Experiments in Transient Nonlinear Energy Pumping in Vibrating Mechanical Systems

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Aim of this Work

- To develop and implement a new approach to passively control vibration and shock in flexible structures.
- This technique is based on passively channeling vibrational energy into nonlinear energy sinks (NESs), where it is confined and dissipated.

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Related Technologies

- Current methods

  Damping

  Linear vibration absorbers

  Active and semi-active control

  Redesign of structure

- Proposed method

  Nonlinear energy sink (NES)

Originality of Our Approach

- NESs are capable of passively absorbing and dissipating **broadband** (transient) **disturbances**
- NESs can **nonlinearly interact with** a **series of structural modes**, extracting a significant amount of energy from each before engaging the next
- In contrast to previous work on nonlinear vibration absorbers, **general transient, strongly nonlinear responses are considered**. The techniques developed directly address the transient problem as well as the steady state.

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Practicality of the NES

- Modular and, hence, can be connected to existing structures with minimal modification
- Lightweight and of simple design
- Passive and does not require power to operate
- Inexpensive, especially when compared to structural redesign
- Although they are local attachments, NESs can affect the global structural dynamics

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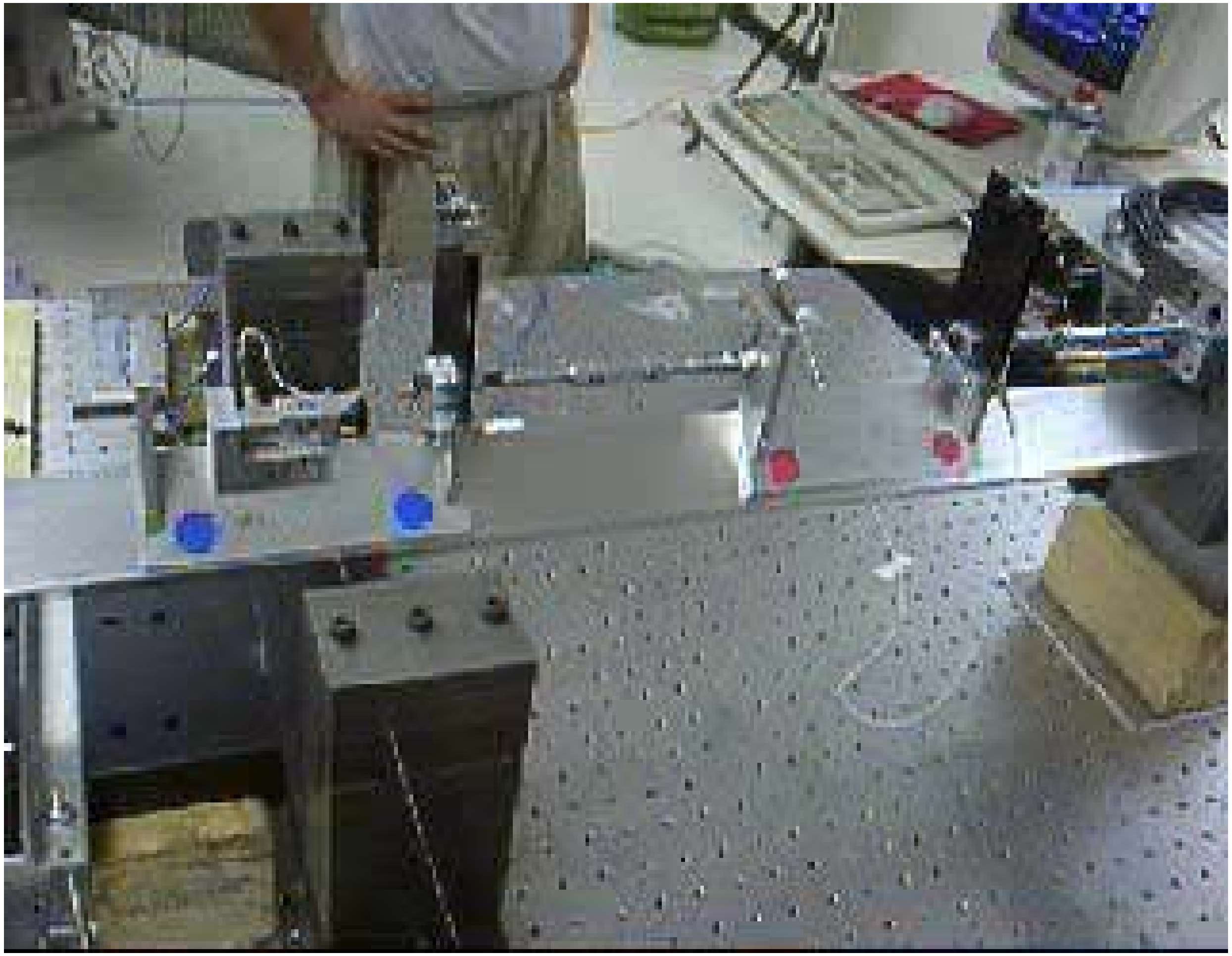
Introduction

- Energy pumping is the passive, one-way, rapid and irreversible transfer of energy from a vibrating main structure to an attached nonlinear energy sink
- The main structure can be either linear or nonlinear, while the coupling with the NES is assumed to be linear and weak
- The NES possesses essential (nonlinearizable) stiffness nonlinearity

Early Experiments

- A system where the NES possesses no damper

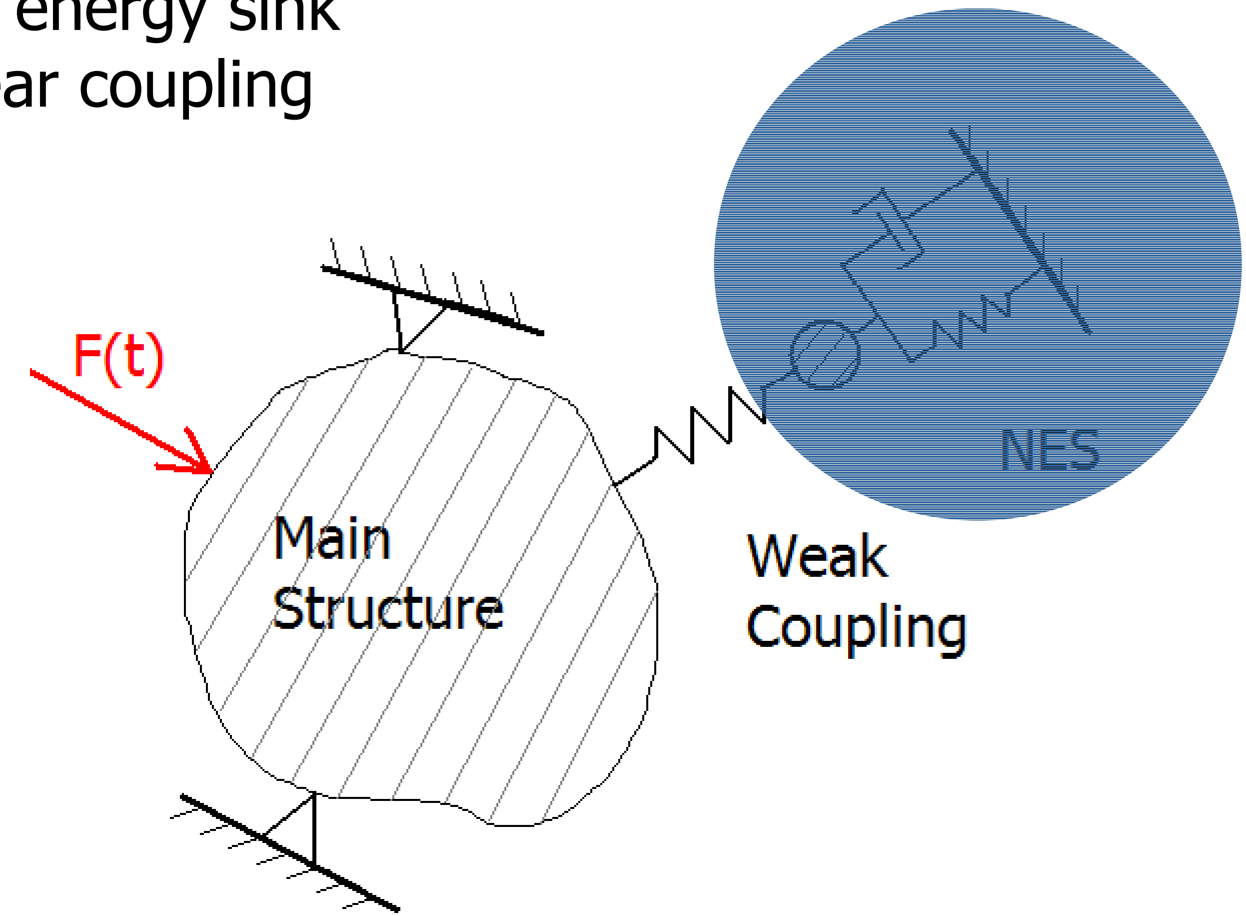


Early Experiments

- A damper has been added to the NES

Basic Configuration

- Linear primary structure
- Nonlinear energy sink
- Weak linear coupling



Theoretical Basis

- Consider an (N+1) DOF linear main structure coupled to an essentially nonlinear NES. In terms of modal coordinates the structural response at the point of attachment O is expressed in the form

$$y_0(t) = \sum_{k=0}^{N} \phi_0^{(k)} a_k(t) \qquad (1)$$

where $\phi_0^{(k)}$ denotes the element at position O of the k-th mass-normalized eigenvector of the uncoupled structure (with $\varepsilon=0$), and $a_k(t)$ represents the k-th modal amplitude of the structure.

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Theoretical Basis

- Then, the equations of motion of the system are expressed as

$$\ddot{v}(t) + Cv^3(t) + \varepsilon\lambda\dot{v}(t) + \varepsilon\left(v - \sum_{k=0}^{N} \phi_0^{(k)} a_k(t)\right) = 0$$

$$\ddot{a}_m(t) + \omega_m^2 a_m(t) + \varepsilon\lambda\dot{a}_m(t) + \varepsilon\left(\sum_{k=0}^{N} \phi_0^{(k)}\phi_0^{(m)} a_k(t) - \phi_0^{(m)} v(t)\right) = 0 \quad ,$$

$$m = 0,1,\ldots,N \qquad (2)$$

- Consider the nonlinear resonance interactions between the NES and an individual mode, say the zero-th one, of the main structure

Theoretical Basis: NNMs

- Dynamics of the unforced, undamped system

$$\ddot{v}(t) + Cv^3(t) + \varepsilon\left(v(t) - \phi_0^{(0)} a_0(t) - \sum_{k=1}^{N} \phi_0^{(k)} a_k(t) \right) = 0$$

$$\ddot{a}_0(t) + \omega_0^2 a_0(t) + \varepsilon\left(\phi_0^{(0)2} a_0(t) - \phi_0^{(0)} v(t) + \sum_{k=1}^{N} \phi_0^{(k)} \phi_0^{(0)} a_k(t) \right) = 0$$

$$\ddot{a}_m(t) + \omega_m^2 a_m(t) + \varepsilon\left(\sum_{k=0}^{N} \phi_0^{(k)} \phi_0^{(m)} a_k(t) - \phi_0^{(m)} v(t) \right) = 0 \ ,$$

$$m = 1,\ldots,N$$

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Theoretical Basis: NNMs

The physical NNM oscillations of the NES and the point of attachment of the structure are approximated as

$$v(t) = \frac{A_v}{\omega_0}\sin\left(\omega_0 t + \gamma_v(t) + O(\varepsilon^2)\right) + O(\varepsilon)$$

$$y_0(t) = \phi_0^{(0)}\frac{A_0}{\omega_0}\sin\left(\omega_0 t + \gamma_0(t) + O(\varepsilon^2)\right) + O(\varepsilon)$$

$$\dot{\gamma}_v = \dot{\gamma}_0 = \frac{\varepsilon}{2\omega_0}\left[\phi_0^{(0)^2} - \phi_0^{(0)}\frac{A_v}{A_0}\right]$$

with frequency

$$\Omega_0 \approx \omega_0 + \dot{\gamma}_v = \omega_0 + \frac{\varepsilon}{2\omega_0}\left[\phi_0^{(0)^2} - \phi_0^{(0)}\frac{A_v}{A_0}\right]$$

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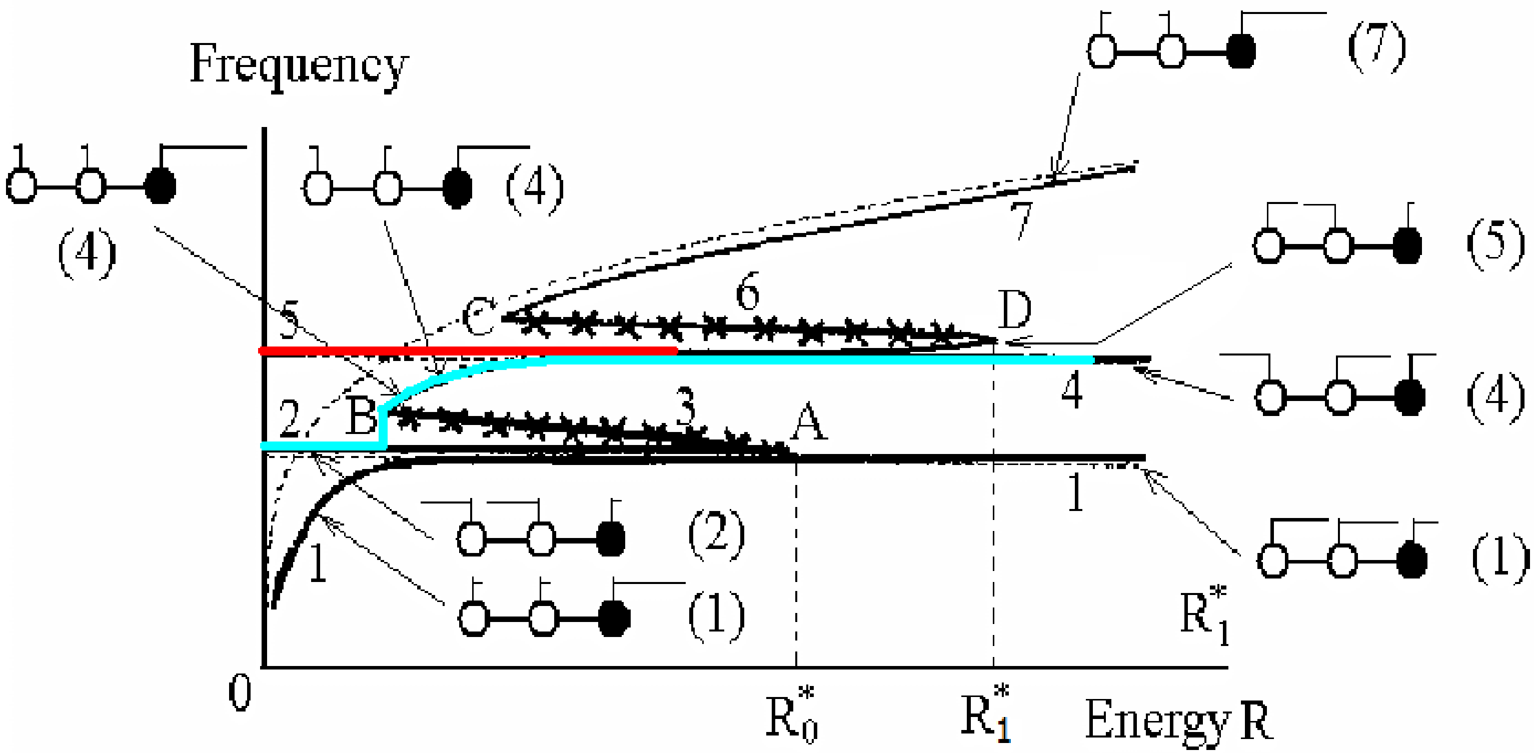
Theoretical Basis

Schematically, we synthesize local results, using the physical energy R as the independent variable.



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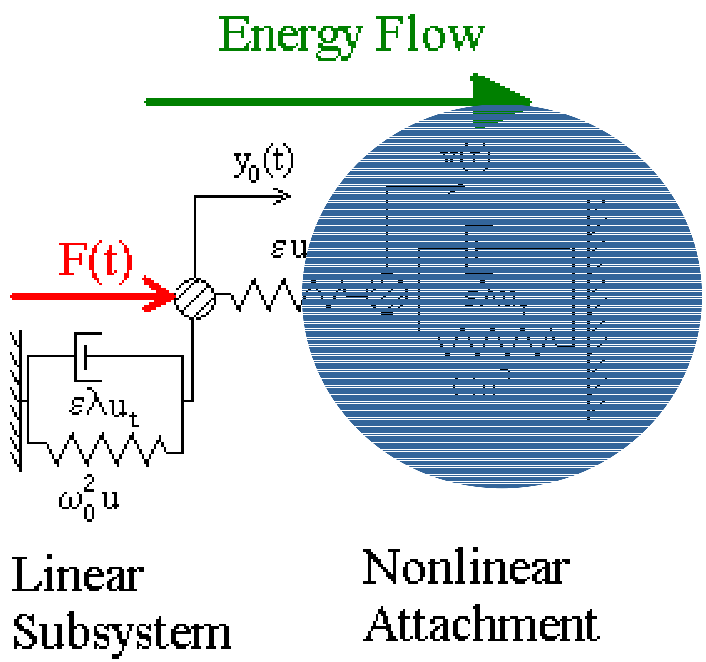
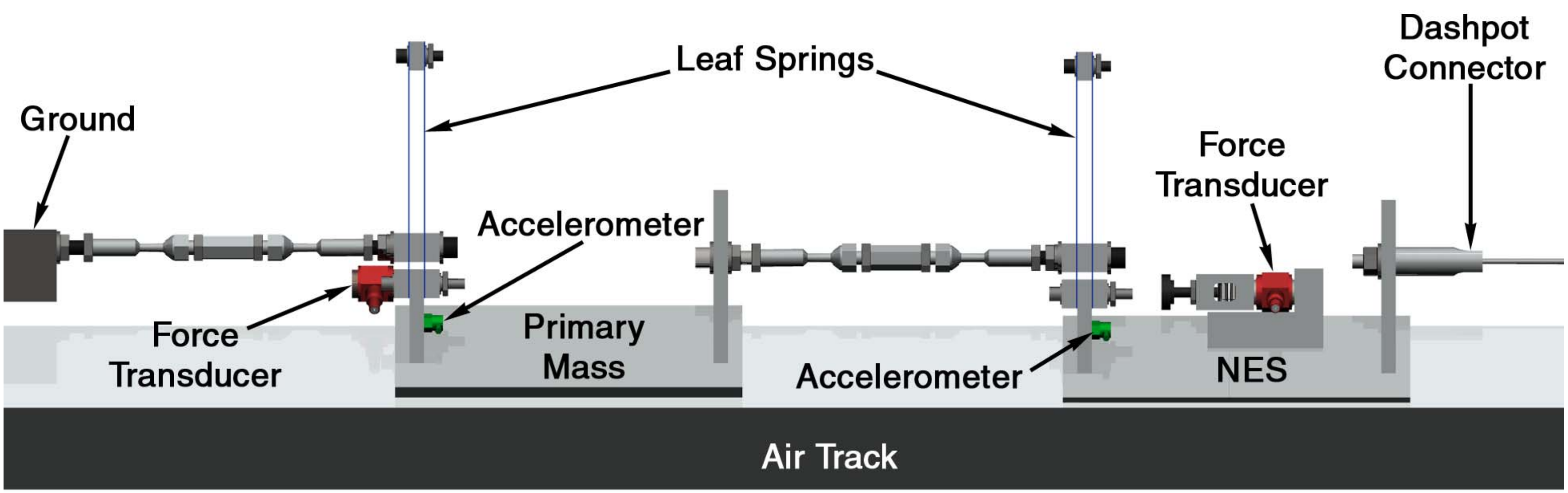
Experimental System: SDOF + NES

Our first experiments were on a 2-DOF system: an SDOF primary structure connected to an NES

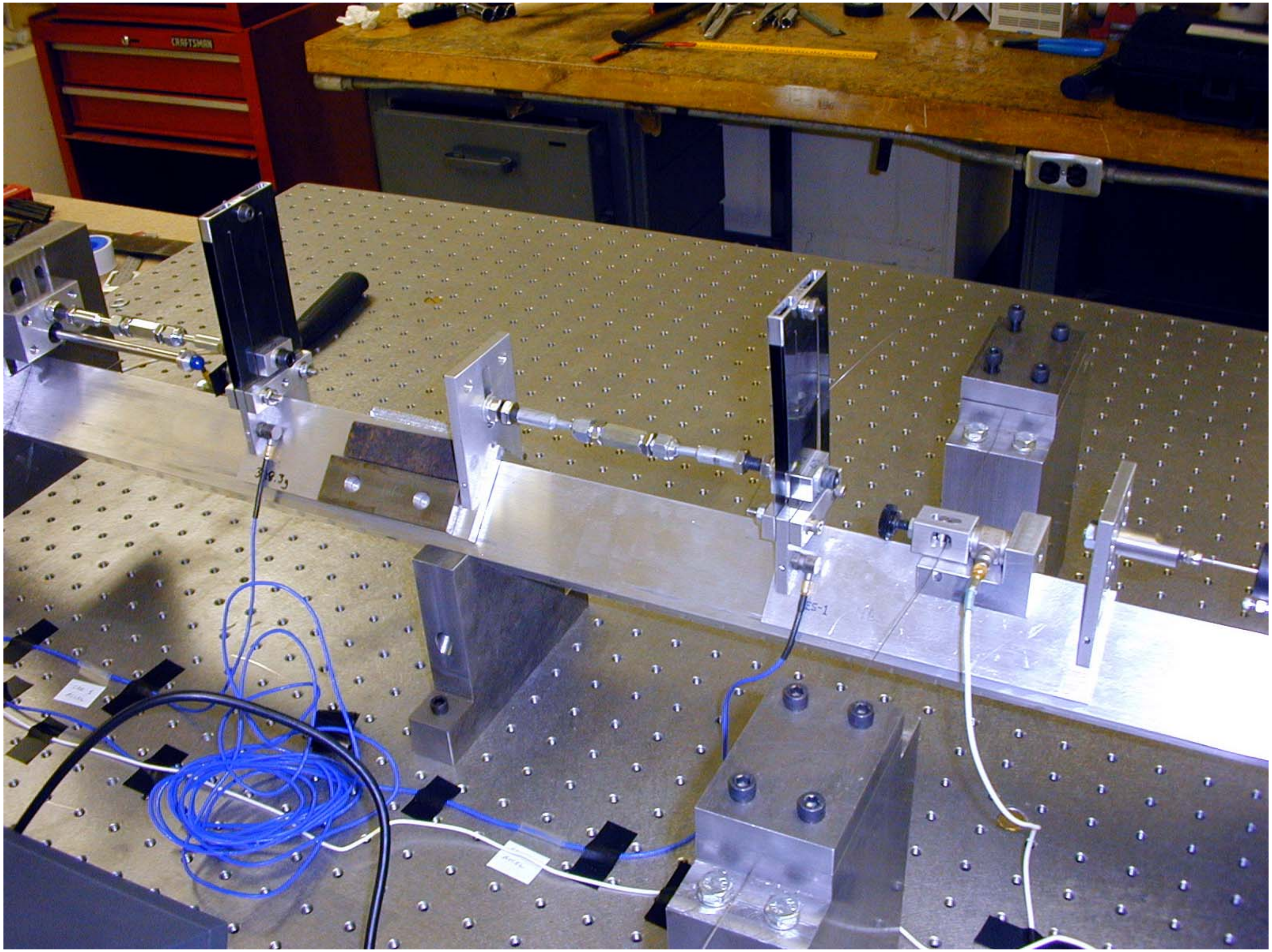


Experimental System: SDOF + NES

- Two "cars" ride on a 1-D air track
- Applied force and response acceleration are measured directly



Experimental System: SDOF + NES



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Experimental System: SDOF + NES

- Linear subsystem

$$M = 0.834\,\text{kg}, \quad K = 993\,\text{N/m}, \quad \varepsilon\lambda = 0.129\,\text{N sec/m}$$

$$\omega_0 = 35.63\ \text{rad/sec} \qquad \zeta = 2.3\times 10^{-3}$$

- Nonlinear energy sink

$$m = 0.393\,\text{kg}, \quad \varepsilon = 114\,\text{N/m}, \quad \varepsilon c = 0.454\,\text{N sec/m}$$

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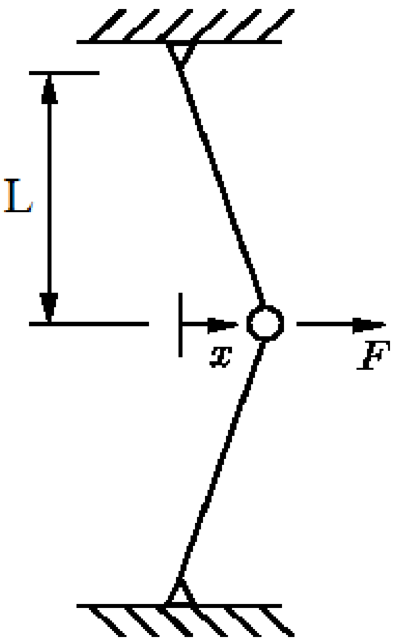
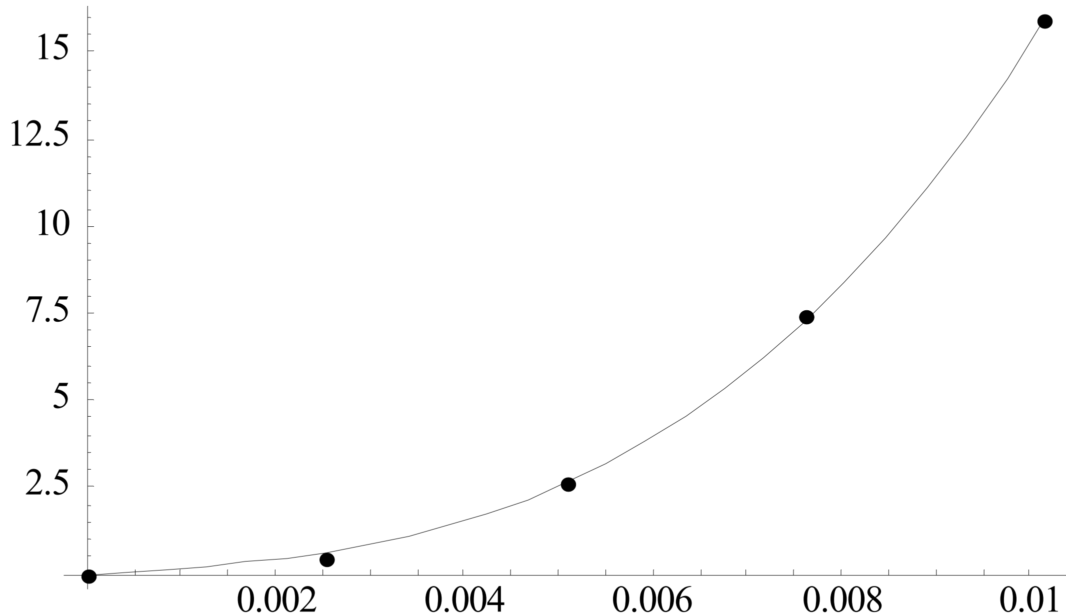
Experimental System

- Nonlinear spring



$$F = \frac{EA}{L^3} x^3 + O(x^5)$$

Fitted polynomial has 1st and 3rd order terms.



$$f(x) = 166x + 1.36\times 10^7 x^3$$

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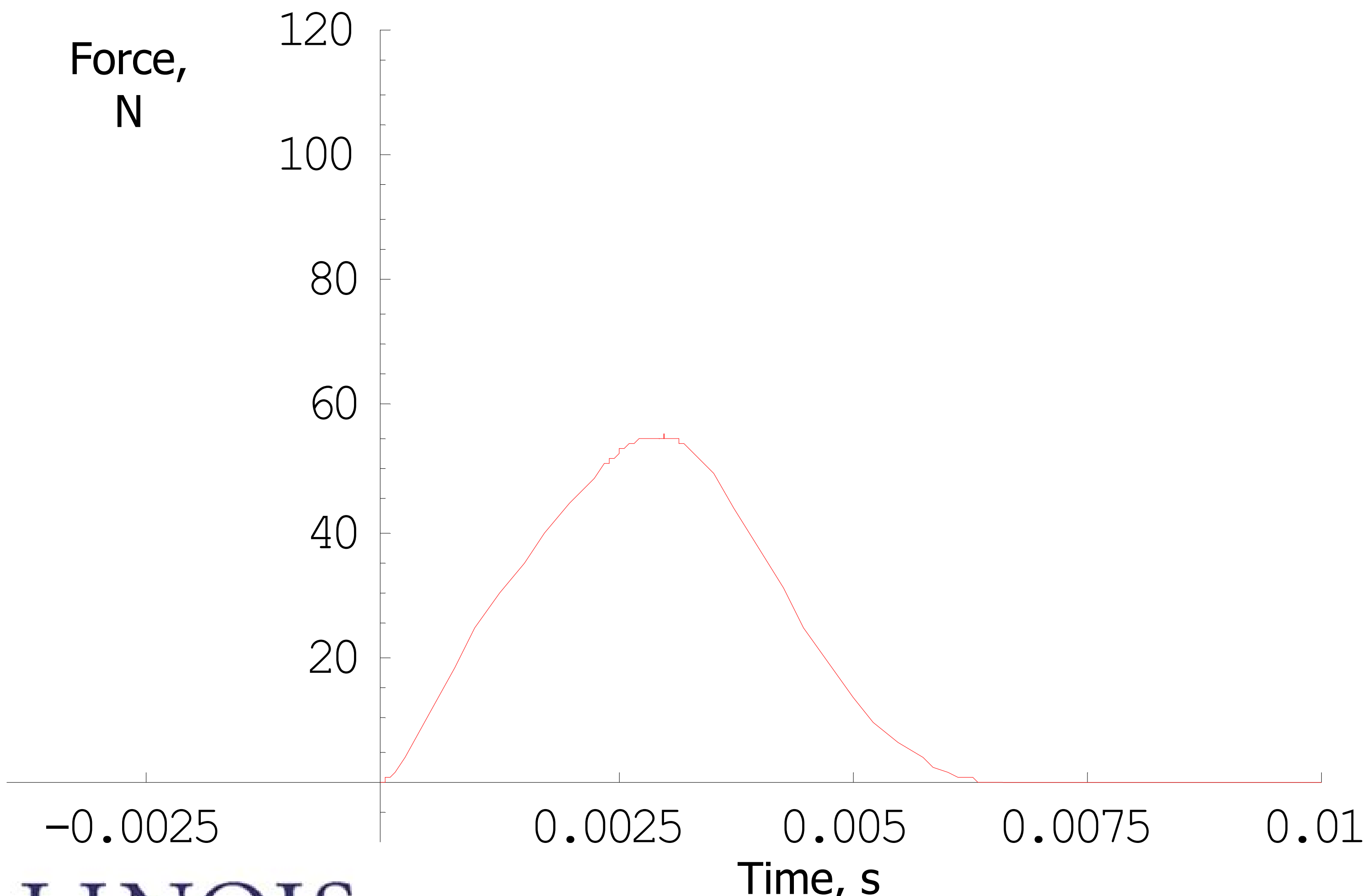
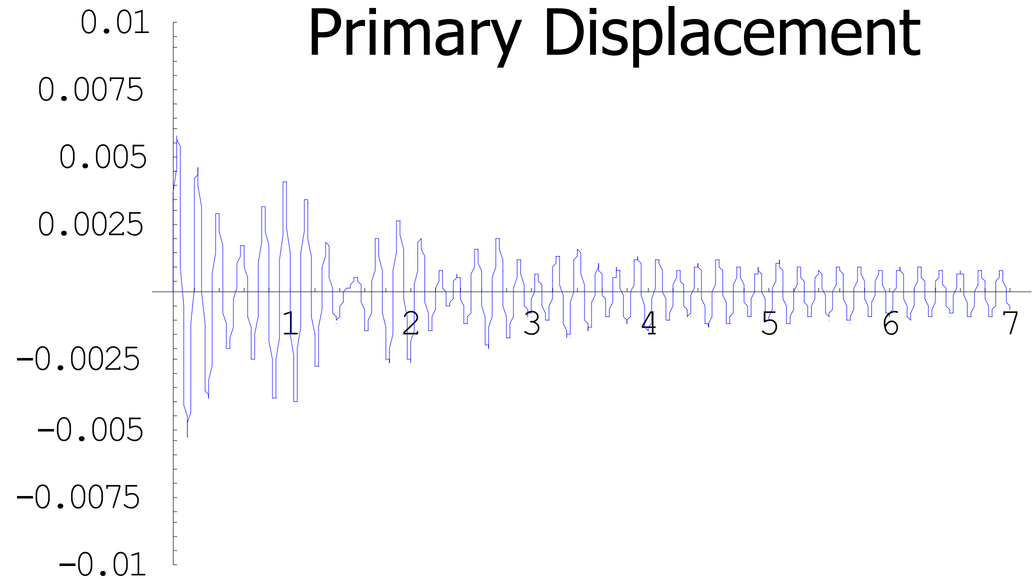
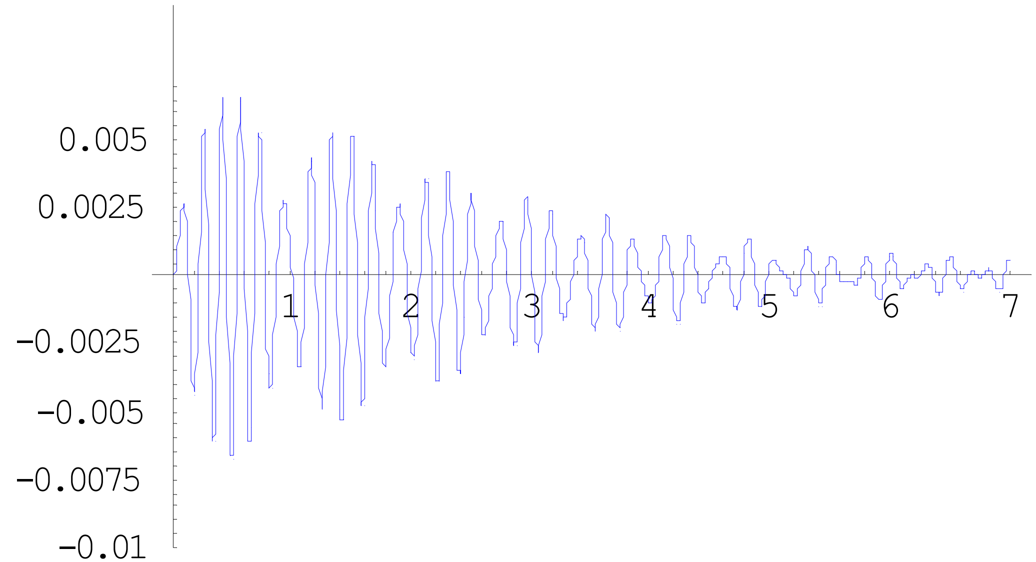
Measured Data

- Typical “strong” force pulse



Simulated Response





Results of numerical integration using measured applied force as input

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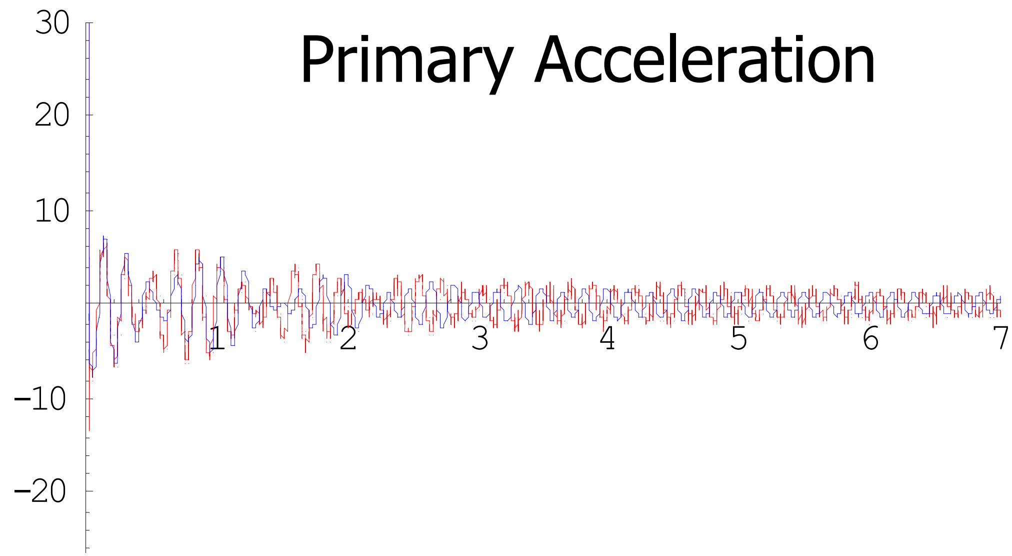
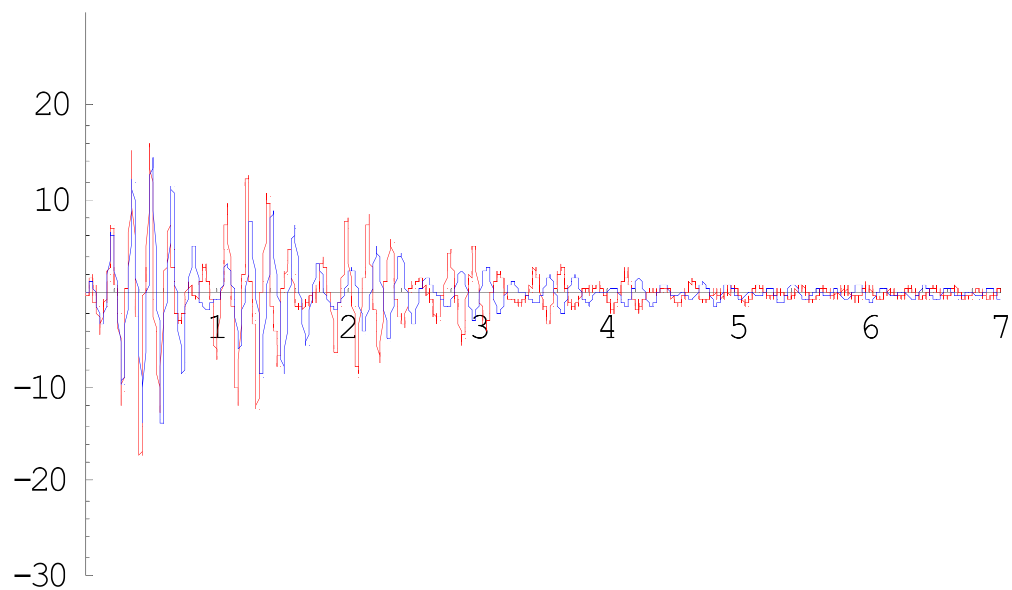
Measured Response & Simulation

- Primary and NES accelerations
- Good agreement of amplitude
- Good agreement of “slow” response
- Distinct transient resonance captures
- Results were repeatable





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Measure of Energy Pumping

- Total energy input

$$E_i(t) = \int_0^t F(\tau)\,\dot{y}(\tau)\,d\tau$$

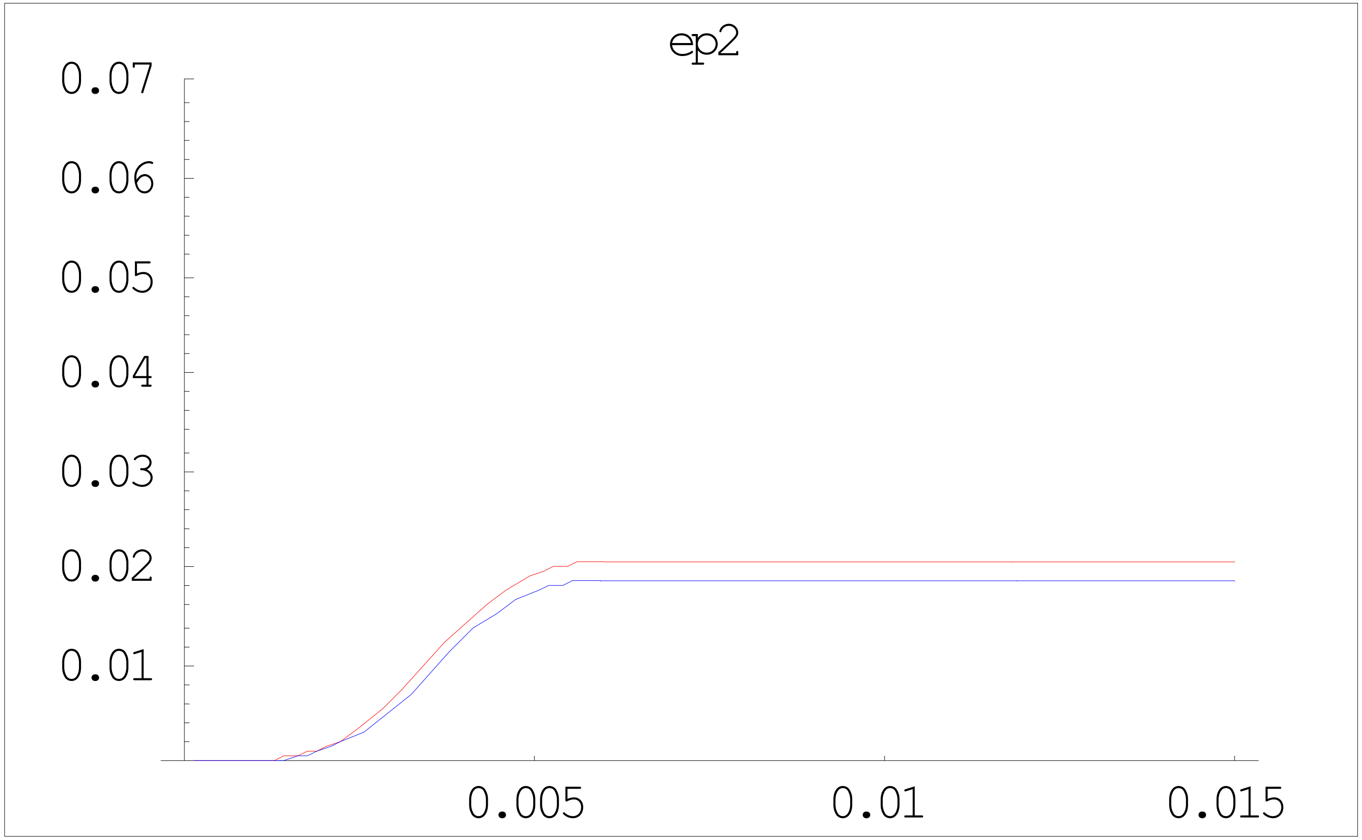
- Energy dissipated in sink dashpot

$$E_{\text{NES}}(t) = \frac{\varepsilon c}{E_i(t_{\max})} \int_0^t \dot{v}^2(\tau)\,d\tau$$

- We compute the normalized dissipation in the NES as a function of time

Measure of Energy Pumping

- All energy input occurs in a few msec.



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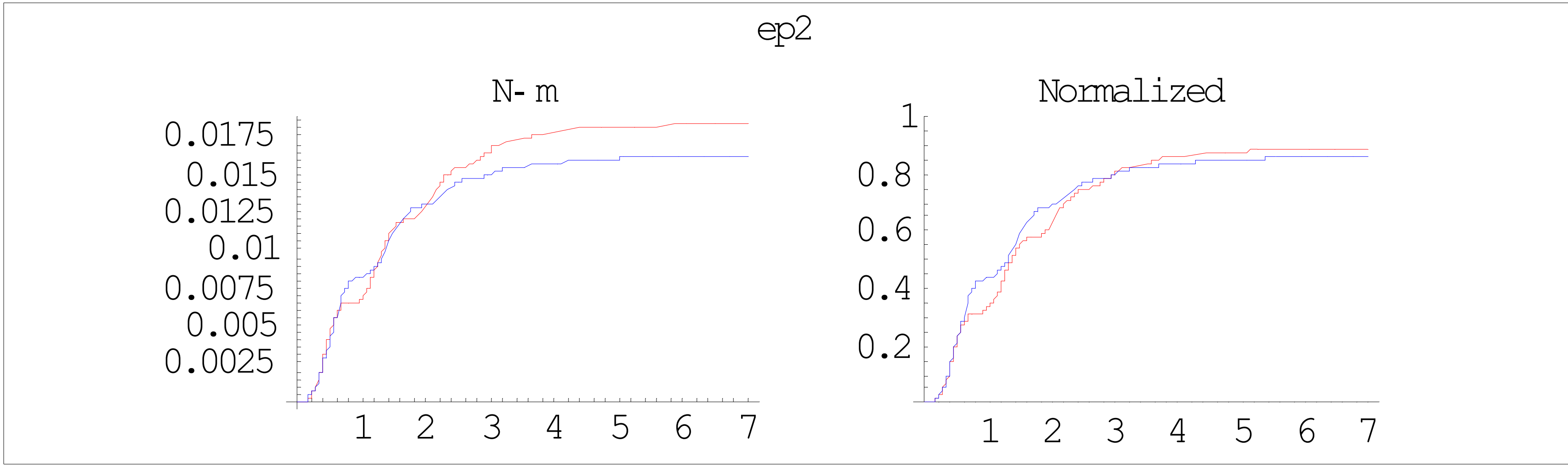
Measure of Energy Pumping

- Dissipation in NES



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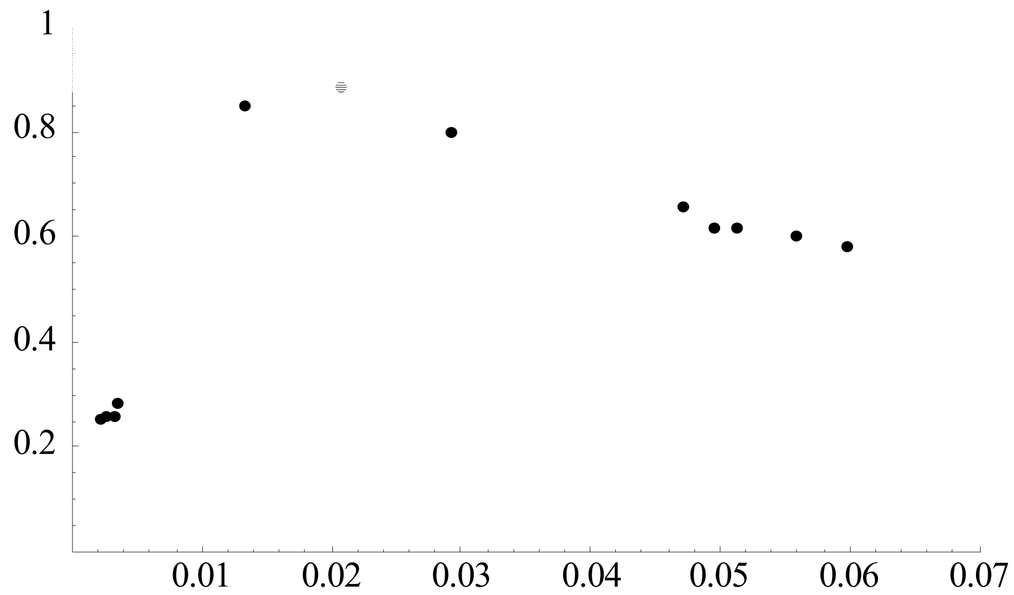
Variation with Level of Forcing

- The existence of a threshold input energy to produce pumping is demonstrated by comparing the results of weak and strong forcing
- Stronger forcing does not always produce more efficient energy pumping

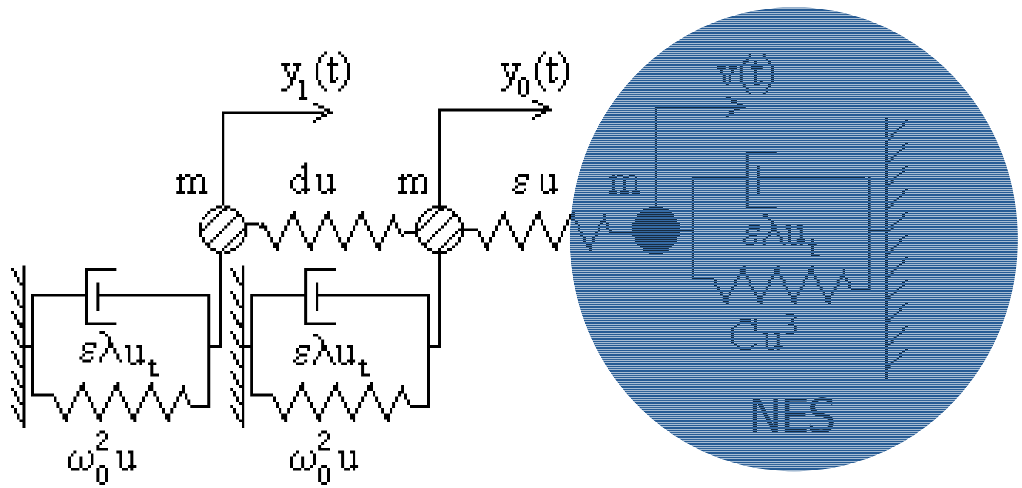
Normalized dissipation in NES



Energy input

Experimental System: 2DOF + NES

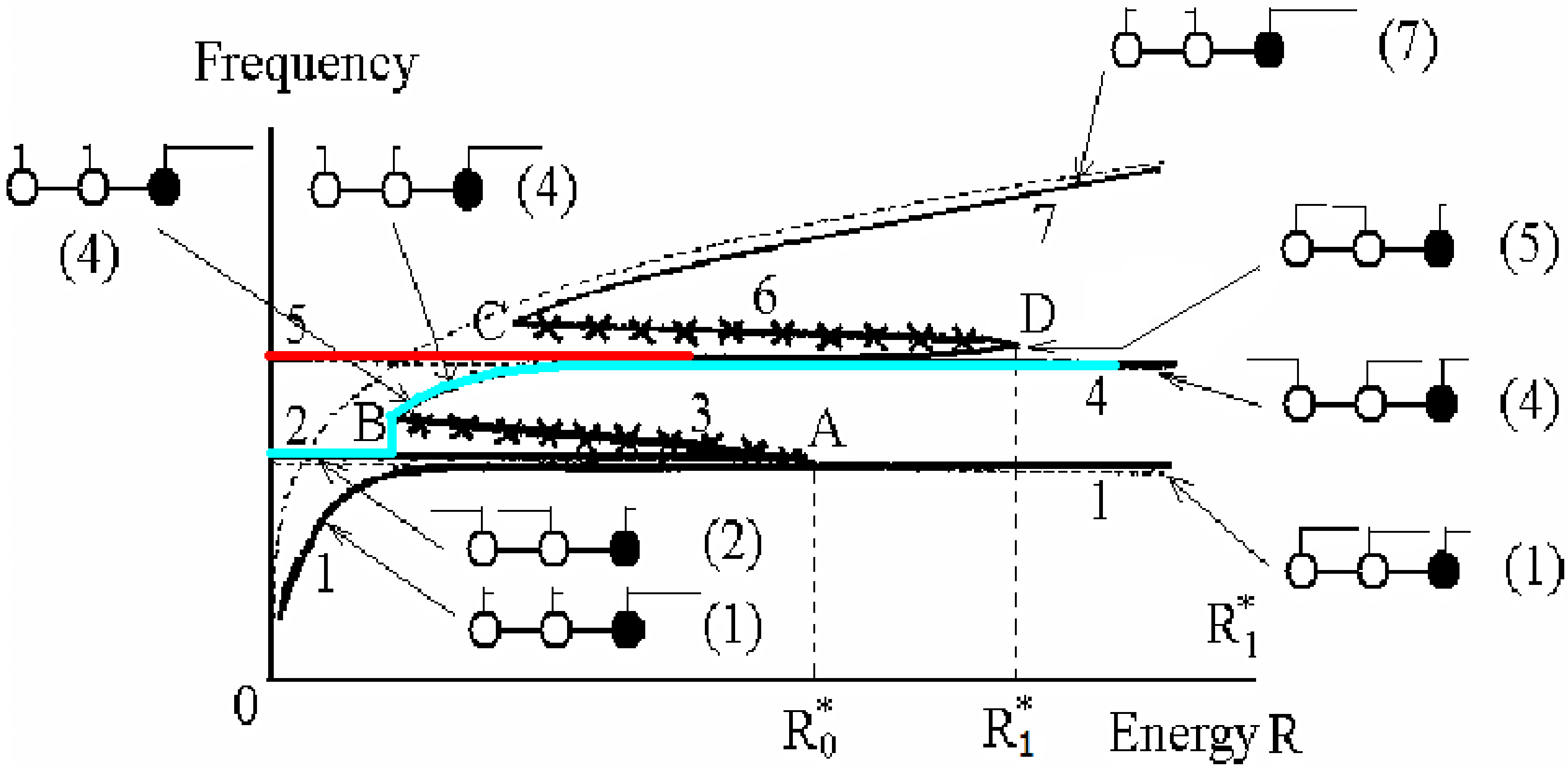
We next consider a 3-DOF system



Linear main system

and seek to demonstrate transient resonance of the NES with both modes of the primary structure

Resonance Capture Cascade



Weak impulse
Strong impulse

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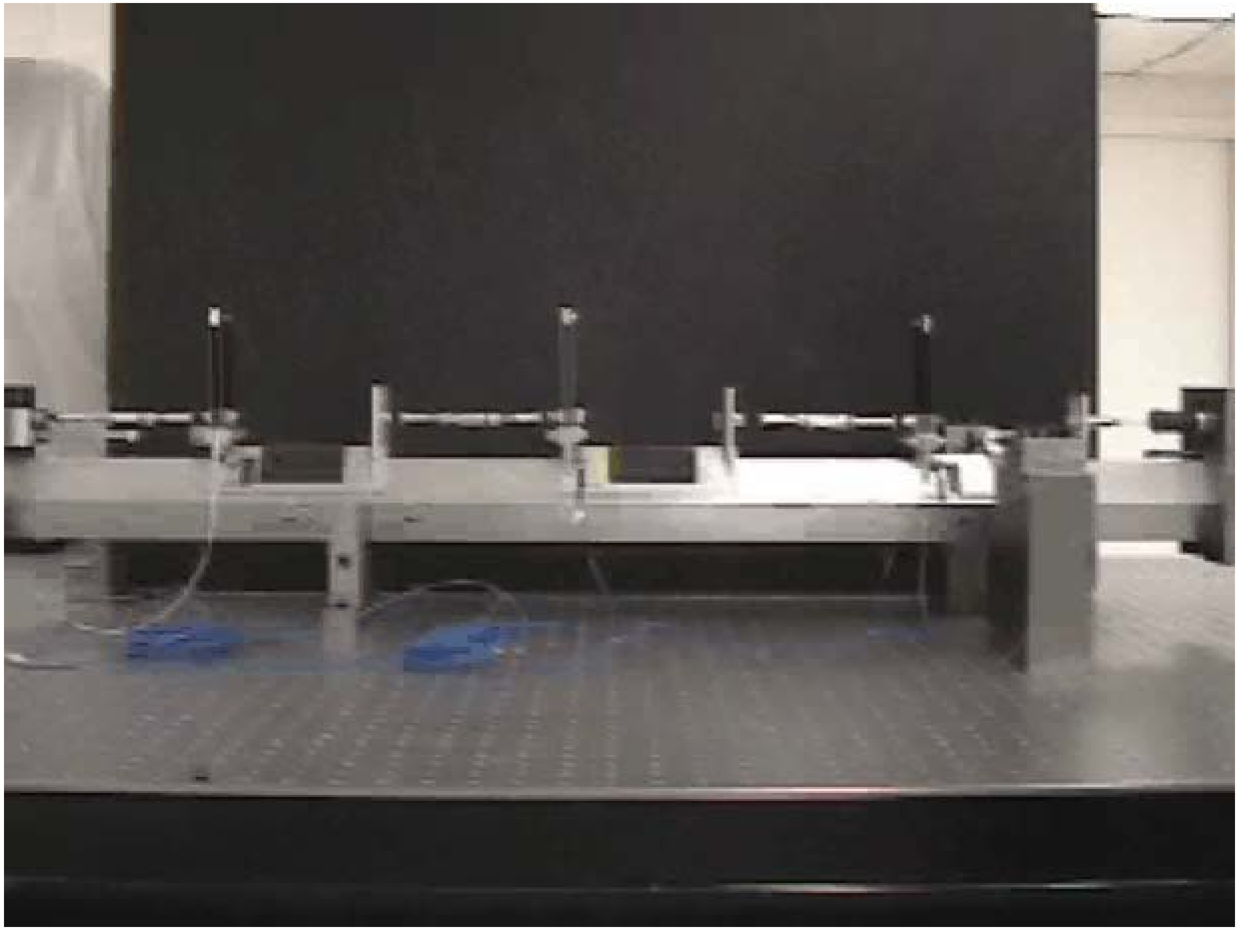
Experimental System: 2DOF + NES

- Additional primary mass
- Same energy sink



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Experimental System: 2DOF + NES

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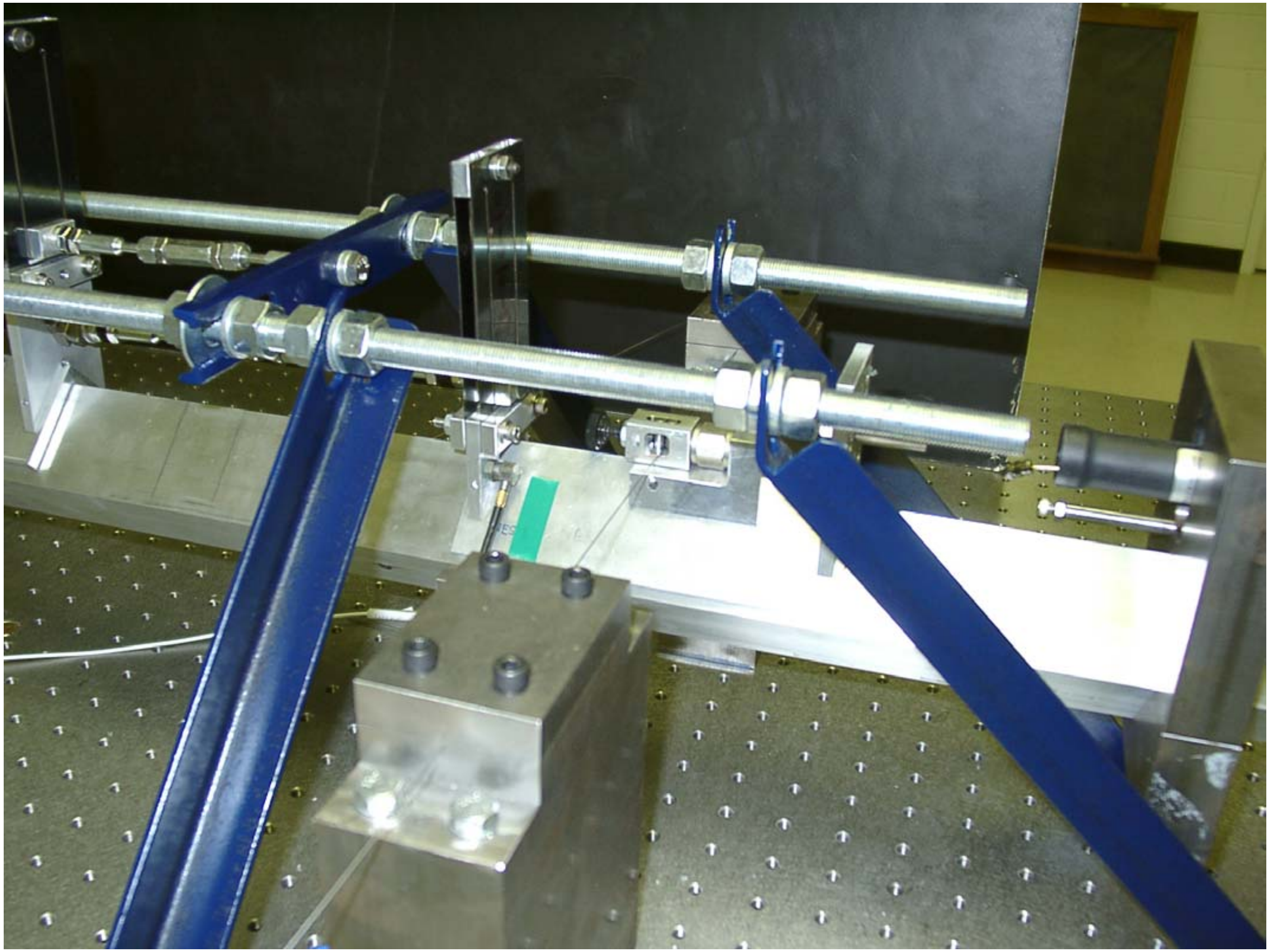

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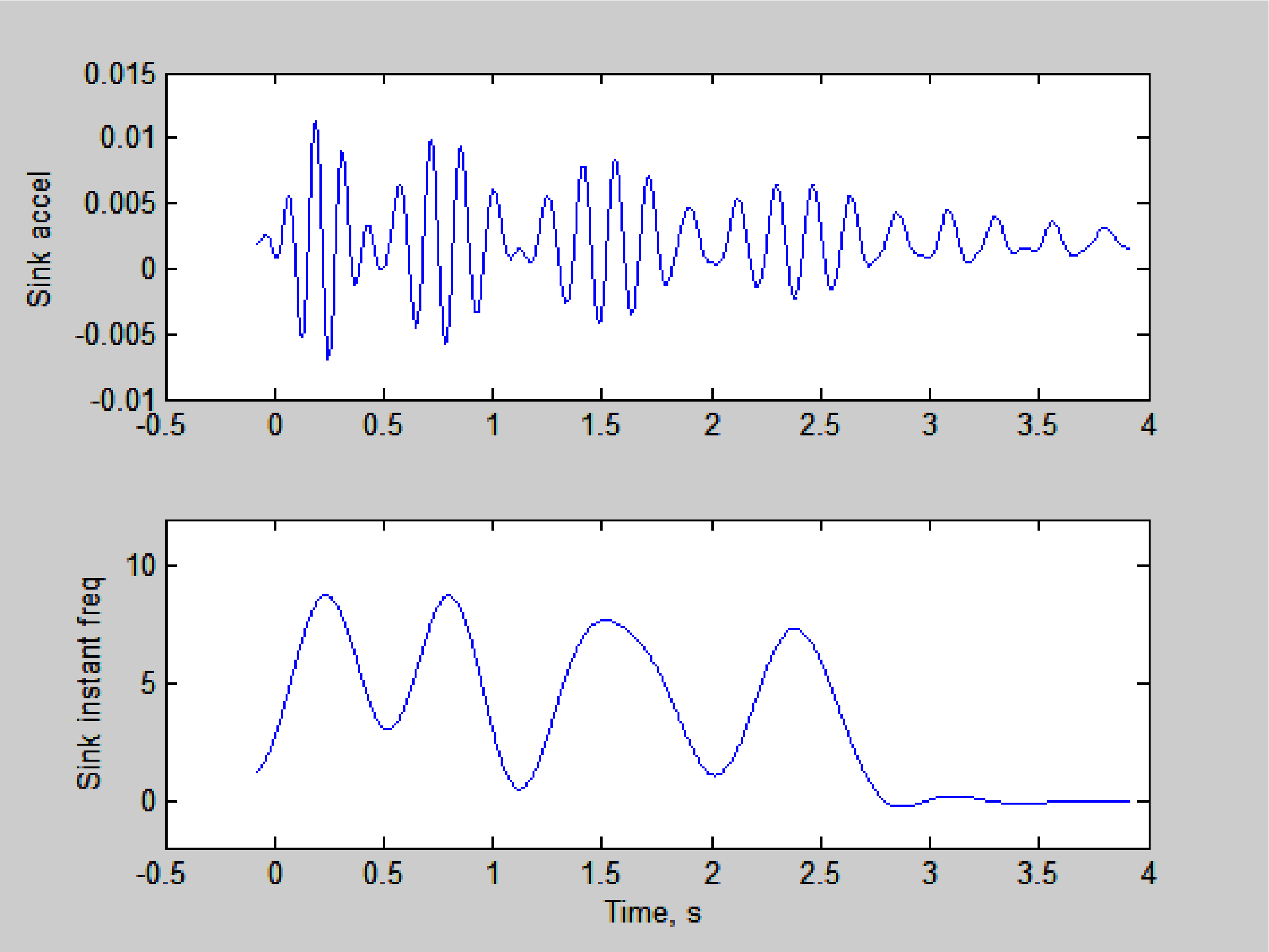
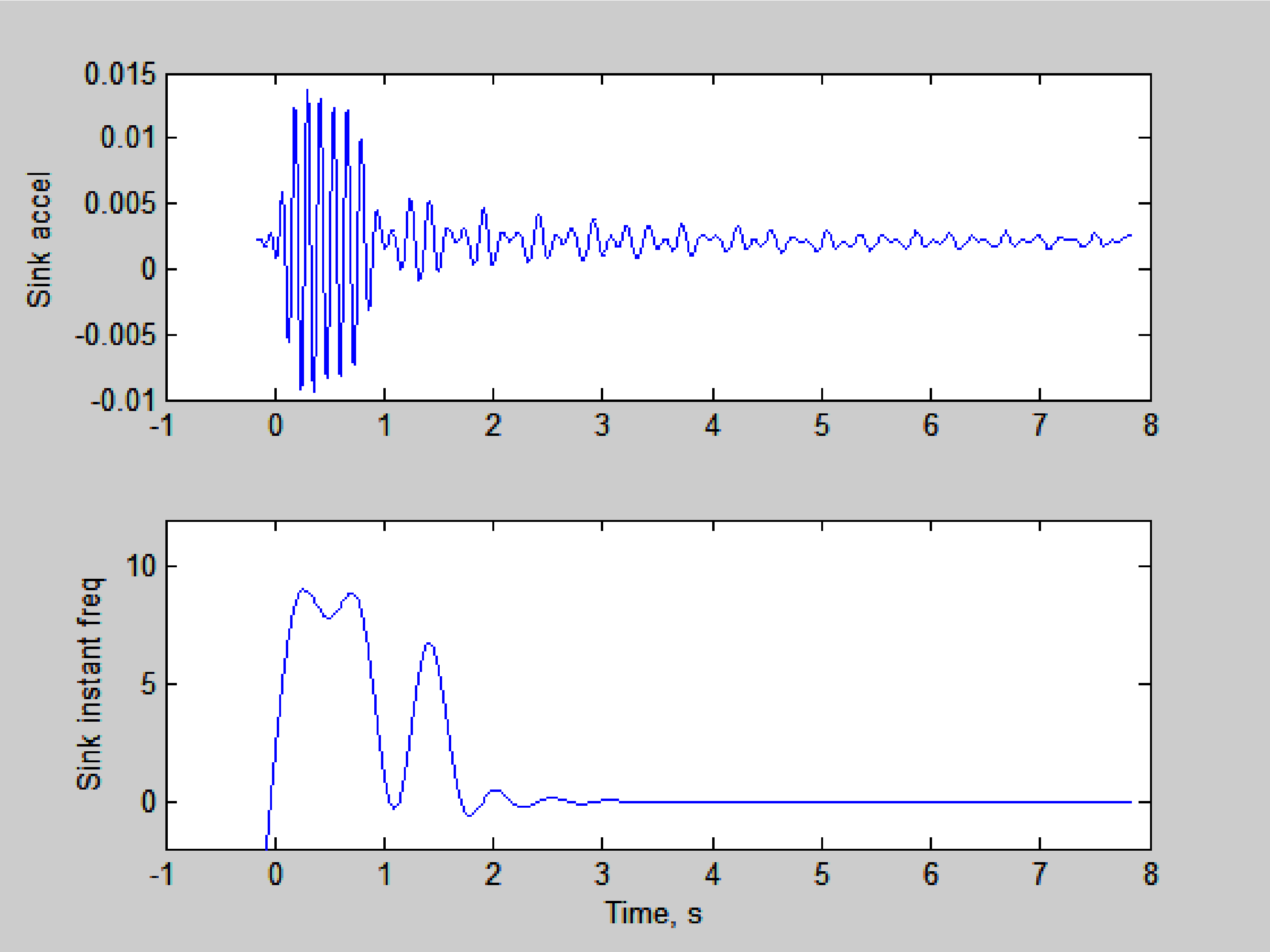
Measured Sink Response

- Pumping from mode 1 only



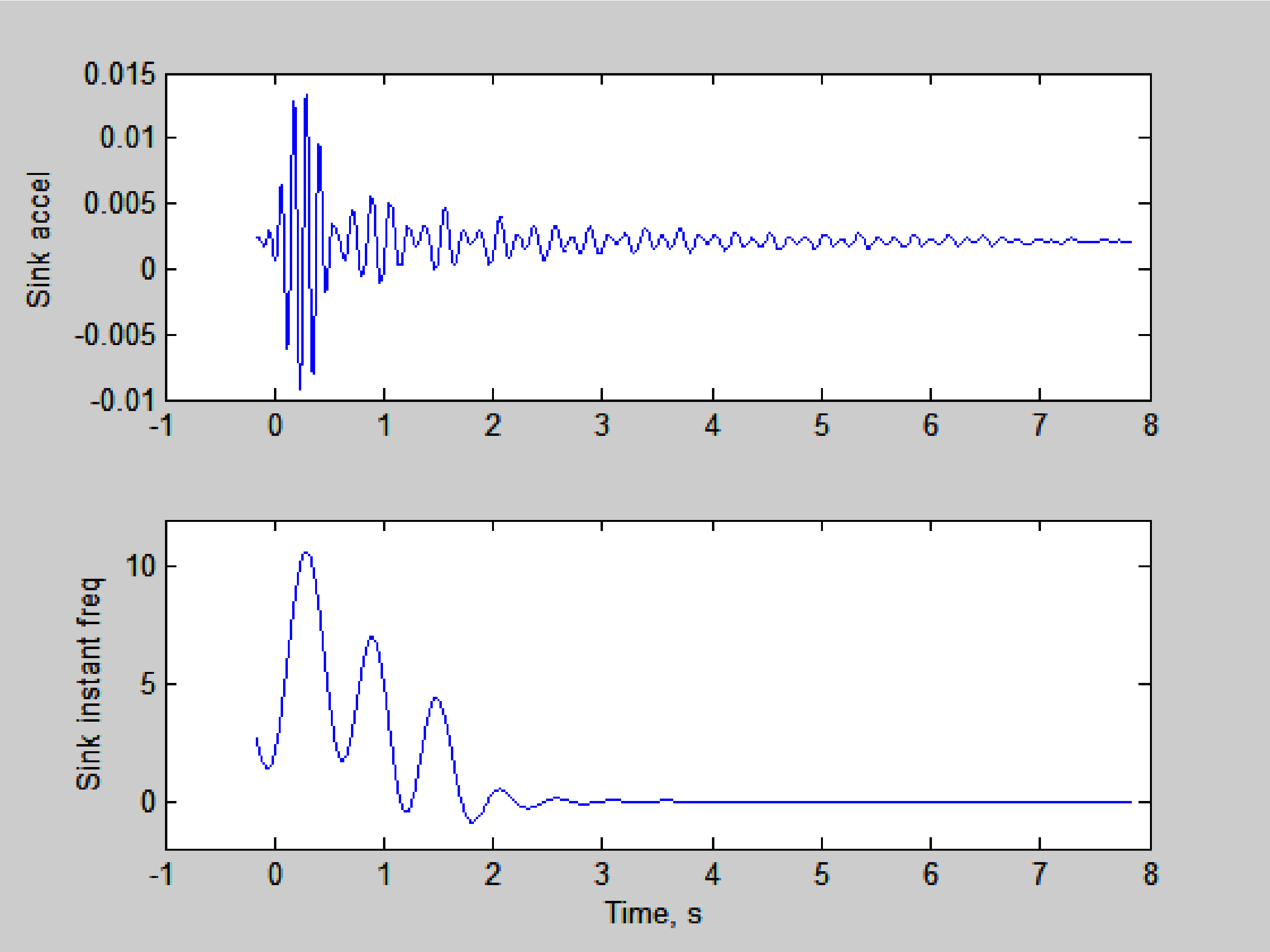
Measured Sink Response

- A brief cascade



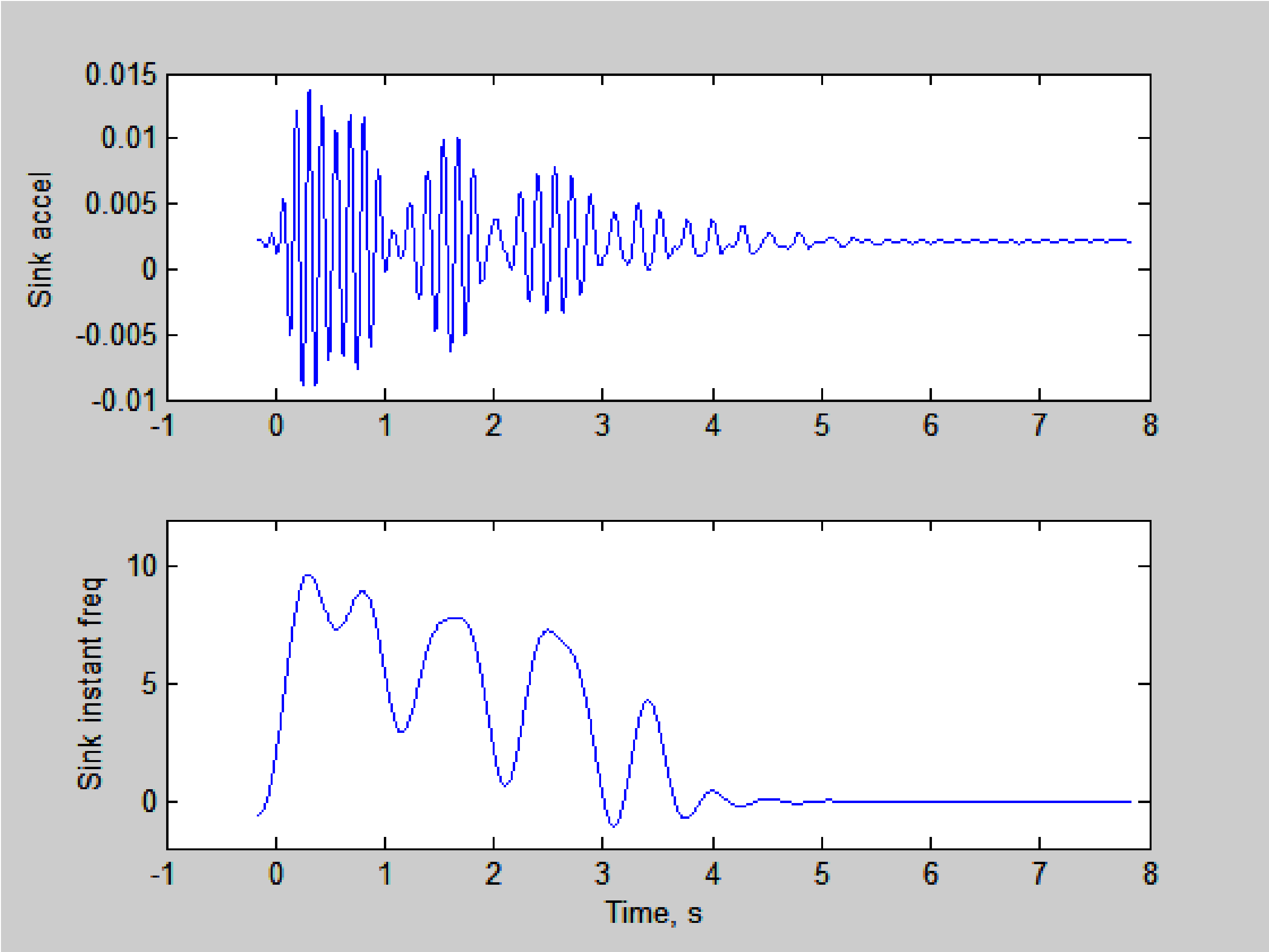
Measured Sink Response

- A sustained cascade (example 1)



Measured Sink Response

- A sustained cascade (example 2)



Conclusions

- Experimental results confirm the existence of nonlinear energy pumping, occurring at a single fast frequency approximately equal to the eigenfrequency of the linear subsystem
- Good agreement between theoretical and experimental results was observed in spite of the strongly nonlinear and transient nature of the dynamics
- Previous analytical and simulation results on the input energy threshold and the variation of efficiency with increasing forcing have been verified
- Evidence of energy pumping cascades from a 2-DOF primary system to a SDOF sink has been found