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Piezoelectric Networking for Structural Vibration Control

K. W. Wang
Diefenderfer Chaired Professor in Mechanical Engineering
Structural Dynamics and Controls Lab
The Pennsylvania State University
University Park, PA 16802
USA

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Background -- Piezoelectric Materials

Mechanically Respond to Electrical Inputs and Electrically Respond to Mechanical Input

Apply electric field

Deform material

$$\begin{bmatrix} \tau \\ E \end{bmatrix} = \begin{bmatrix} E_s & -h_{31} \\ -h_{31} & \beta_{33} \end{bmatrix} \begin{bmatrix} \varepsilon \\ D \end{bmatrix}$$

Induce electrical field or displacement

Induce mechanical strain or stress

- τ : stress
- ε : strain
- β_{33} : dielectric const.
- h_{31} : piezo coefficient
- D : electrical displacement
- E : electrical field
- E_s: elastic modulus

Background

Piezoelectric Materials for Structural Control

- Good bandwidth and authority
- Integration with host structures to form smart structures with distributed actions
- Combined sensor and actuator functions -- Self-sensing and collocated control

Has Potential for Various Applications

Piezoelectric Materials for Structural Control

- **FAQ & Challenges**
  - Higher authority and efficiency?
  - Better controllability and precision?
  - Fail-safe property - can we limp home?

→ **Piezoelectric tailoring for better combination of actions**

- **Electrical tailoring**
- **Mechanical tailoring**

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Some Research Highlights

- **Hybrid Damping and Control**
- **Adaptable Narrowband Disturbance Rejection**
- **Vibration Delocalization of Nearly Periodic Structures**

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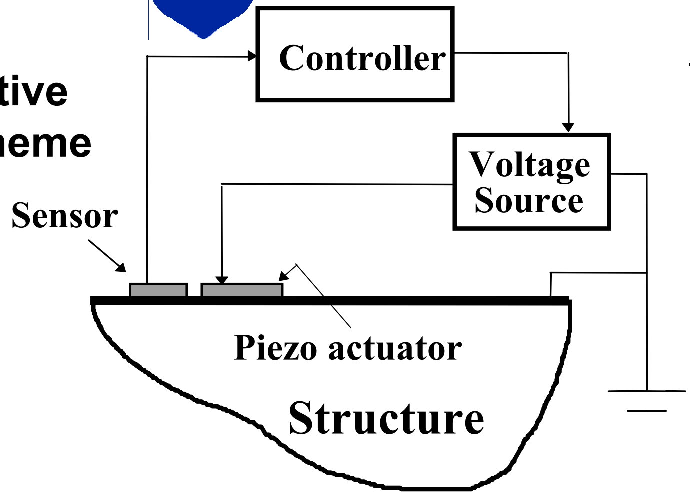
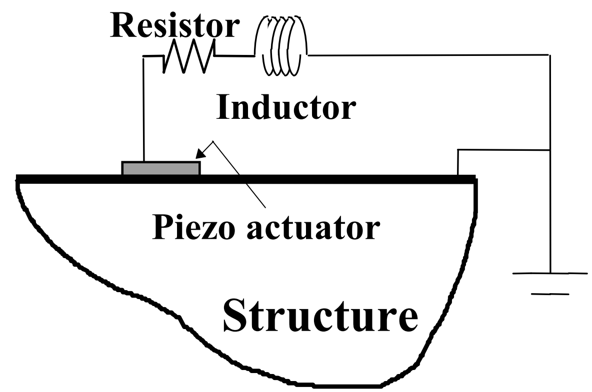
Background on Active-Passive Piezoelectric Actions

Active Scheme

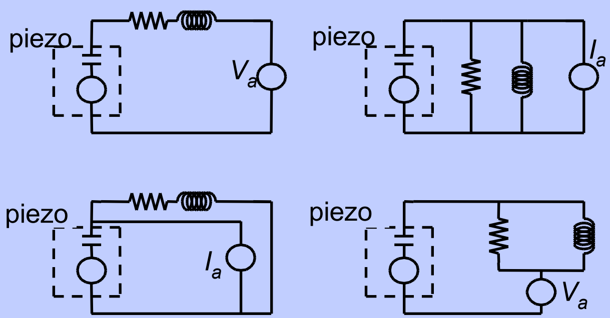


Passive Scheme [Hagood and Von Flotow, 1991]

- Tuned electrical damper/absorber



Self-Contained Active-Passive Hybrid Piezo Networks (APPN)



Different topologies

- Different arrangement of the active and passive elements
- Different types of active source

Passive elements

- Provide passive damping and fail-safe property
- Enhance active authority or efficiency

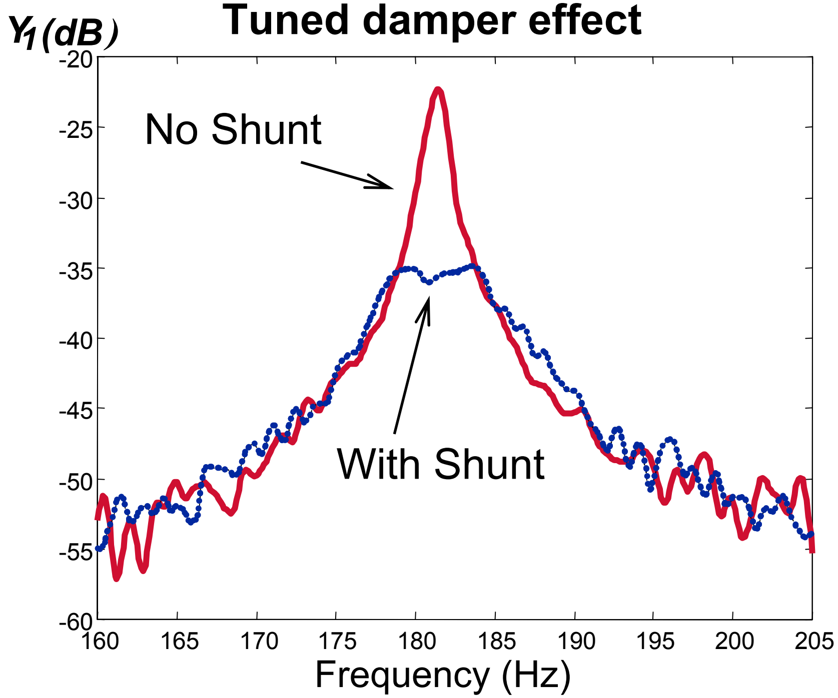
Open Loop Experiments

Shunt circuit together with active source will introduce passive damping *(tuned damper effect)* as well as enhance active authority *(resonant driver effect)*

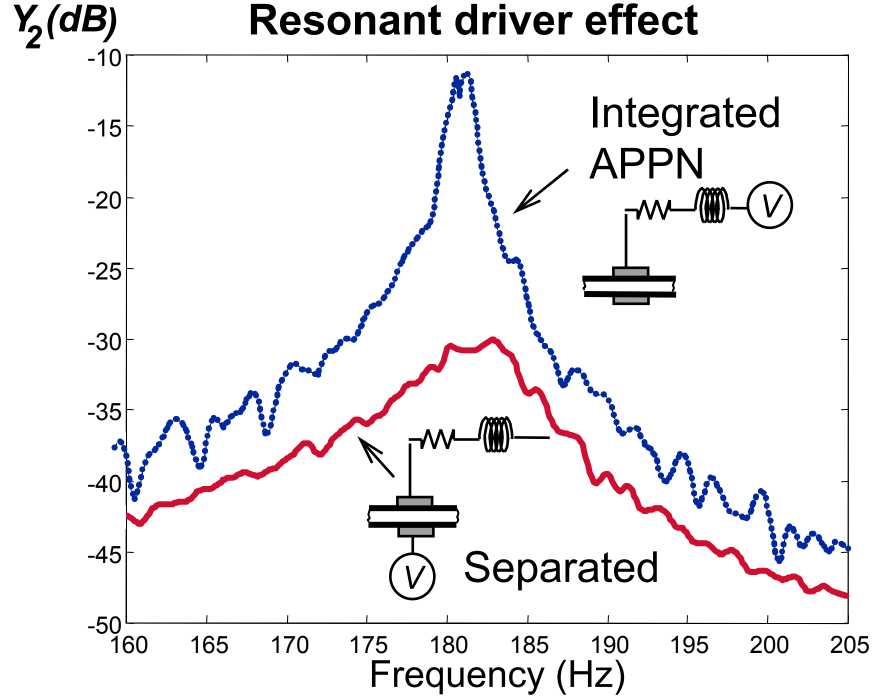
Y_1 (passive damping index) = vibration amplitude / disturbance force

Y_2 (active authority index) = vibration amplitude / control voltage

Tuned damper effect



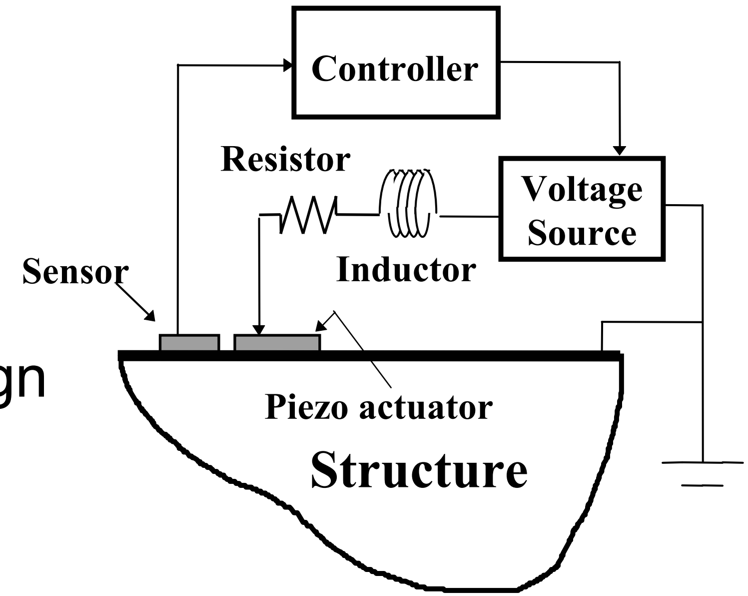
Resonant driver effect



Active-Passive Hybrid Piezo Networks (APPN)

Observation

- Passive circuit could enhance passive damping and active authority in APPN - integrated design is better than separated design



Issues

- Circuit parameters tuned for purely passive systems might not be optimal for the hybrid configuration
  - ➢ ***Need a simultaneous controller and circuit synthesis process***

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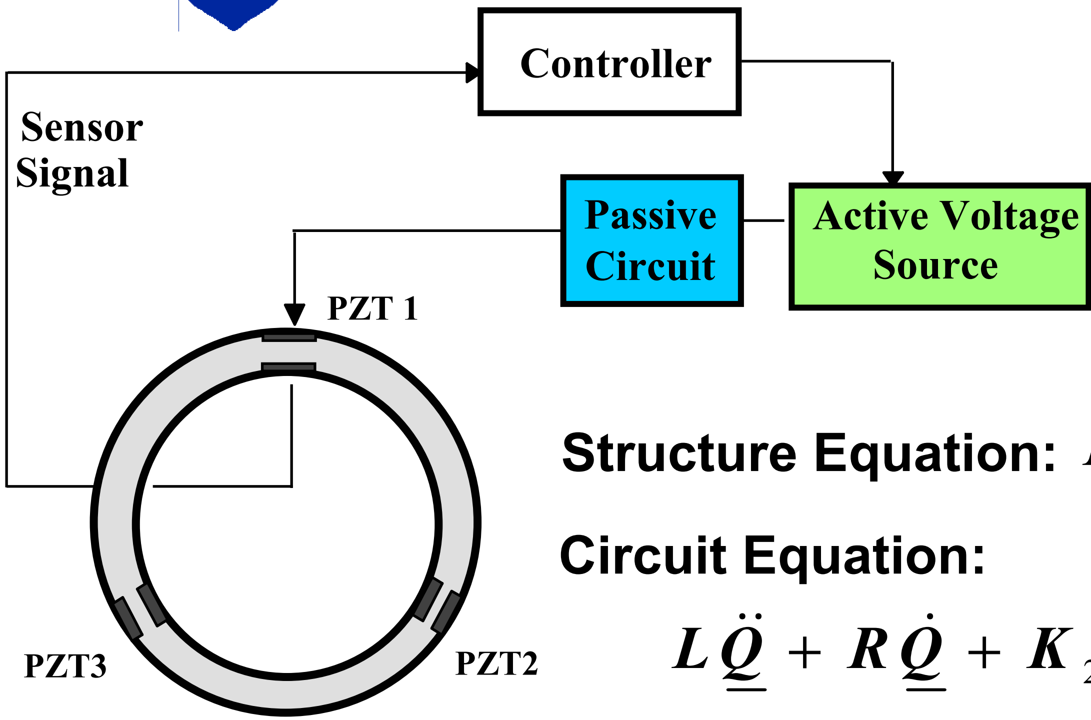
APPN- Ring Structure



Structure Equation: $M\underline{\ddot{q}} + C\underline{\dot{q}} + K\underline{q} + K_1\underline{Q} = \underline{F}$

Circuit Equation:

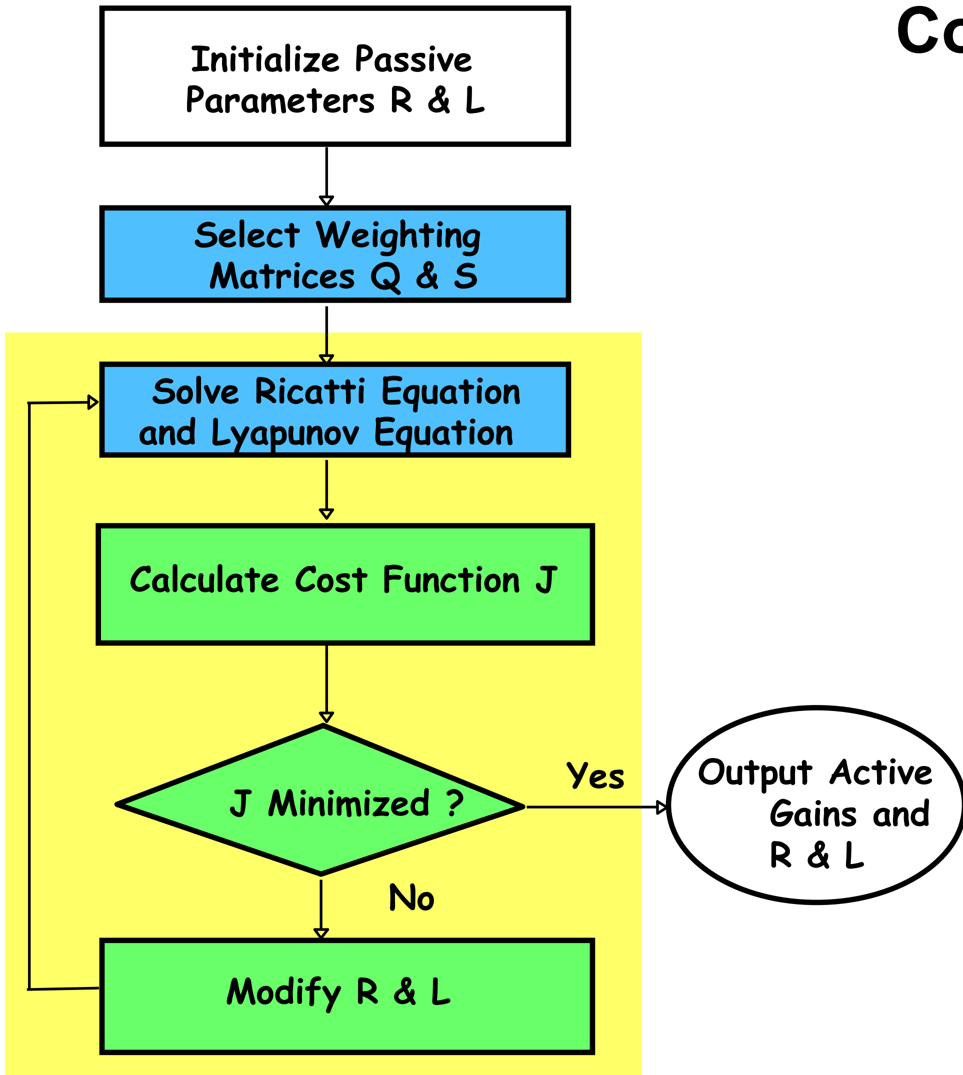
$$L\underline{\ddot{Q}} + R\underline{\dot{Q}} + K_2\underline{Q} + K_1^T\underline{q} = \underline{V}_c$$

Sensor Equation: $\underline{V}_o = K_{ob}\underline{q}$

State-Space Formulation: $\underline{\dot{y}} = A(R_i, L_i)\underline{y} + B(R_i, L_i)\underline{u}$

State matrix and input matrix are dependent functions of the passive control parameters

A Simultaneous Controller/ Circuit Design Method

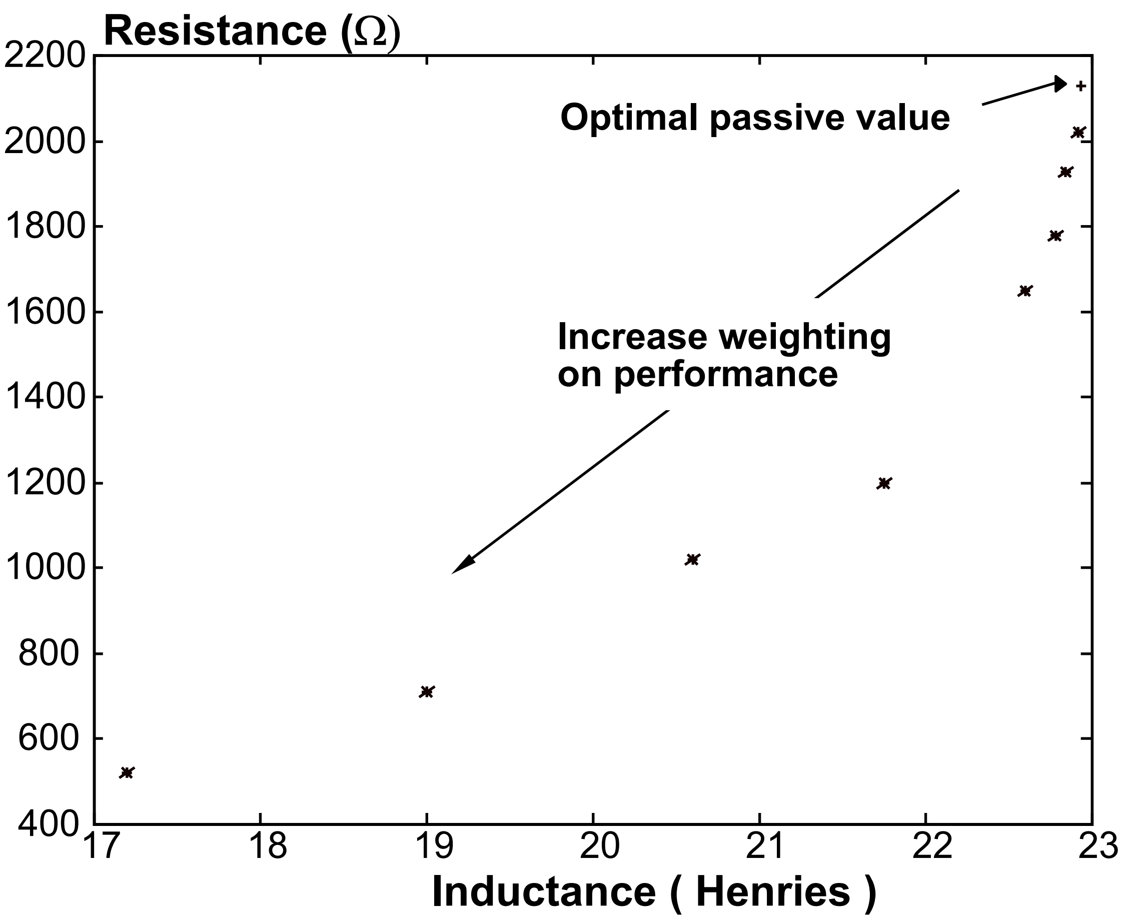


Coupled Optimal Control-Optimization Process

– Given passive R and L, determine active gains via optimal control: minimize objective function *J* reflecting vibration reduction and control effort

– Modify R and L via optimization scheme to further reduce *J* while updating active gain: search for *optimum among optimum*

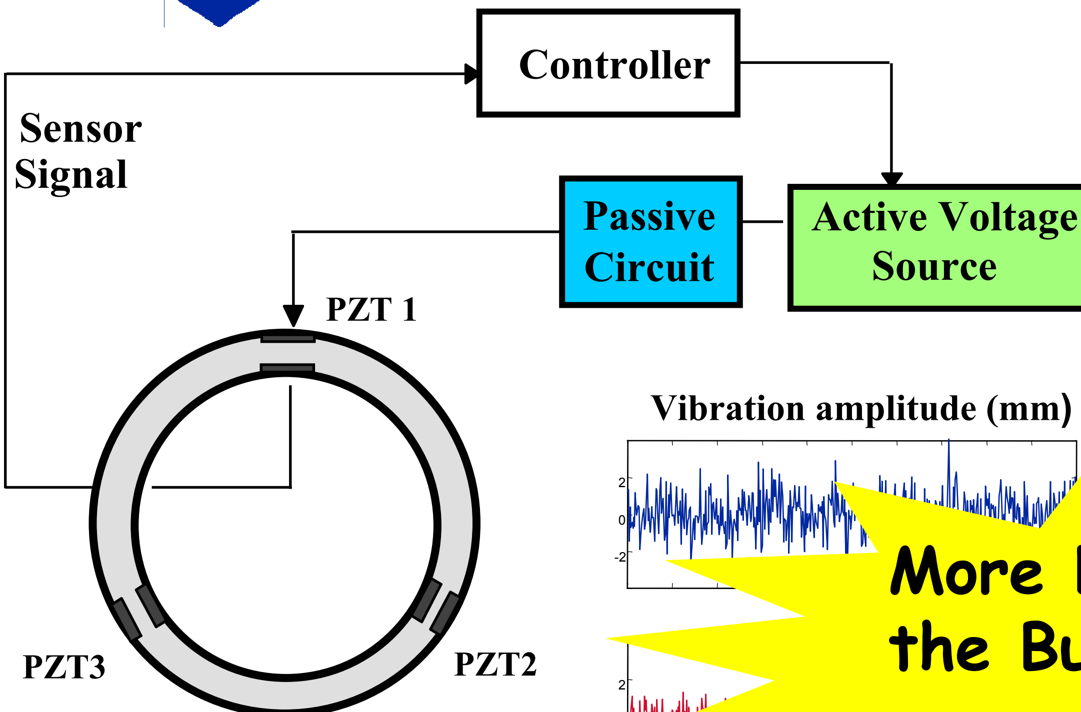
Optimal RL Values vs. Performance Weighting

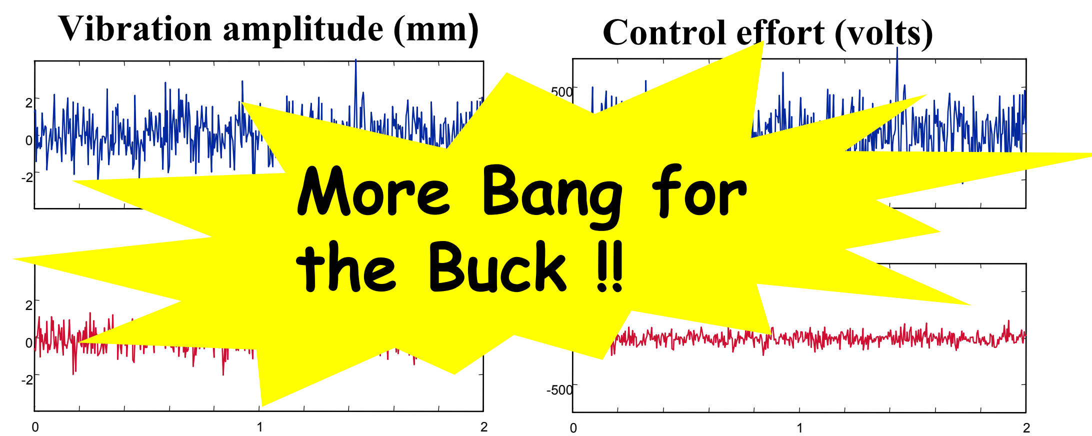


- Optimal *RL* values for the hybrid system can be quite different from the *RL* values optimized for the purely passive system as the demand on performance increases

Concurrent method is useful for hybrid system

APPN-Ring Structure





	σ_w *(mm)*	σ_{V1} *(volts)*	σ_{V2} *(volts)*	σ_{V3}*(volts)*
Uncontrolled	10.23	-	-	-
Purely Active Case	1.08	379	244	244
Active-Passive Hybrid Case	0.67	79.3	61.8	97.4

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Some Research Highlights

- Hybrid Damping and Control
- Adaptable Narrowband Disturbance Rejection
- Vibration Delocalization of Nearly Periodic Structures

Adaptable Narrowband Disturbance Rejection -- *Background and Motivation*

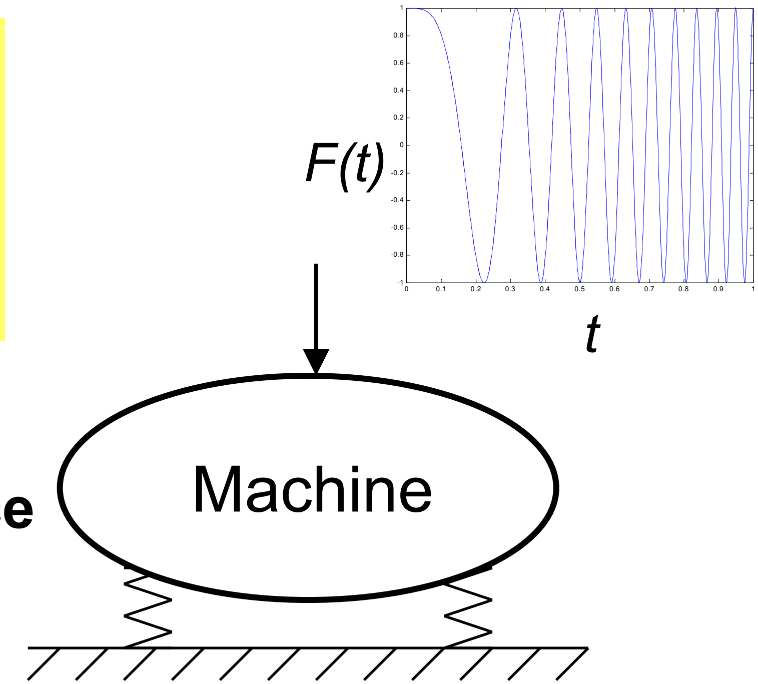
Problem of Interest

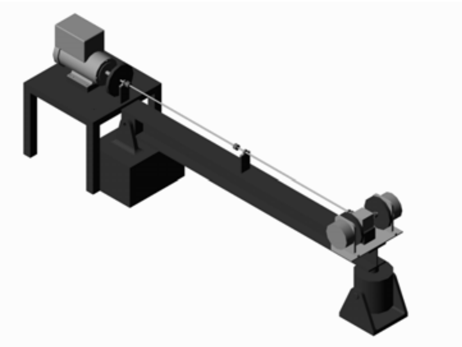
- **Machine or structure under harmonic disturbance**
- **Disturbance frequency varies**

Example

- **Machine with rotating unbalance**
  - Rapid frequency variation during spin-up and spin-down
  - Slow frequency variation due to changing operating conditions

➢ **Passive approach would not be effective**

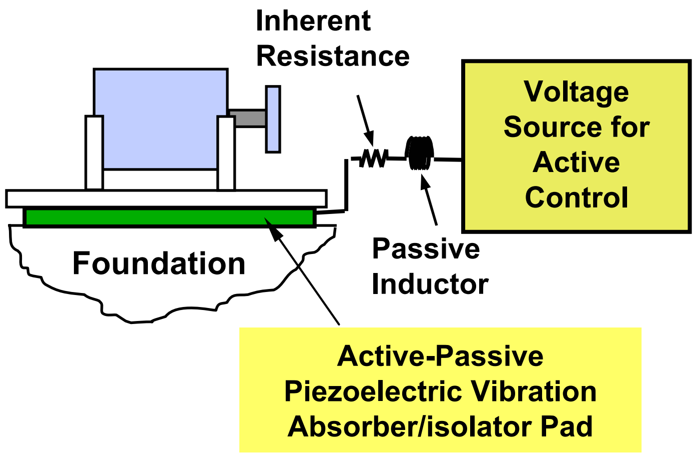




Active-Passive Absorption/ Isolation

A High Performance Active-Passive Hybrid Approach

- Based on the concept of absorber dynamics
- Tune passive inductor to nominal frequency
- Use active feedback for frequency tracking
  - Active variable inductor
- Use active feedback to increase performance and robustness
  - Negative resistance
  - Active coupling enhancement



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Active Inductance and Resistance

- General System Model:

$$M\underline{\ddot{q}}+C\underline{\dot{q}}+K^{D}\underline{q}+K_{C}Q=\underline{\hat{F}}\cdot f(t)$$

$$L_{p}\ddot{Q}+R_{p}\dot{Q}+\frac{1}{C_{p}^{S}}Q+K_{C}^{T}\underline{q}=V_{C}$$

- Active Inductor Control Law

$$\boxed{\left(V_{C}\right)_{inductor}=-L_{a}(t)\cdot\ddot{Q}}$$

- Want to simulate an ideal variable inductor (no internal resistance)
  - Add negative resistance action to reduce overall resistance

$$\boxed{\left(V_{C}\right)_{resistor}=R_{a}\dot{Q}}$$

K^{D} = open-circuit stiffness matrix
K_{C} = coupling vector
L_{p} = passive inductance
R_{p} = internal resistance of passive inductor
C_{p}^{S} = capacitance of PZT at constant strain
V_{C} = control voltage

Implement through a PI control on voltage across passive inductor

$$V_{C}(t)=-\frac{L_{a}(t)}{L_{p}}V_{L}(t)+\frac{R_{a}}{L_{p}}\int_{0}^{t}V_{L}(t)dt$$

Active Coupling Enhancement

Effective bandwidth of absorber is related to electro-mechanical coupling

- higher coupling = higher performance, more robustness

$$M\ddot{\underline{q}} + C\dot{\underline{q}} + K^{D}\underline{q} + K_{C}Q = \underline{\hat{F}} \cdot f(t)$$

$$L_p\ddot{Q} + R_p\dot{Q} + \frac{1}{C_p^S}Q + K_C^T\underline{q} = V_C$$

Can't change K_c for a given system
Can increase <u>effective coupling</u> in circuit equation

$$V_C = -\left(G_{ac} - 1\right)K_C^T q \ , \quad G_{ac} \geq 1$$

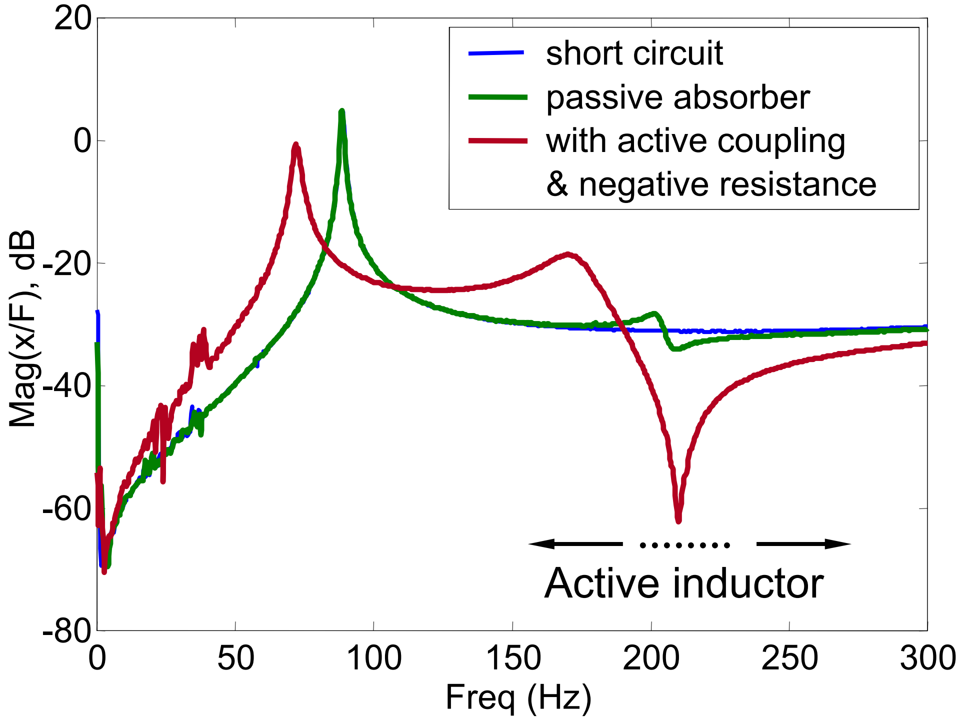
Active Coupling Control Law

Closed-Loop Circuit Equation:

$$\left(L_p + L_a(t)\right)\ddot{Q} + \left(R_p - R_a\right)\dot{Q} + \frac{1}{C_p^S}Q + G_{ac}K_C^T\underline{q} = 0$$

Effects of Active Actions

Experimental Results



- Active coupling and negative resistance increase both performance and robustness
- Active inductance tracks the operating frequency

Stability Analysis of Adaptive Absorber

- Use Lyapunov's method to find stability criteria for adaptive (time-varying) absorber
- System Equations in matrix form:

$$\underbrace{\begin{bmatrix} M & 0 \\ 0 & G_{ac}L_t(t) \end{bmatrix}}_{\bar{M}}\ddot{z} + \underbrace{\begin{bmatrix} C & 0 \\ 0 & G_{ac}R_t \end{bmatrix}}_{\bar{C}}\dot{z} + \underbrace{\begin{bmatrix} K^D & G_{ac}K_C \\ G_{ac}K_C^T & \dfrac{G_{ac}}{C_p^S} \end{bmatrix}}_{\bar{K}} z = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

where $z = \begin{Bmatrix} q \\ \tilde{Q} \end{Bmatrix}$

$$\tilde{Q} = \frac{1}{G_{ac}} Q$$

- Lyapunov Function Candidate:

$$V(z) = \frac{1}{2} z^T \bar{K} z + \frac{1}{2} \dot{z}^T \bar{M} \dot{z}$$

Physical meaning:

Total energy of system

Stability Analysis of Adaptive Absorber

- Condition for $V(z)$ to be a valid Lyapunov Function:

$$\left| K^D - G_{ac} C_p^S K_C K_C^T \right| > 0$$

  - For SDOF system, reduces to $\boxed{G_{ac} < 1 + \frac{1}{K_{ij}^2}}$

Bound on coupling gain

- With $\dot{V}(z) \leq 0$ and use Invariance Principle, derive sufficient condition for stability:

$$[\bar{C} - \dot{\bar{M}} / 2] > 0 \quad \Rightarrow \quad -R_t C_p^S \delta^2 \omega^3 < \dot{\omega}$$

Bound on frequency reduction rate for given resistance

Closed-loop system is stable if coupling gain is not too large and frequency is not decreasing very rapidly

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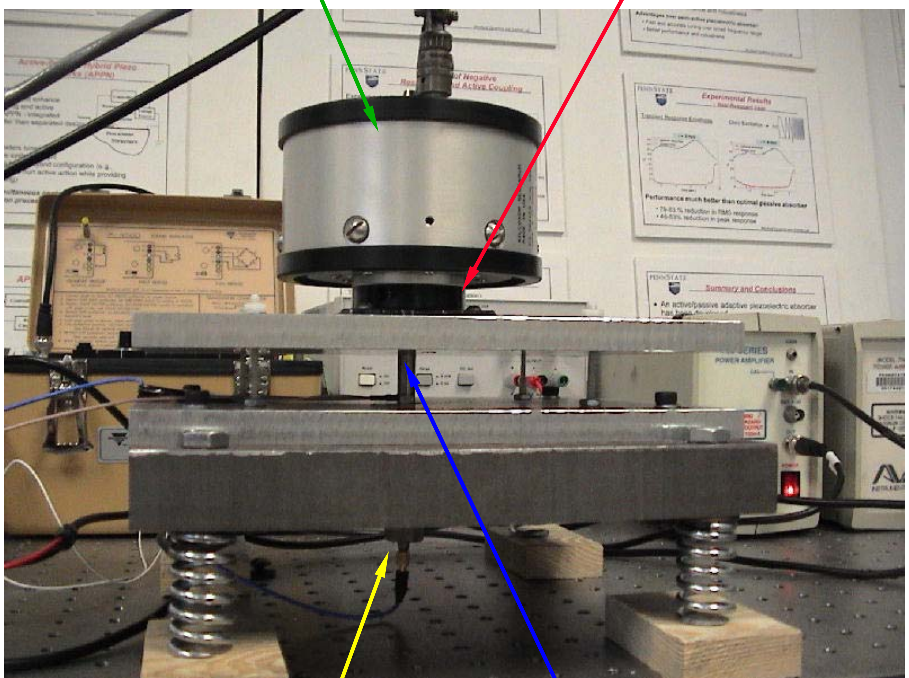
Vibration Isolation – Experimental Setup

Shaker (input force)

Load cell

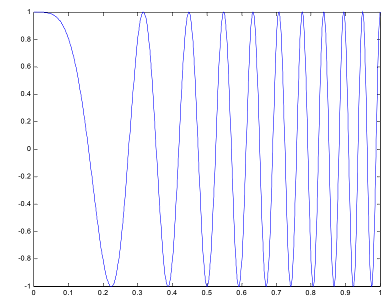


Accelerometer on foundation

Piezo stack actuator and sensor

- Linear Chirp Excitation Parameters:
  - ω_o = 430 Hz (resonant)
  - $\dot{\omega} / \omega_o = .005, .02, .05, .10$

f(t)



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- Passive Baseline: optimally damped passive piezoelectric absorber

Vibration Isolation – Experimental Transient Performance

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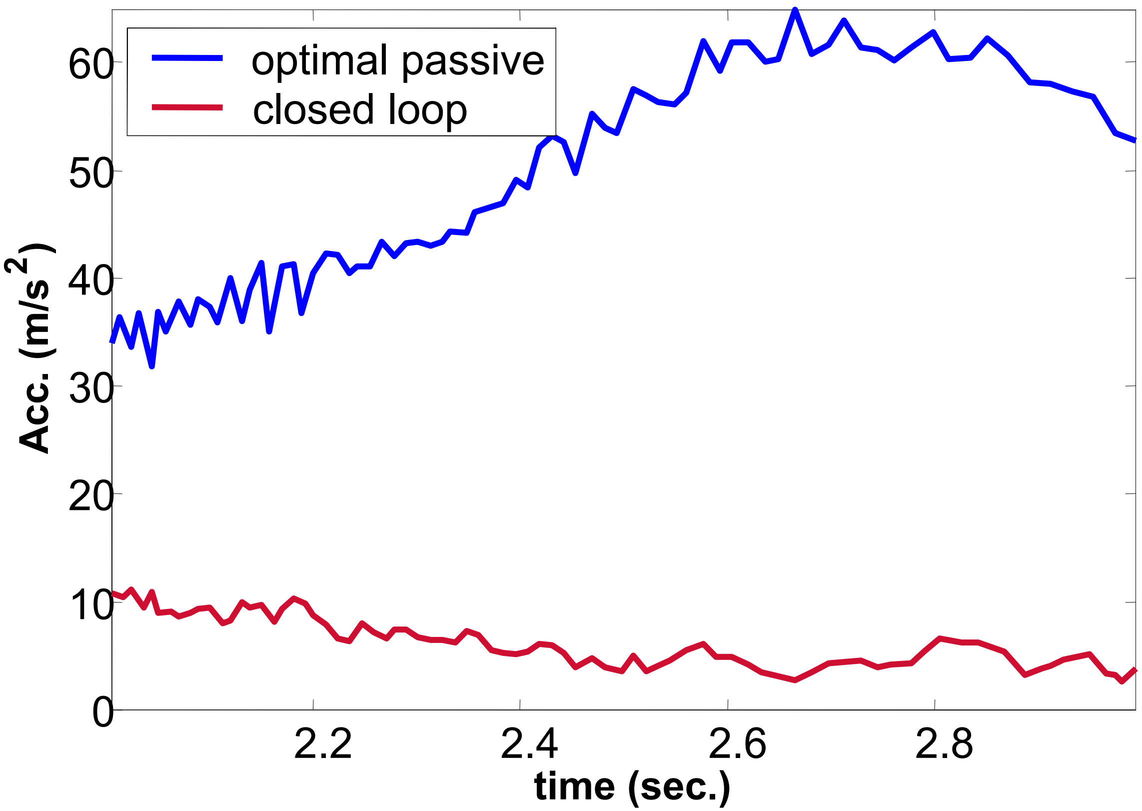
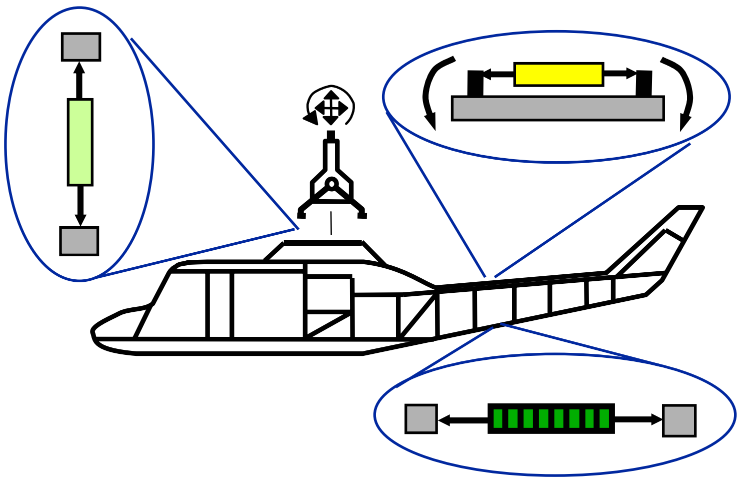
Transient Response Envelope ($\dot{\omega} / \omega_o = .05$)



$\dot{\omega} / \omega_o$	% Reduction RMS Response
0.005	83.7
0.020	83.6
0.050	86.5
0.100	82.7

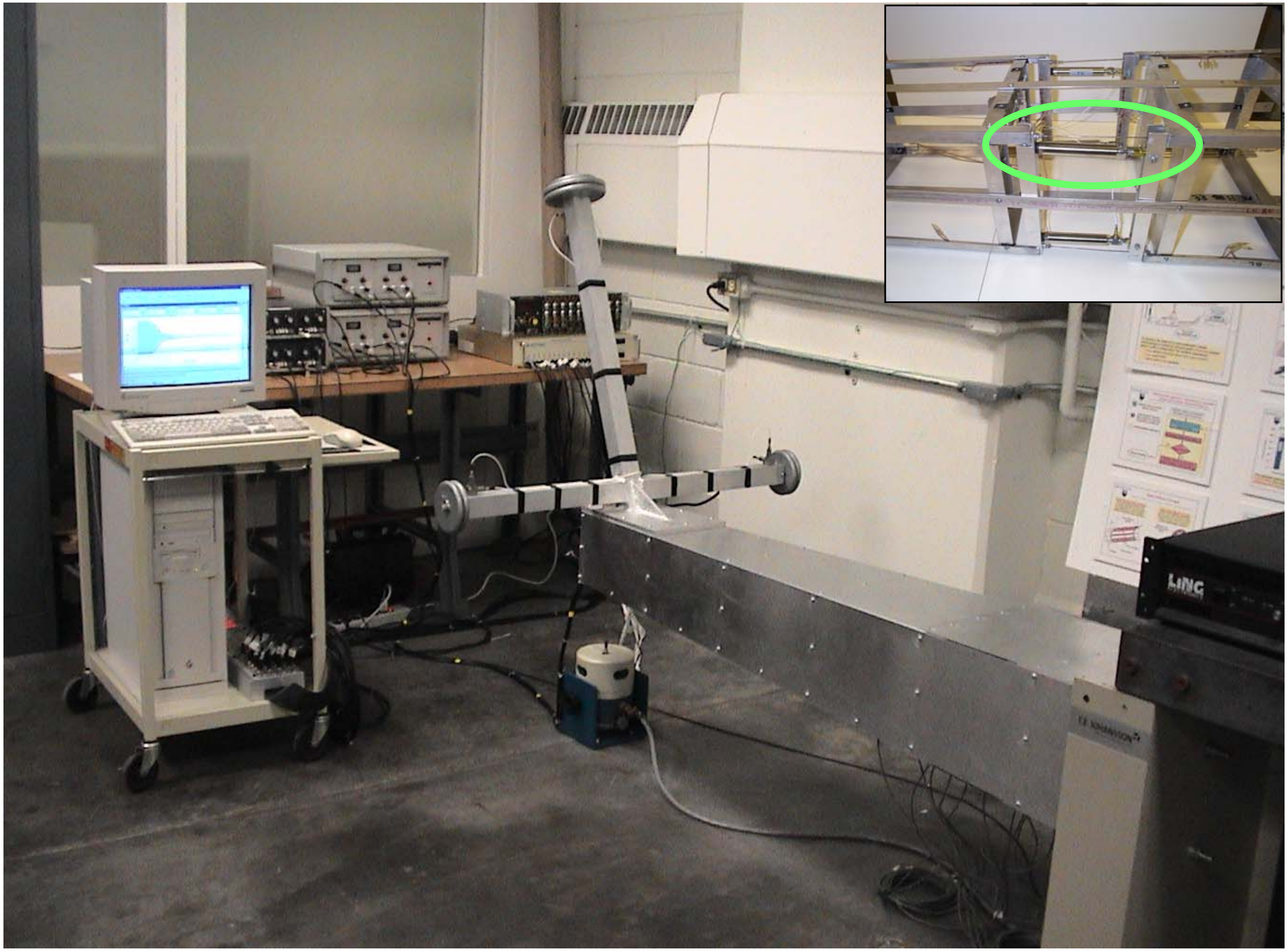
Vibration transmissibility is greatly reduced (80% reduction) with active-passive piezoelectric absorber/isolator

Smart Airframe

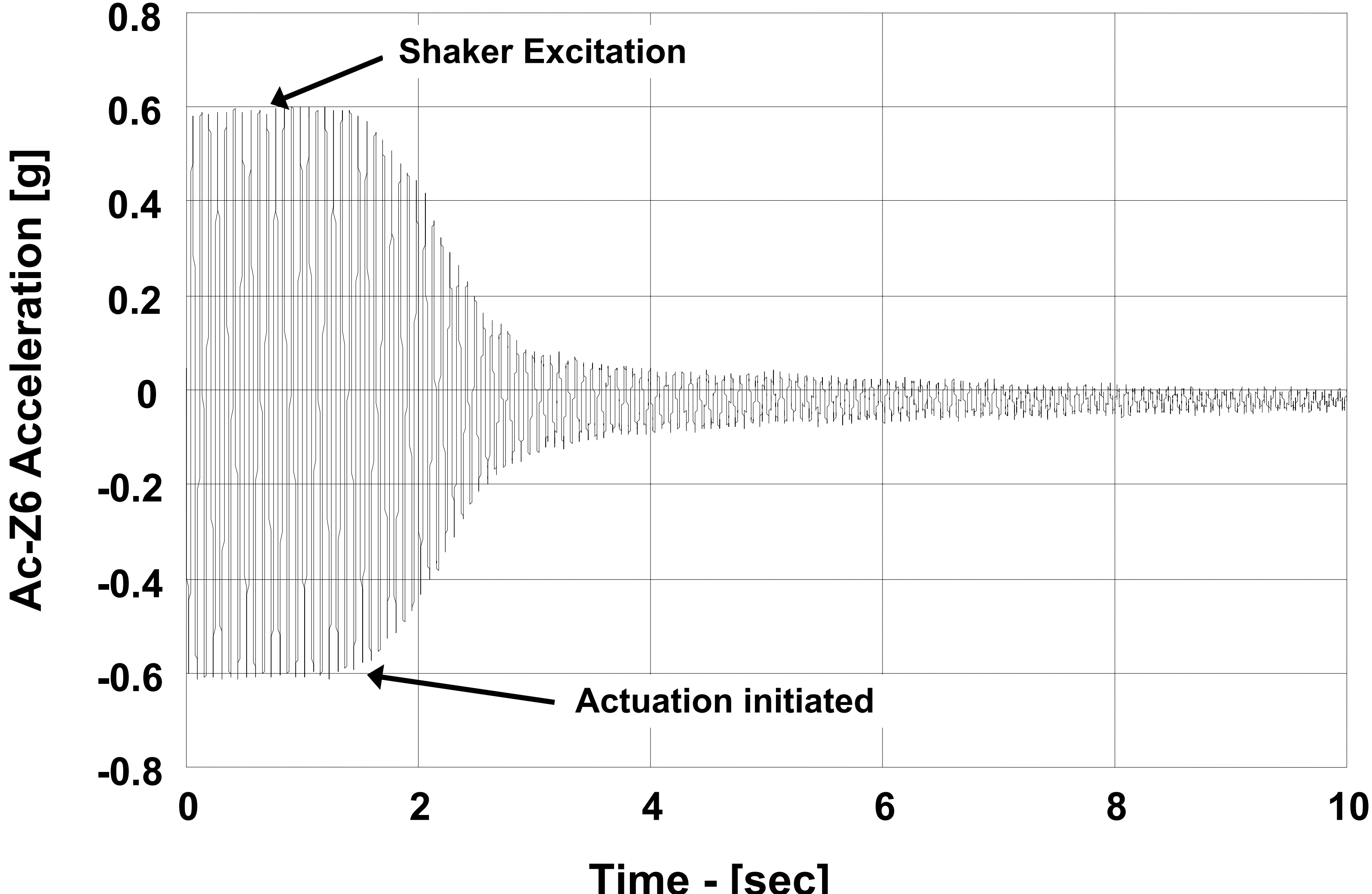


- Traditional active airframe vibration control -- centralized actuator configuration
  - Not the best distribution
  - Large effort required - heavy hydraulic actuators
- New smart structure configuration -- distributed actuation “built-in” throughout the airframe
  - Optimal location of actuators for various types of disturbance
  - Better performance with less control effort

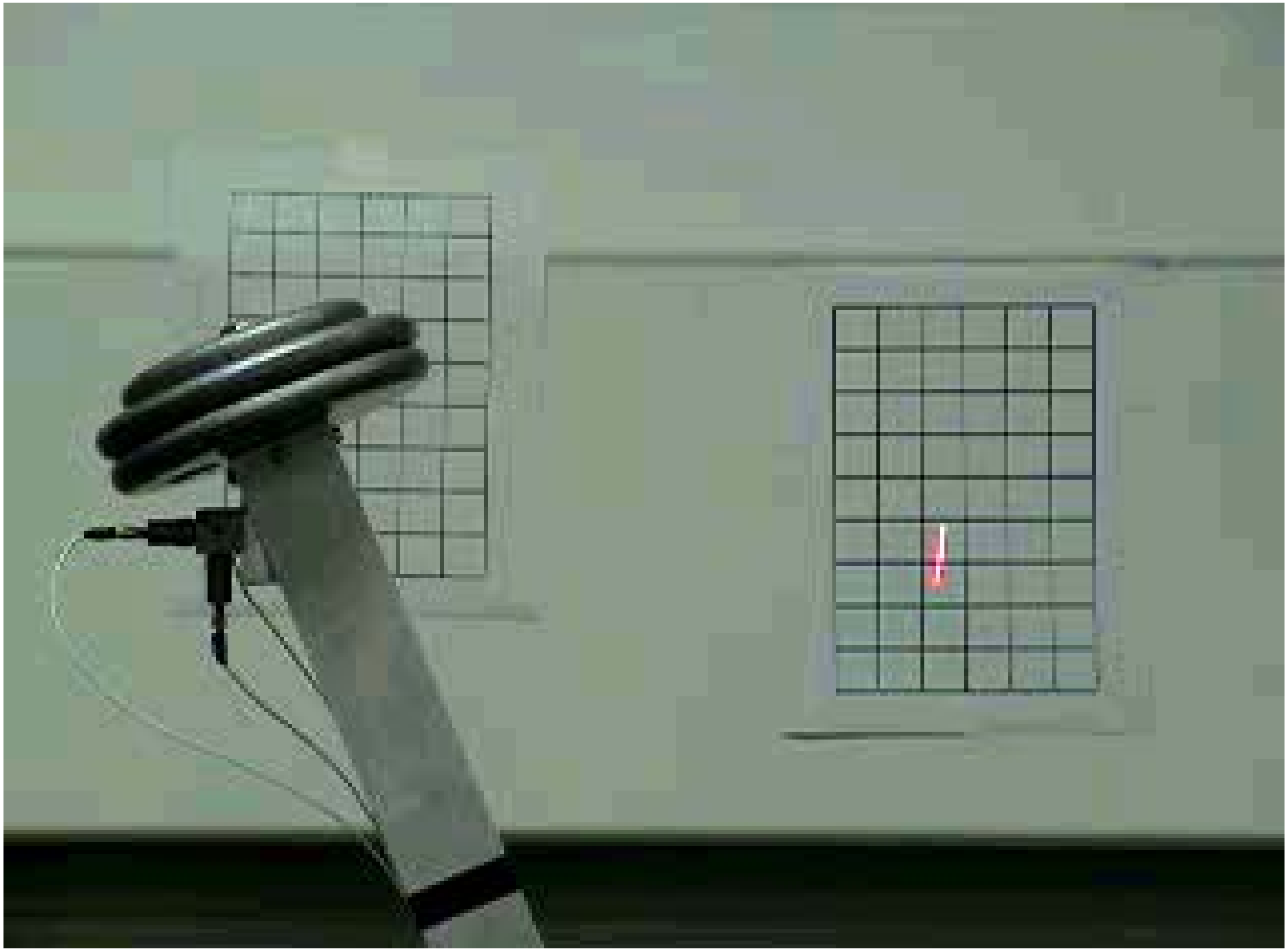
Experimental Setup



Active Vibration Control Demonstration



Active Vibration Control Demonstration



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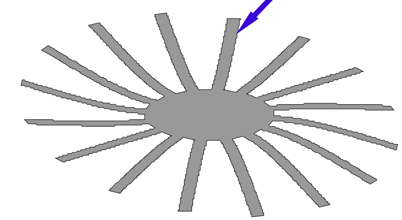
Some Research Highlights

- **Hybrid Damping and Control**
- **Adaptable Narrowband Disturbance Rejection**
- **Vibration Delocalization of Nearly Periodic Structures**

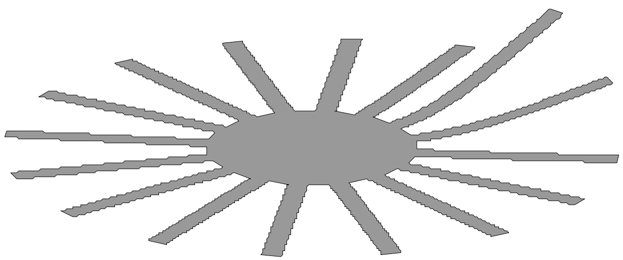
Background: Periodic and Nearly Periodic Structures

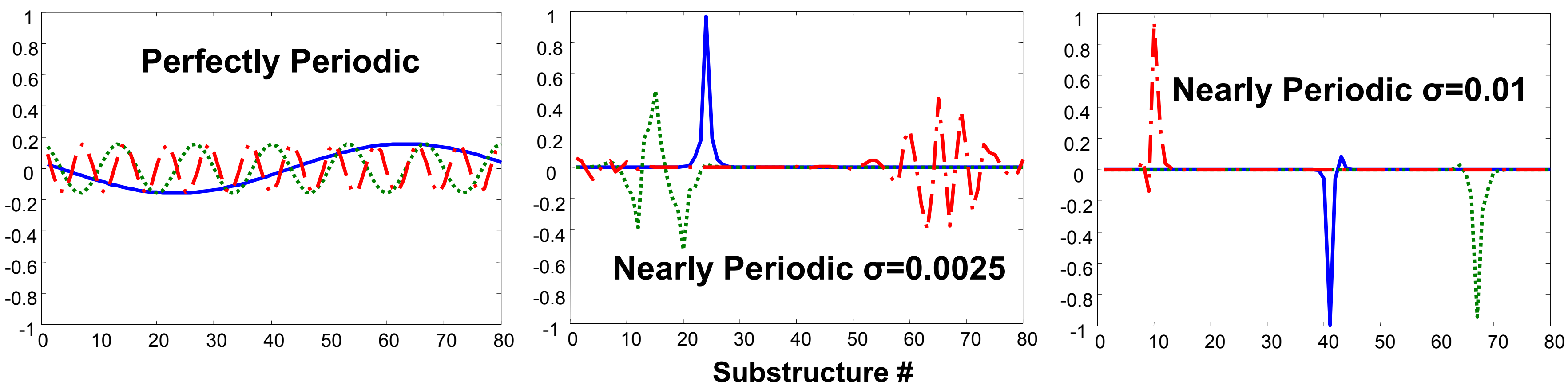


- **Periodic structure examples:** bladed-disks, space structures, satellite antenna, etc.
- **Perfectly periodic structure (ideal case)**
  - In a vibration mode, energy/amplitude are uniformly distributed and extended throughout the substructures
- **Nearly (mistuned) periodic structure (in reality) --** small differences among substructures
  - When the coupling between substructures is weak, mistuning can cause vibration localization
    - ➢ **Energy is confined in a small region - increase amplitude and stress locally**
    - ➢ **Detrimental to structure health**

Vibration Localization

- The *mistuned* system is a small perturbation to the ideal case – stiffness variation among substructures with standard deviation σ
- Modes are drastically changed !!





Background: Vibration Localization Study

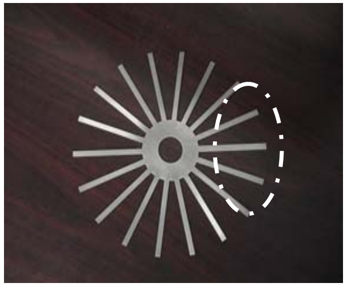
- **Previous work focused on predicting and exploring the cause of vibration localization** (Hodges, 1982; Mester and Benaroya, 1995; Pierre, et al., 1996; Slater, et al., 1999).

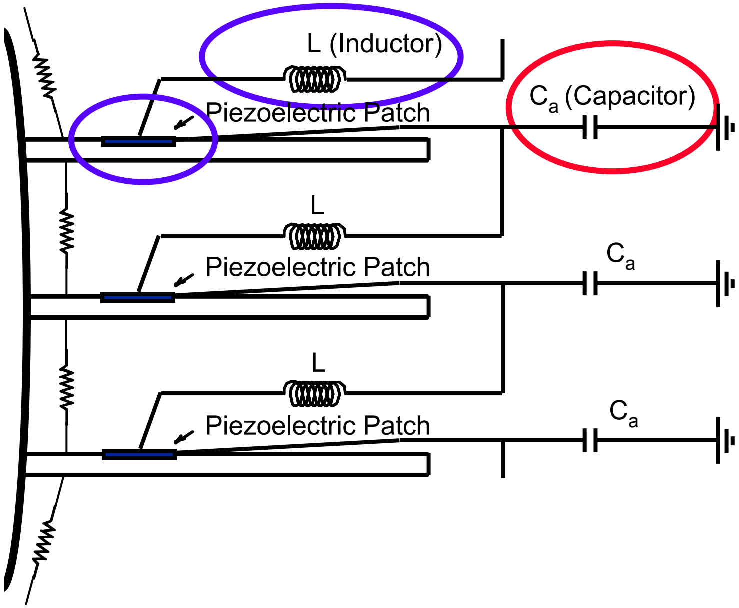
- **Little work has been done on reducing/eliminating vibration localization**
  - Modify the nominal mechanical stiffness of some of the substructures (Castanier and Pierre, 1997): not trivial to implement
  - Shorting piezoelectric patches on blades to increase coupling (Agnes, 1999; Gordon and Hollkamp, 2000): improvement not obvious
  - **New method?**

New Idea

Create *piezoelectric networks* to destroy localization *(delocalization)* by forming an additional strong wave channel

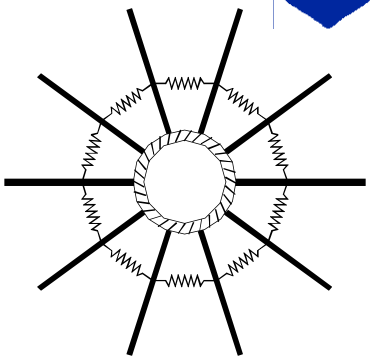
- Individual resonant piezoelectric shunts on local substructures can absorb vibration energy in electrical form;
- The resonant shunts are coupled through capacitors to create an additional electro-mechanical wave/energy channel with strong coupling → Localized energy can now propagate in electrical form.





- Easier to implement than mechanical tailoring
- Can achieve high performance through fully utilizing electrical network dynamics

System With or Without the Piezoelectric Circuits



Original Mechanical Structure (harmonic motion)

$$-\Omega^2 x_j + (1+\Delta s_j)x_j + R_c^2(x_j - x_{j-1}) + R_c^2(x_j - x_{j+1}) = 0$$

Δs_j : stiffness mistuning, R_c^2 : mechanical coupling

Mechanical Structure integrated with piezoelectric circuits

$$-\Omega^2 x_j + (1+\Delta s_j)x_j + R_c^2(x_j - x_{j-1}) + R_c^2(x_j - x_{j+1}) + \delta\xi y_j = 0$$

$$-\Omega^2 y_j + \delta^2 y_j + R_a^2(y_j - y_{j-1}) + R_a^2(y_j - y_{j+1}) + \delta\xi x_j = 0$$

x_j : mechanical displacement

y_j : electrical displacement (charge flow in the circuits)

δ : tuning ratio related to circuitry inductance

ξ : piezoelectric electro-mechanical coupling coefficient

R_a^2 : coupling capacitance between circuits (electrical coupling)

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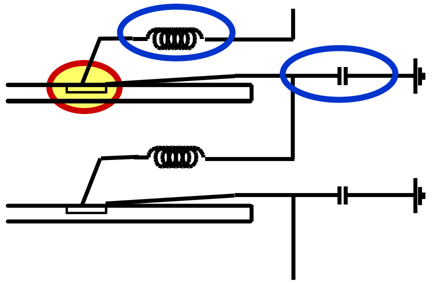
Electromechanical Coupling Enhancement

Recent Observations

- Promising delocalization results via piezo-networking
- Treatment can be easily tuned via circuitry elements
- Effectiveness is limited by the level of ***electromechanical coupling*** of the piezo patch – How much mechanical energy can be transferred?
    - **Not strong enough to achieve global delocalization**
    - **Determined by the piezoelectric material property, size and location of patches, and stiffness of host structures**
    - **Difficult to vary via passive design**

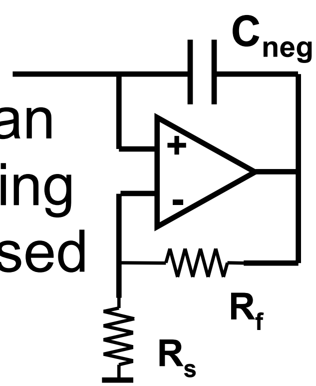


Electromechanical coupling of piezoelectric patches -- **Bottle Neck ?!**

Electromechanical Coupling Enhancement -- New Idea

Utilizing an active coupling enhancement approach to increase the system electromechanical coupling → improve the vibration delocalization performance of the piezoelectric networks

- The generalized electromechanical coupling coefficient (ξ) can be increased by ***negative capacitance circuit***, added in series to the piezoelectric networks
- The negative capacitance can be realized using an op-amp based negative impedance converter (NIC) circuit

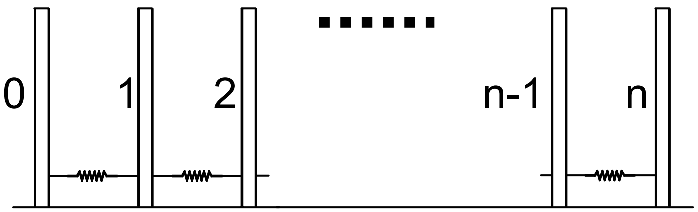


$$\xi = \kappa / \sqrt{k(k_p - k_{nc})}$$

k : related to substructure mechanical stiffness
κ : related to piezo material properties (e.g., d_{13})
k_p : electrical stiffness due to piezo capacitance
k_{nc} : electrical stiffness change due to negative capacitance

Transfer Matrix and Wave Analysis

- Transfer matrix description for general periodic structures



$$\mathbf{v}_j = \mathbf{T}_j^m \mathbf{v}_{j-1} \qquad \mathbf{v}_j = \begin{Bmatrix} x_{j+1} \\ x_j \end{Bmatrix}$$

- Transfer matrix

$$\mathbf{T}_j^m = \begin{bmatrix} (1-\Omega^2+\Delta s_j+2R_c^2)/R_c^2 & -1 \\ 1 & 0 \end{bmatrix}$$

- Amplitude after propagating through n substructures

$$\mathbf{v}_n = \prod_{j=n}^{1} (\mathbf{T}_j^m)\mathbf{v}_0$$

- The average exponential decay is determined by the Lyapunov exponent

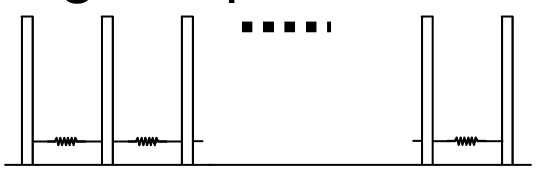
$$\gamma(\mathbf{v}_0) = \lim_{n\to\infty} \frac{1}{n} \log \|\mathbf{v}_n\| = \lim_{n\to\infty} \frac{1}{n} \log \left\| \prod_{j=n}^{1} (\mathbf{T}_j^m)\mathbf{v}_0 \right\|$$

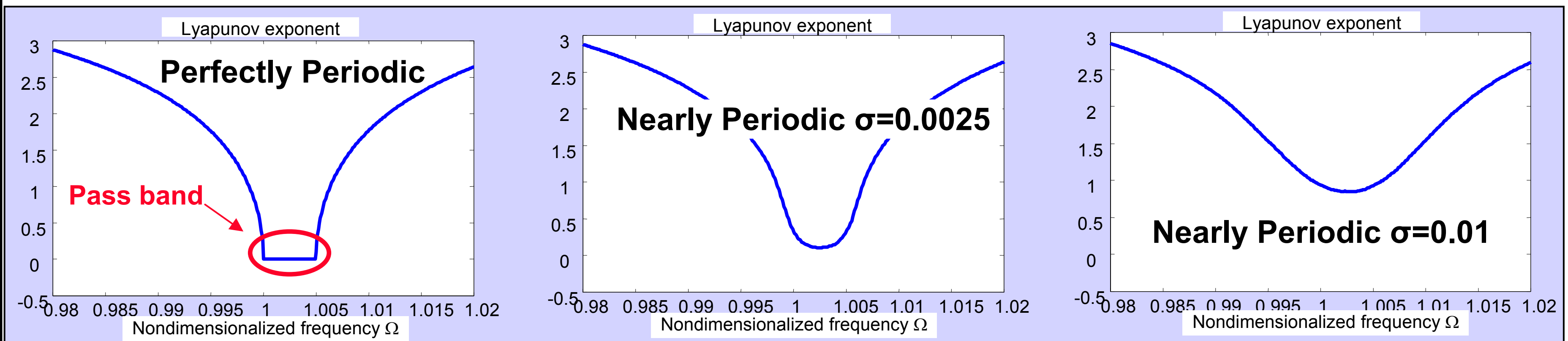
Lyapunov Exponents

- Lyapunov exponent is a measure of the asymptotic exponential decay rate of the wave vector → each wave decays **spatially** at the rate of $e^{-\gamma k}$ (γ_k is the k-th L. exp.)
  — Localization index
- Perfectly periodic structure has frequency **passband**:

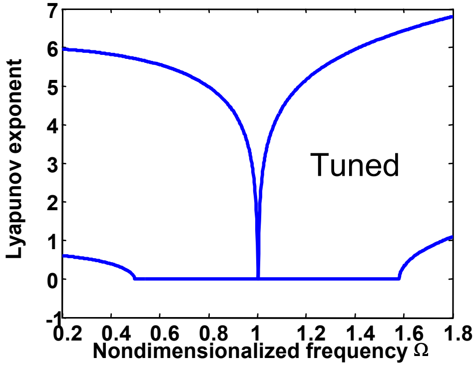
  Passband: L. exp. is zero → no decay (Vibration modes);
- When system has mistuning, there is no longer a passband

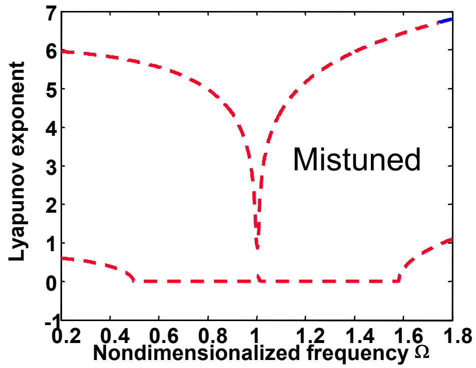
Larger L. exp = more vibration localization

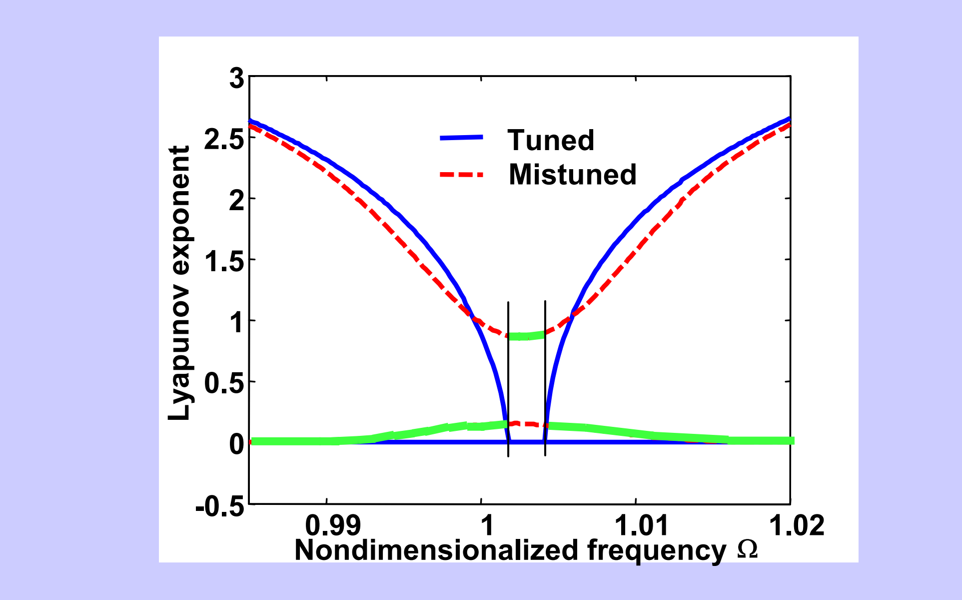




Localization Index: Bi-coupled System

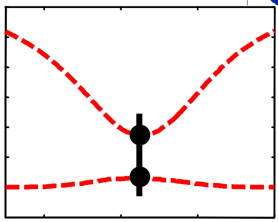




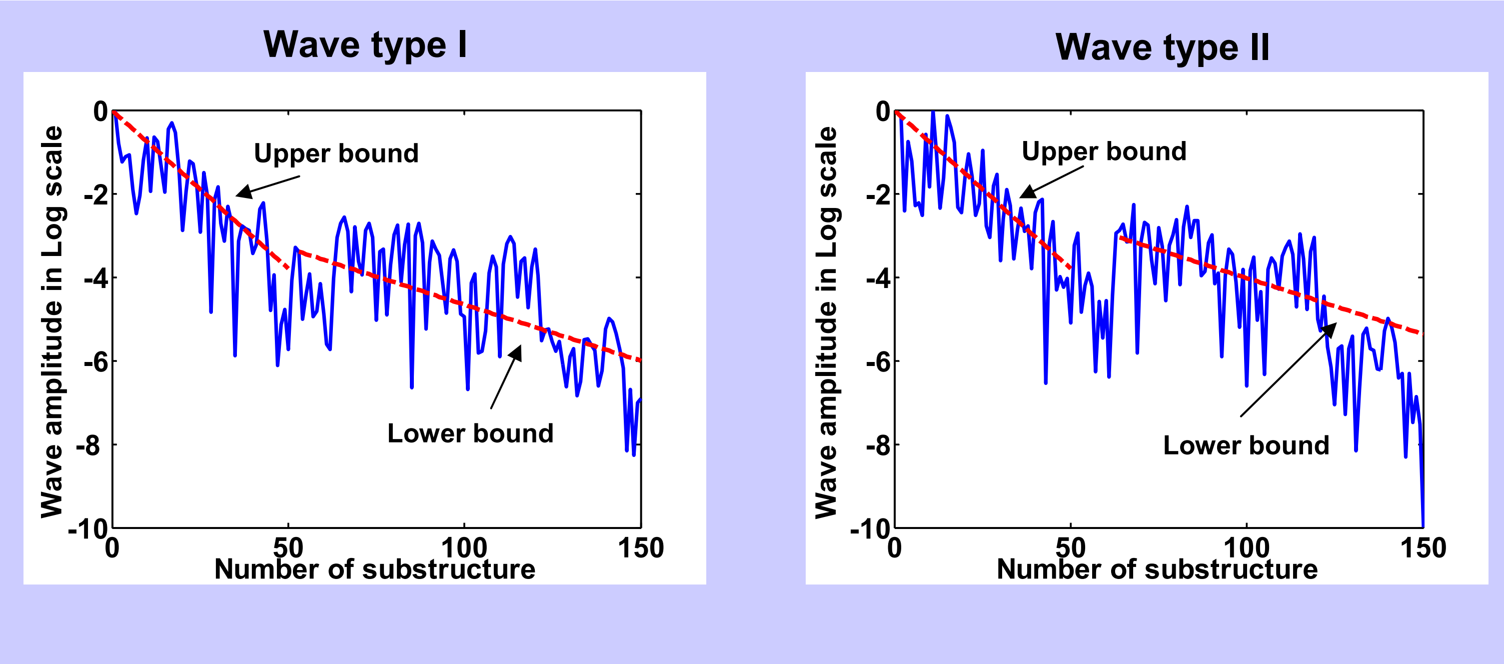


- Two L. exponents corresponding to two wave types
  - In the double passband, the *upper branch* of L. exp. plot will be the worst case → Localization Index
  - In single passband range, the lower branch will govern the vibration modes → Localization Index

Localization Index: Wave Conversion

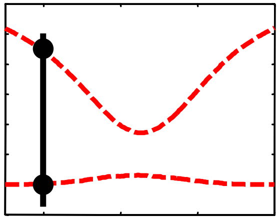


Double passband (Ω=1.0035)

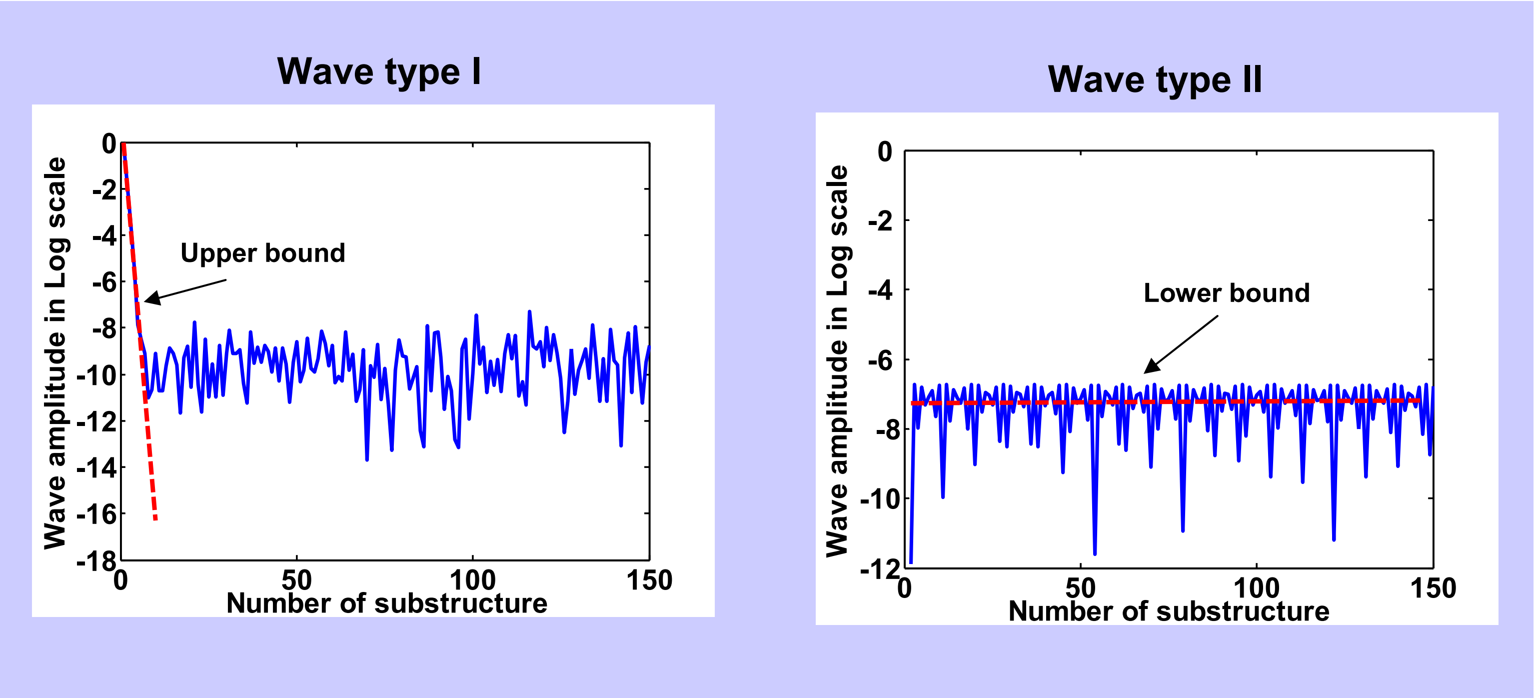


- A wave incident is applied to mistuned system
- Both wave types decay at similar fashion, first at the large rate (upper branch), then at the small rate (lower branch)
- **localization Index : upper bound of L. exp.**

Localization Index: Wave Conversion



One wave is in passband; the other is in stop band (Ω=0.995)



- **Each wave decays at its respective L. exp.**
- **Lower branch L. exp. governs the mode shape**
- **Localization index: Lower bound of L. exp.**

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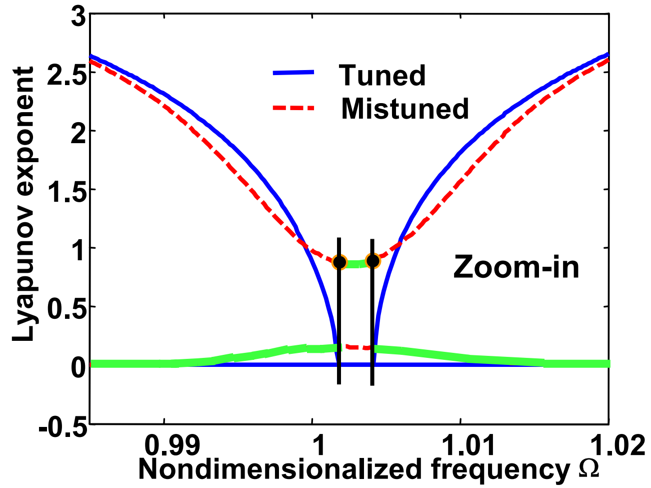
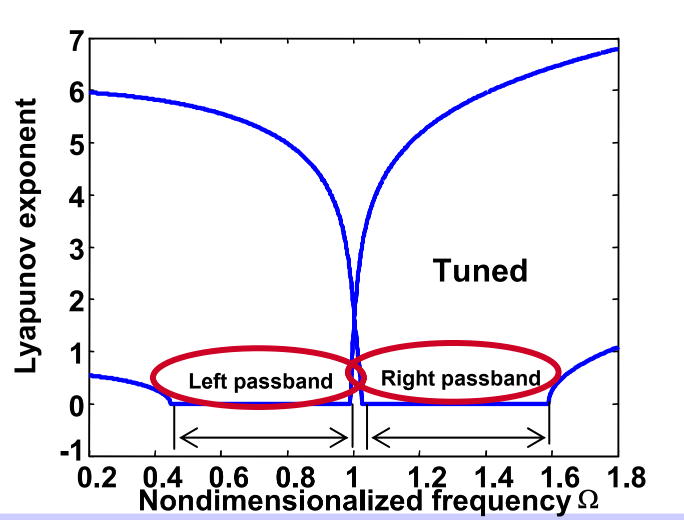
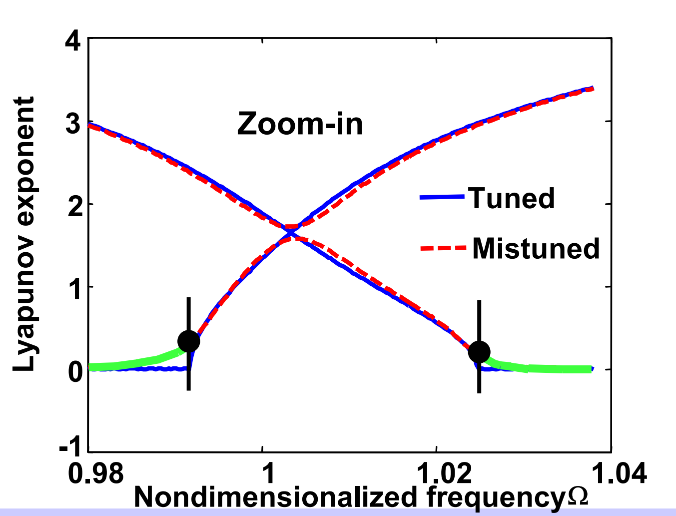
System w/o Negative Capacitance

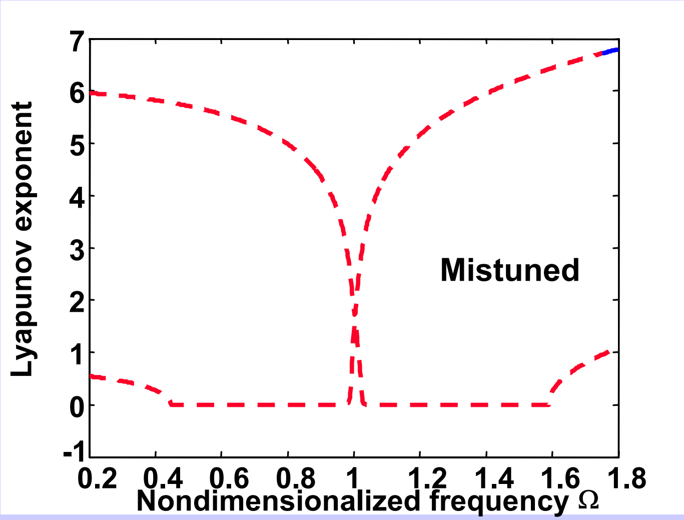


- **Green lines – localization index**
- **The worst case is the mode in the double passband with the largest L. exp.**

System with Negative Capacitance







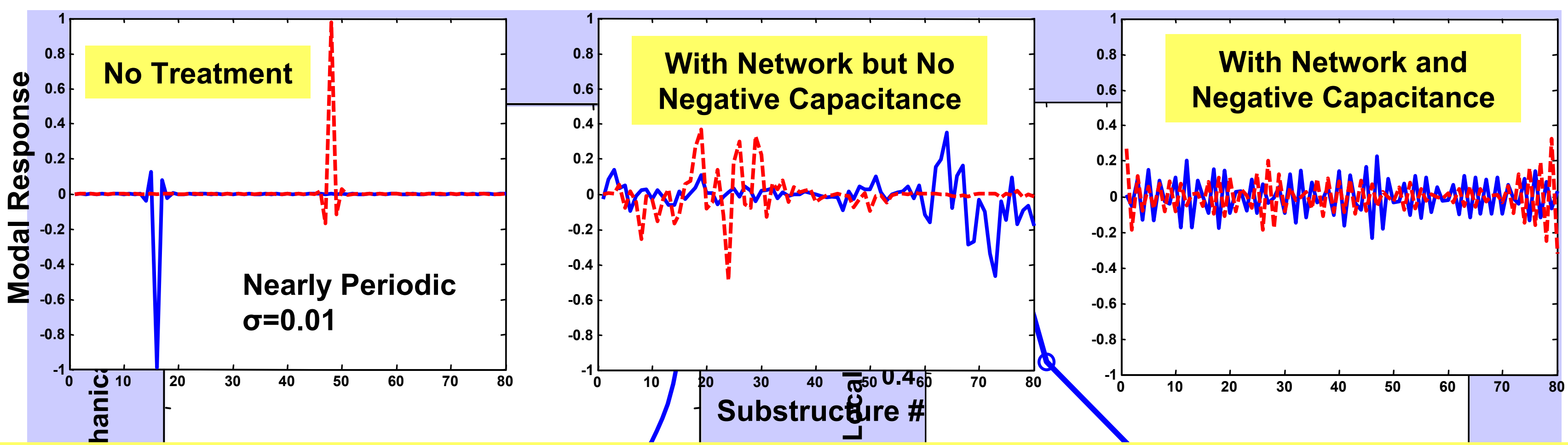
- **With negative capacitance, the double passband is evolved into two passbands**
- **The worst cases for mode localization are the modes at the edge of the passbands**

System with Negative Capacitance

With the design of negative capacitance,

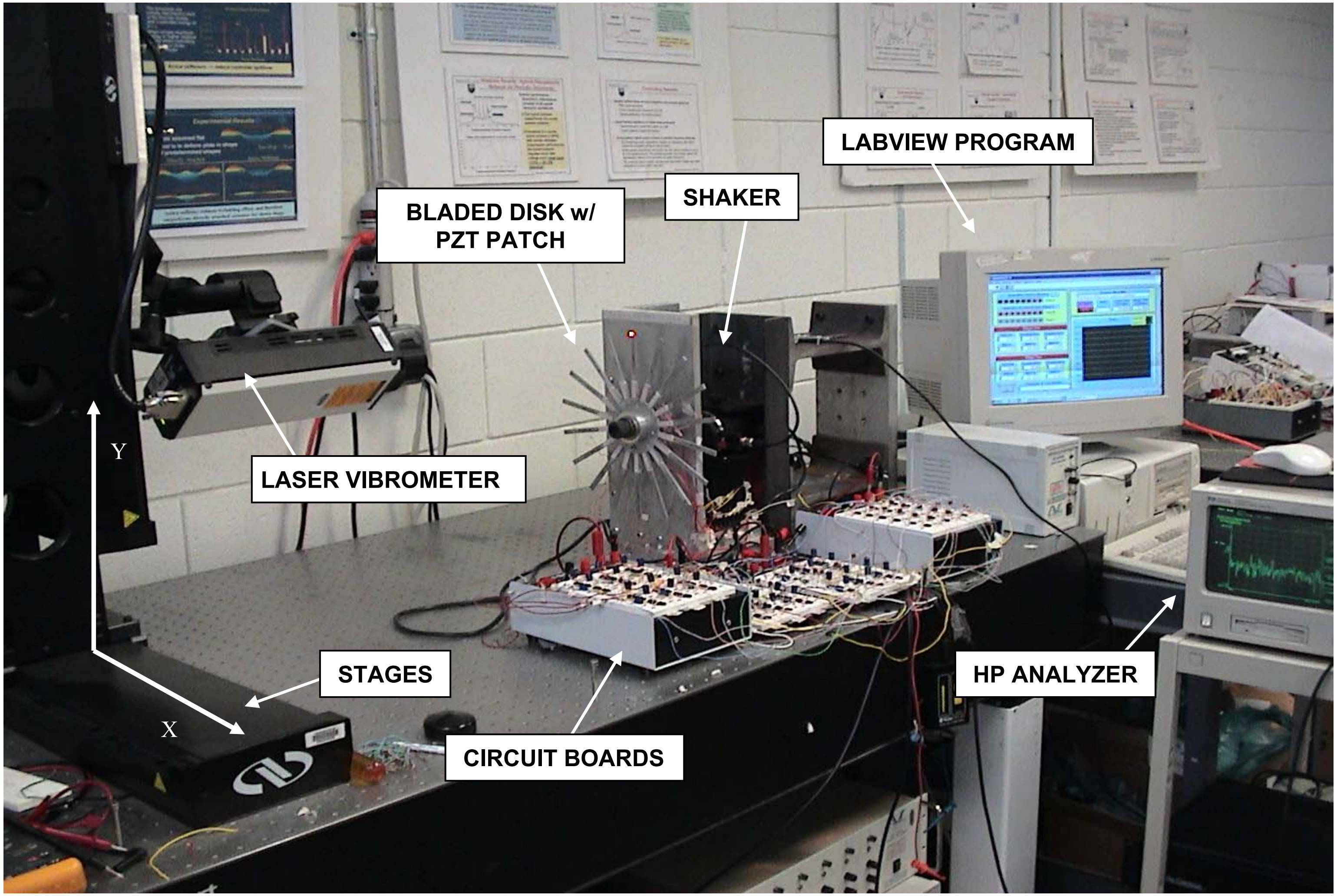
- Electromechanical coupling coefficient can be increased
- Localization index is reduced

→ **Modes are less localized with active coupling enhancement**



- **Vibration localization is obviously reduced with the introduction of piezoelectric network and further greatly improved with negative capacitance**

Experimental Setup



Bladed Disk and PZT Patch

Dimensions (unit : inch)
Material: aluminum alloy Total diameter: 12 Inner (hole) diameter of the hub disk: 1.5 Outer diameter of the hub disk: 3.5 Blade number, length, width, and thickness: 18; 4.25; 0.305; 0.125

PZT Geometry (unit: inch)	PZT Material Property
Length: 1.0 Width:0.30 Thickness:0.04	Piezoelectric material: Type 5A Relative dielectric constant K^T: 1750 Electromechanical coupling factor k_{31}: 0.36 Piezoelectric charge constant: $175*10^{-12}$ (C/N or m/V) Young's modulus :$6.3*10^{10}$ (N/m^2)

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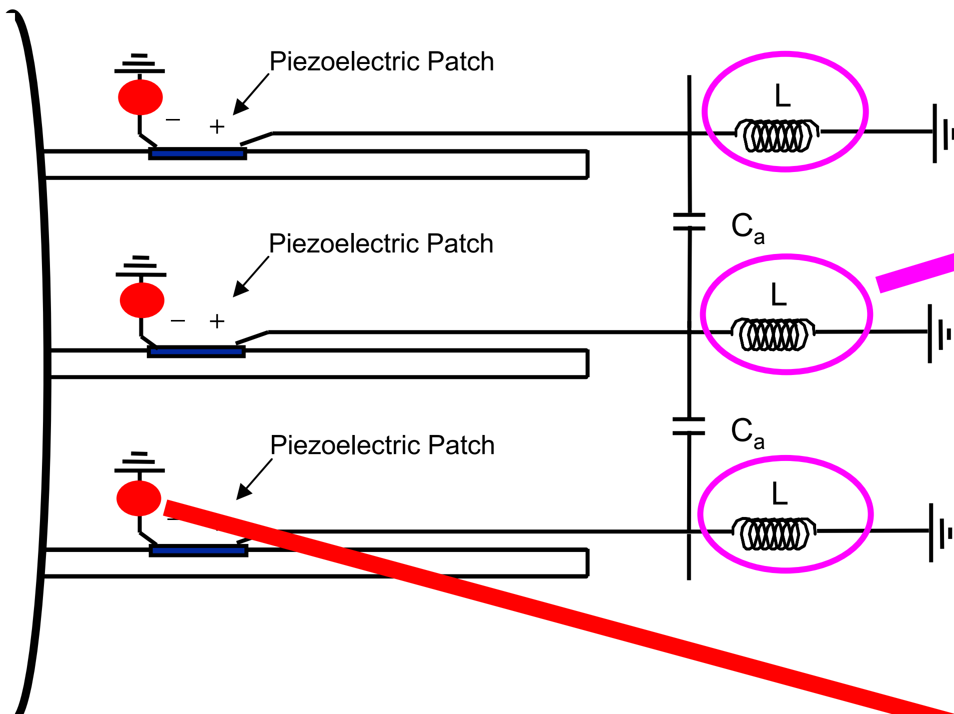
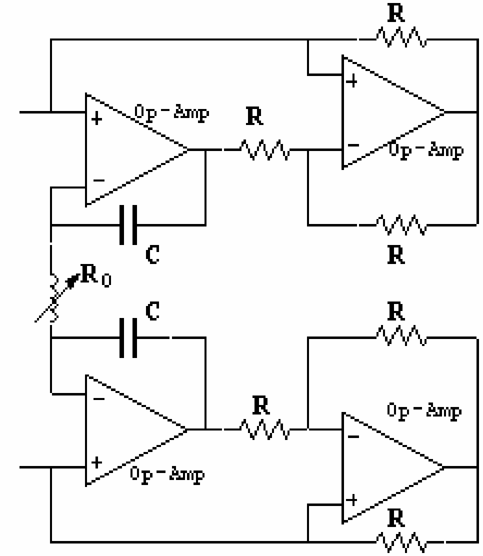
Synthetic Inductor and Negative Capacitor

Piezoelectric network without negative capacitance



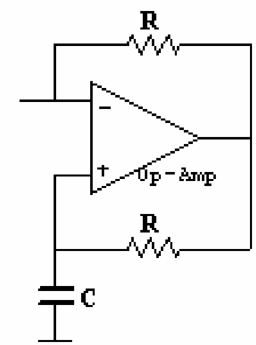


Synthetic inductor

$L=RR_0C$ (Henry)

Negative capacitor

$C^* = -C$ (Farad)



Negative Impedance Converter

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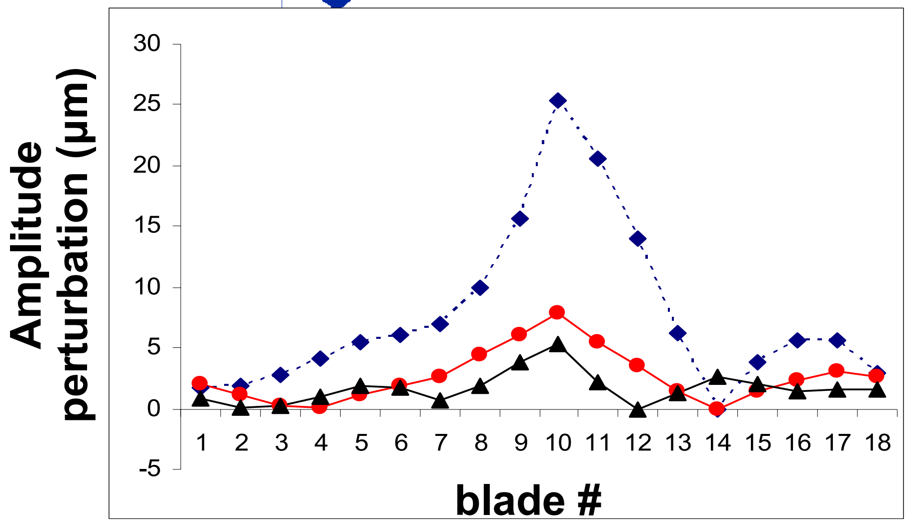
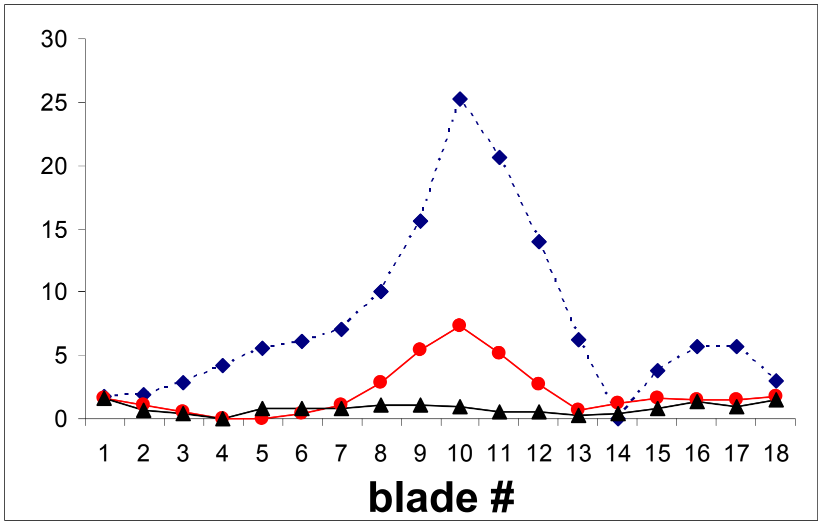
Experimental Results



f_e=201Hz



f_e=212Hz

◆ : system without piezoelectric network;

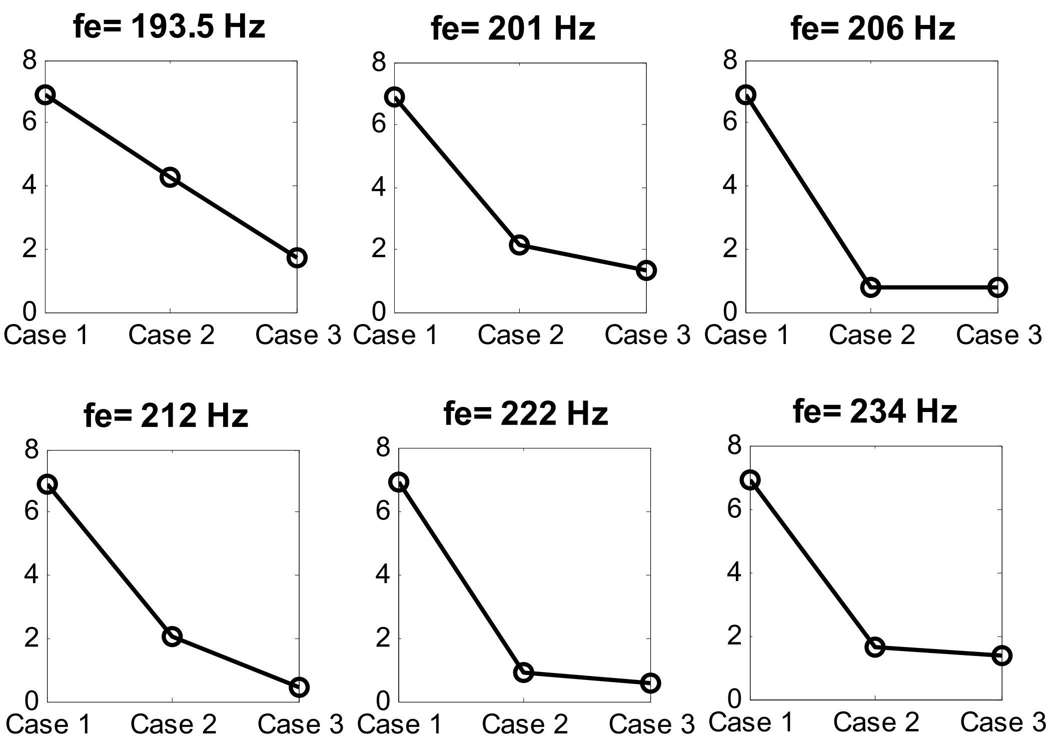
● : system with piezoelectric network but without negative capacitance;

▲ : system with piezoelectric network and with negative capacitance;

Vibration localization can be effectively reduced via piezoelectric networking and further improved by negative capacitance circuits

Experimental Results

Standard Deviations of Blade Amplitudes



<u>Case 1</u>: system without piezoelectric network

<u>Case 2</u>: system with piezoelectric network but w/o negative capacitance

<u>Case 3</u>: system with piezoelectric network w/ negative capacitance

Blade amplitude distribution is more uniform with piezoelectric network, and is further improved by the addition of negative capacitance

Overall Conclusion

- **Piezoelectric networking can be utilized for different types of structural control enhancement -- vibration energy absorption, dissipation, and redistribution**
  - General modal damping and control
  - Adaptable narrowband disturbance rejection
  - Vibration delocalization

- **Effective structural control can be achieved through electrical tailoring of the piezoelectric networks**
  - With careful design, active-passive hybrid networks could outperform purely active and passive systems
    - **Better performance than both**
    - **Less control effort, robust, and failsafe than purely active systems**

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The End

Questions?