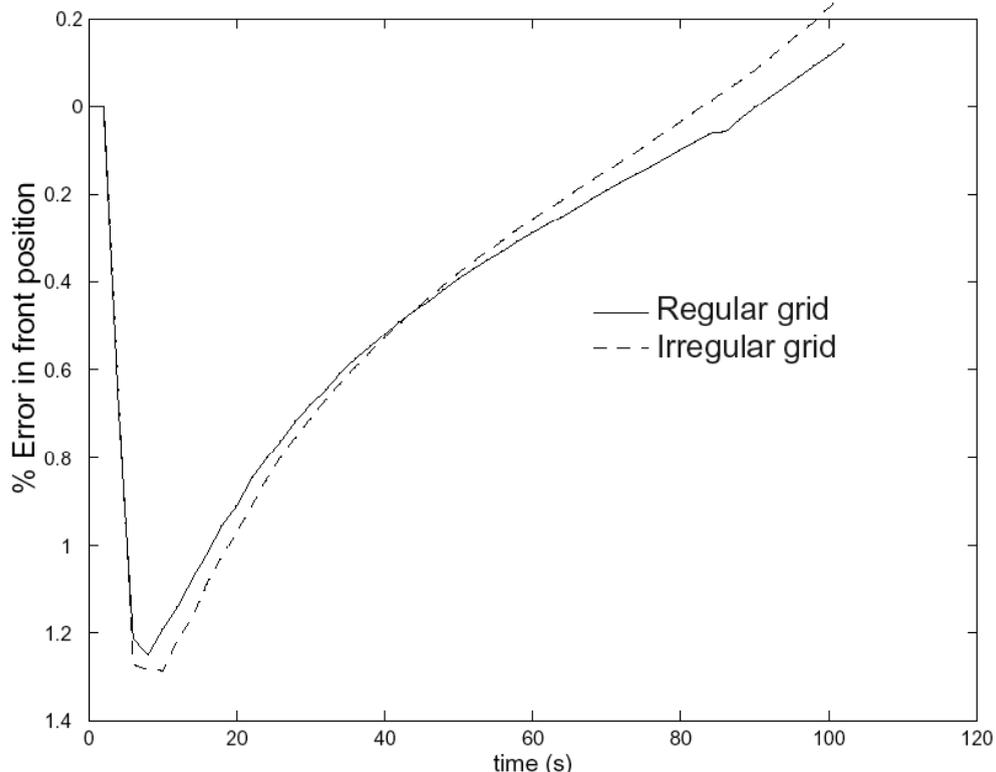
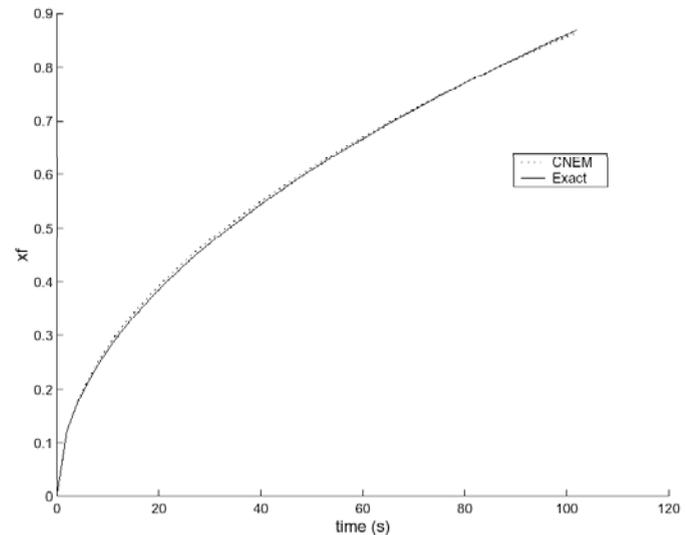
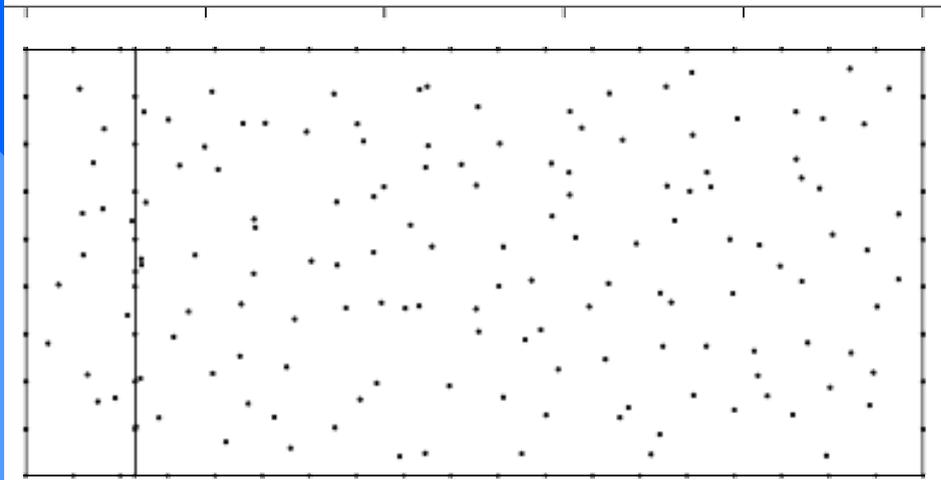
Position of the front:

..... Exact solution

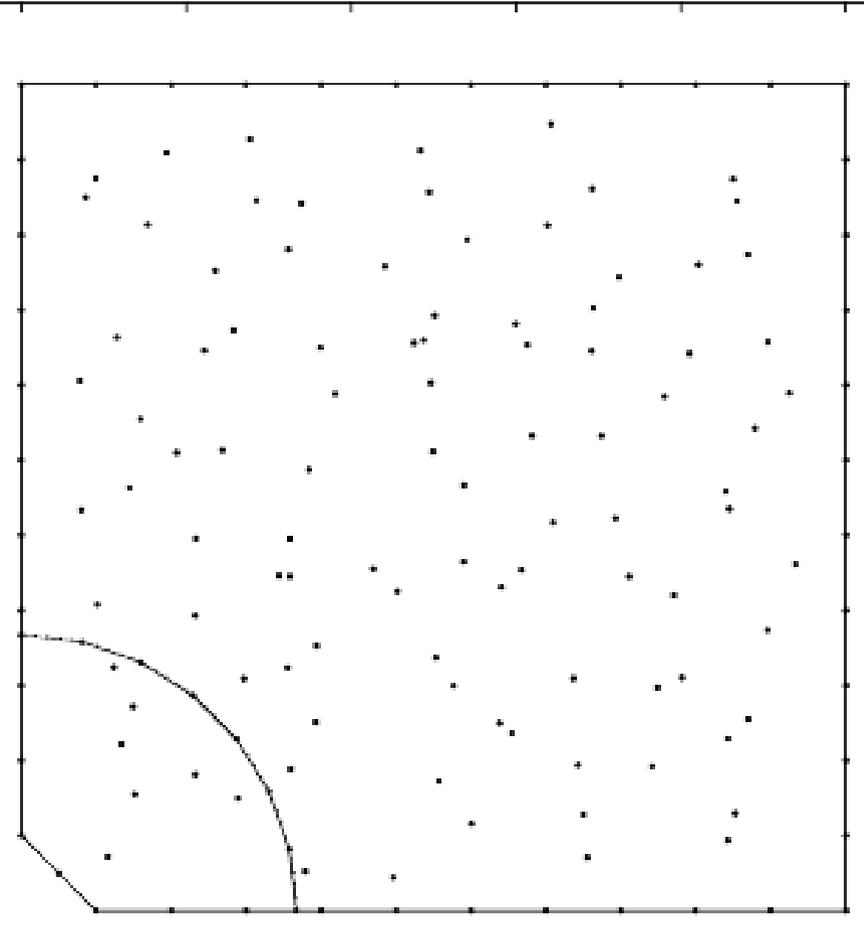
—●—●—●—●— C-NEM solution



$t = 2$ s



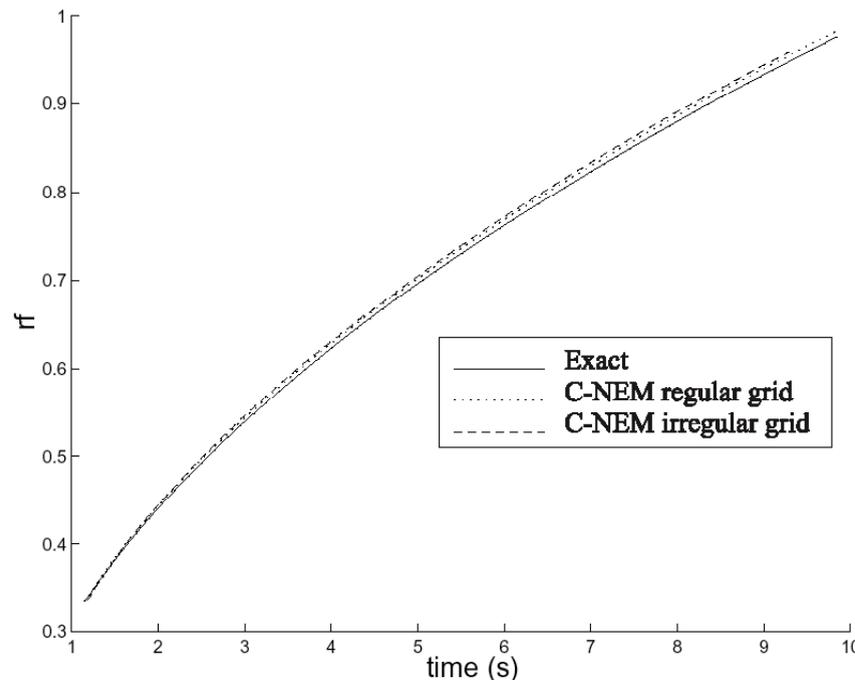
$t = 1.15$ s



Exact solution



C-NEM solution



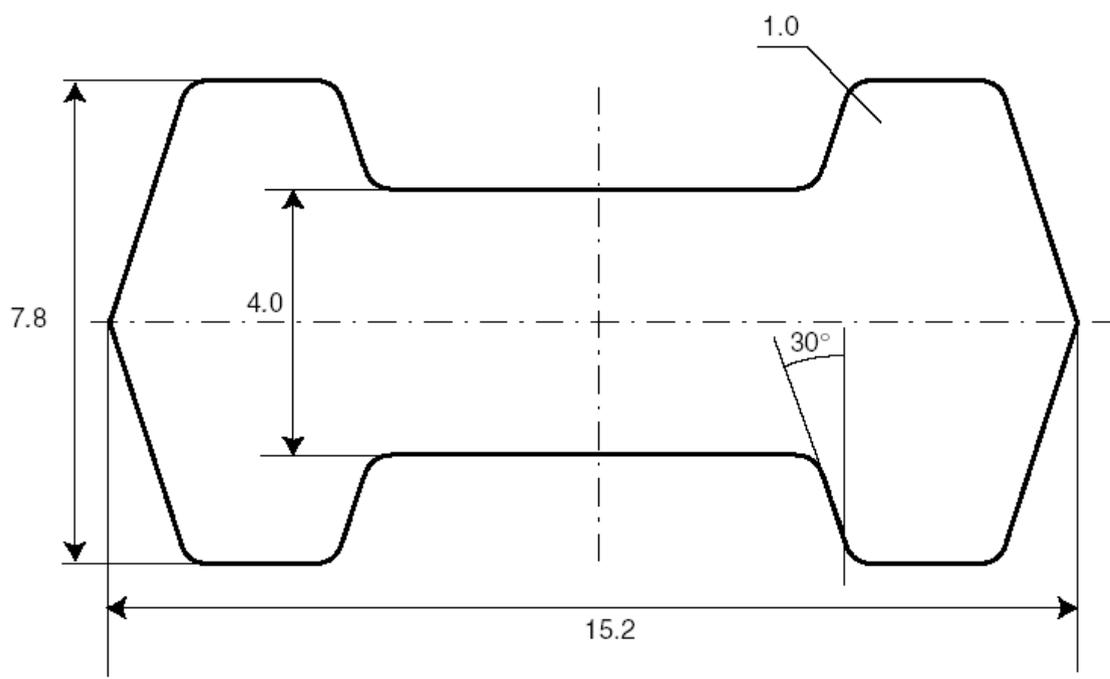
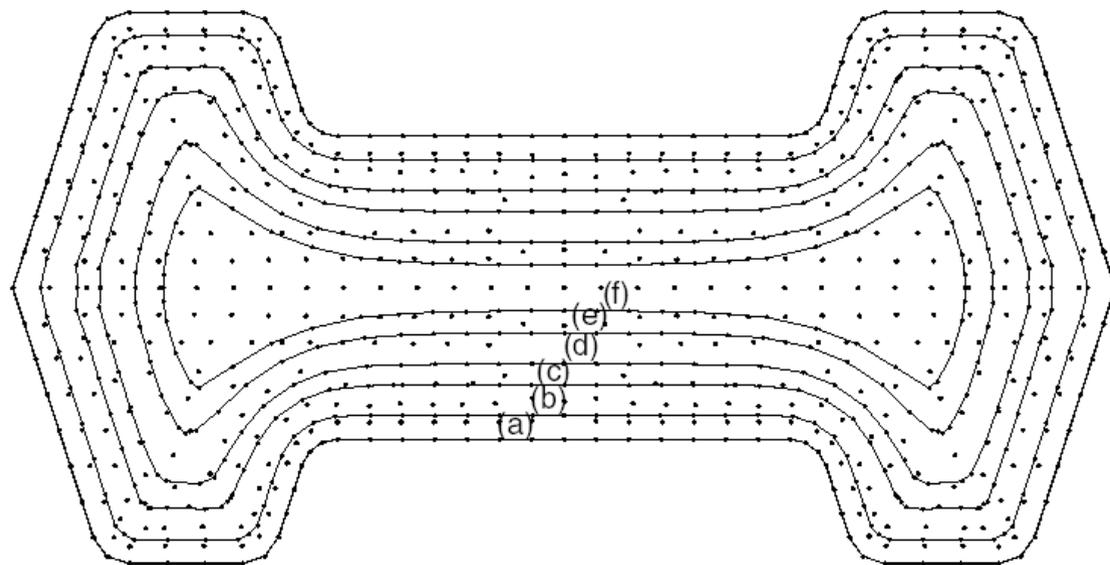
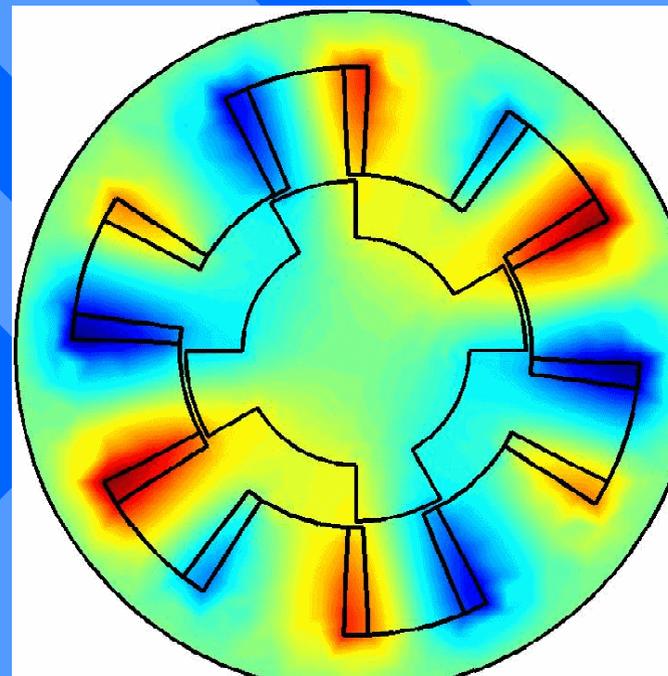
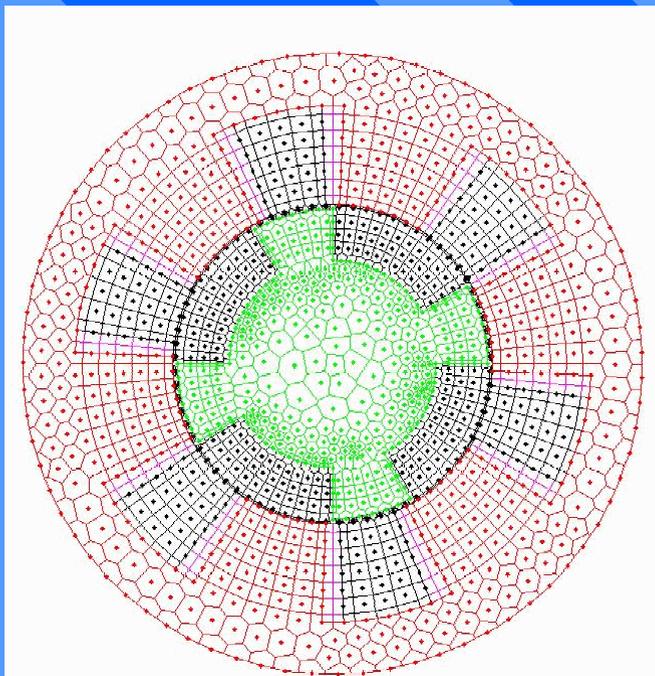
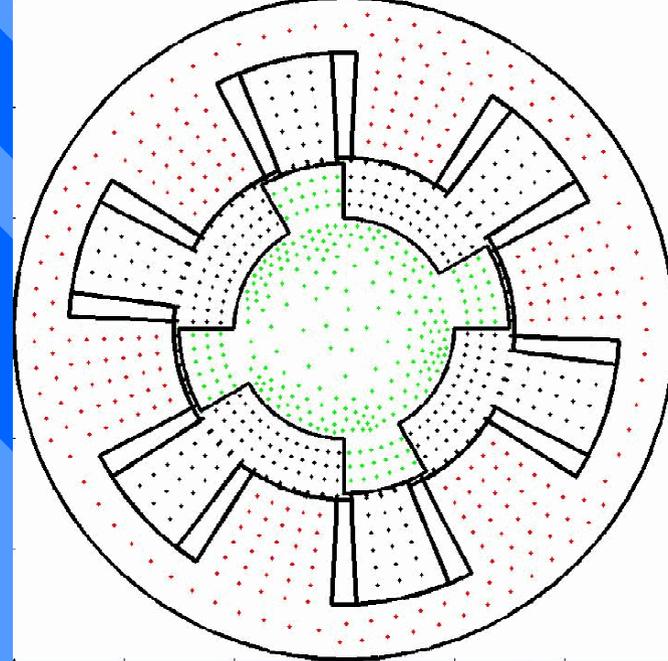


Figure 12. Aluminium part.



Machine électrique

$$\text{Div}(\nu \text{Grad} A) + J = 0$$

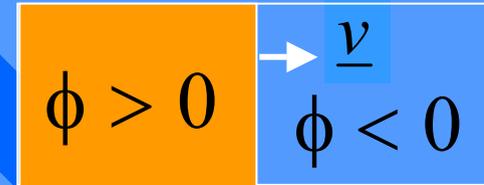


Exemples numériques : (VII) Diphasique

L'approche Level-Set

$$\frac{\partial \phi}{\partial t} + \underline{V} \text{Grad} \phi = 0$$

$$\underline{V}|_{\phi=0} = \underline{v}|_{\phi=0}$$



$\phi = 0$

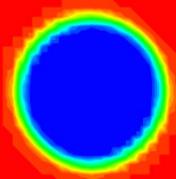
Higher order solvers to compute distance functions

Osher, Sussman, Smereka,
Fatemi, Sethian, Paredes, ...

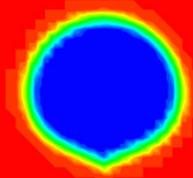
$$\frac{\rho_f}{\rho_g} = \frac{10}{1.2} \quad \frac{\eta_f}{\eta_g} = 1000$$

$$\frac{\partial \tilde{\phi}}{\partial t} + \text{sgn}(\phi) \frac{\text{Grad} \tilde{\phi}}{\|\text{Grad} \tilde{\phi}\|} \text{Grad} \tilde{\phi} = \text{sgn}(\phi)$$

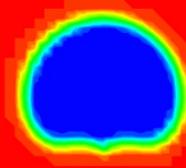
$t = 0.1$



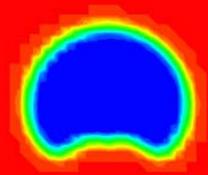
$t = 0.4$



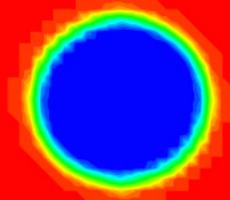
$t = 0.7$



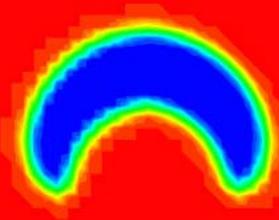
$t = 1$



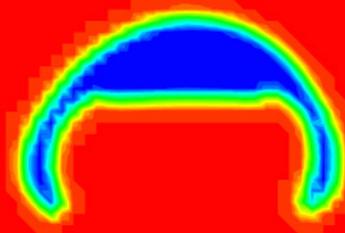
$t = 0.1$



$t = 0.4$



$t = 0.7$

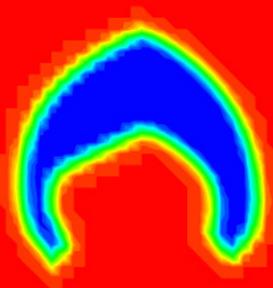


$$\frac{\rho_f}{\rho_g} = \frac{100}{1.2}$$

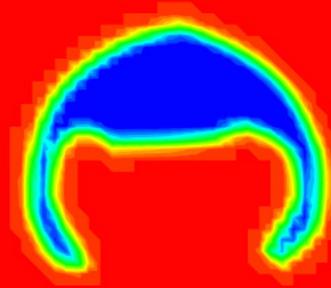
$$\frac{\eta_f}{\eta_g} = 1000$$

$$\frac{\rho_f}{\rho_g} = \frac{1000}{1.2}$$

$t = 0.4$



$t = 0.54$



$$\int_{\Omega} \rho(\phi) \frac{d\underline{v}}{dt} \underline{v}^* d\Omega - \int_{\Omega} p \operatorname{Div} \underline{v}^* d\Omega + 2 \int_{\Omega} \eta(\phi) \underline{\underline{D}} : \underline{\underline{D}}^* d\Omega =$$

$$= \sigma \int_{\Omega} H(\phi) \operatorname{Grad} k \underline{v}^* d\Omega + \sigma \int_{\Omega} H(\phi) k \operatorname{Div} \underline{v}^* d\Omega + \int_{\Omega} \rho(\phi) \underline{g} \underline{v}^* d\Omega$$

$$- \sigma \int_{\Omega} k \underline{v}^* \operatorname{Grad} H(\phi) d\Omega$$

$$\int_{\Omega} \rho \frac{d\underline{v}}{dt} \underline{v}^* d\Omega = \int_{\Omega} \rho \frac{\underline{v}^n(\underline{x}) - \underline{v}^{n-1}(\underline{X})}{\Delta t} \underline{v}^* d\Omega$$

$$\underline{x} = \underline{X} + \underline{v}^{n-1}(\underline{X})$$

Exemples numériques : (VIII) Dynamique

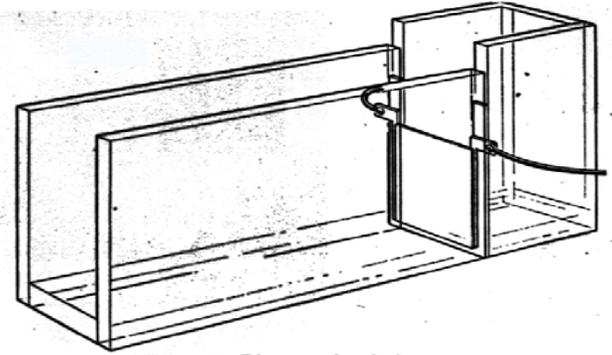
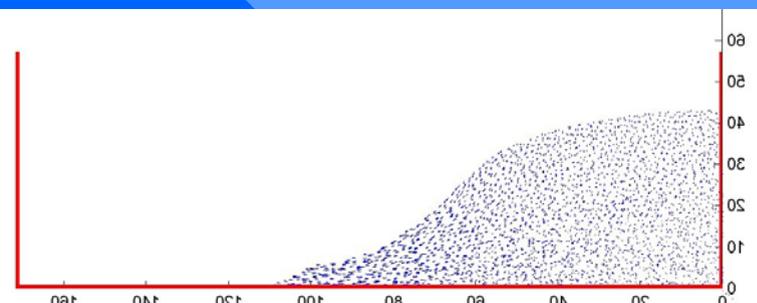
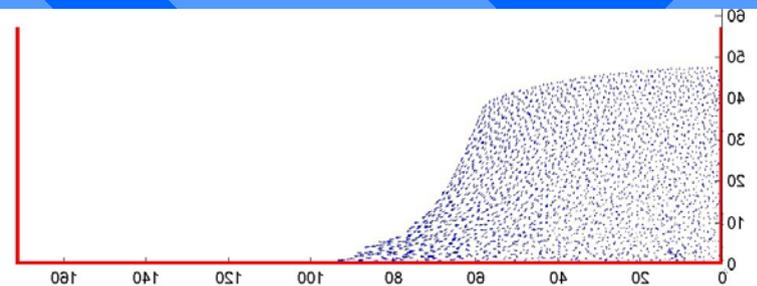
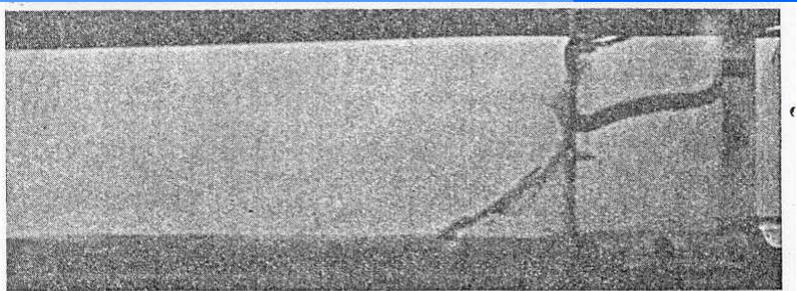
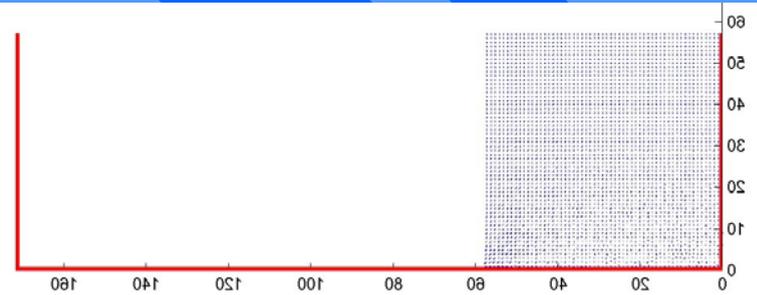
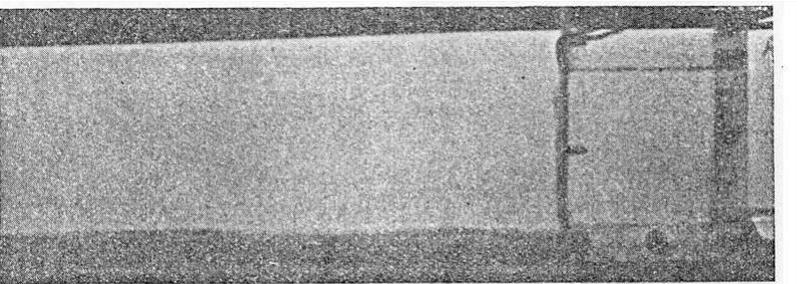
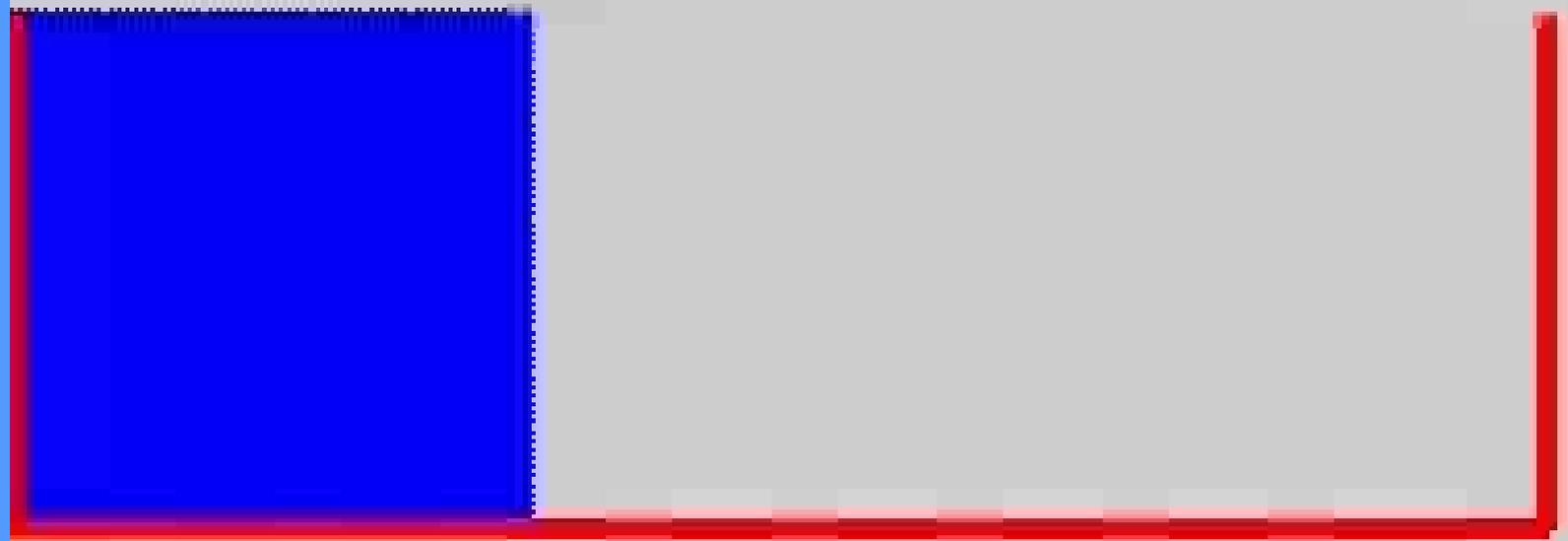


FIGURE 1. Diagram of typical apparatus.





Et les recherches en cours ...

« Capturing » les discontinuités mobiles:

C-NEM - Level Sets

3D  Nessy

Définition d'une dérivée nodal diffuse

Le blocage dans la limite de l'incompressibilité

Multi-échelle (réduction de modèles, P.U., ...)

Thermomécanique couplée: endommagement, ...

Adaptativité, ...