Characterisation and simulation of porous foams

Some recent results

ROYAL INSTITUTE OF TECHNOLOGY

Rémi Guastavino
Peter Göransson

Background

Most porous materials are non-isotropic in elasticity/viscoelasticity/viscoacoustic properties

To derive adequate simulation models, these properties must be known with a relevant precision

Work has been on-going since the BRAIN project ended in the mid-90's.

Fibrous wools are modelled with a high degree of accuracy

Open cell porous foams are still an open point

Contents of lecture

Review results for fibrous wools

Review our early work on porous foams

Recent results (experimental/numerical) on foams

Constitutive modelling

$$\boldsymbol{\sigma}_f = \hat{\mathbf{H}}\boldsymbol{\varepsilon}_f$$

$$\boldsymbol{\sigma}_f^{\ T} = \begin{bmatrix} \sigma_{11} & \sigma_{22} & \sigma_{33} & \sigma_{12} & \sigma_{13} & \sigma_{23} \end{bmatrix}$$

$$\hat{\mathbf{H}} = \mathbf{H} + d_\lambda(s)\mathbf{H}_\lambda + d_G(s)\mathbf{H}_G$$

$$d_\lambda(s) = \sum_{l=1}^{N_a} \frac{\left(3\varphi_l^2 + 4\varphi_l\mu_l\right)}{\delta_l(s+\beta_l)} s$$

$$d_G(s) = \sum_{l=1}^{N_a} \frac{2\mu_l^2}{\delta_l(s+\beta_l)} s$$

$$s = \pm i\omega$$

$$\mathbf{H}^{-1} = \begin{bmatrix} a & d & g & 0 & 0 & 0 \\ & a & g & 0 & 0 & 0 \\ & & b & 0 & 0 & 0 \\ & & & s & 0 & 0 \\ & & & & t & 0 \\ & & & & & t \end{bmatrix}$$

$$a = \frac{1}{E_1}; b = \frac{1}{E_2}; d = \frac{-\nu_1}{E_1}; g = \frac{-\nu_2}{E_2}; t = \frac{1}{G_2}; s = \frac{1}{G_1}$$

$$\mathbf{H}_\lambda = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ & 1 & 1 & 0 & 0 & 0 \\ & & 1 & 0 & 0 & 0 \\ & & & 0 & 0 & 0 \\ & & & & 0 & 0 \\ symm & & & & & 0 \end{bmatrix} \qquad \mathbf{H}_G = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ & 2 & 0 & 0 & 0 & 0 \\ & & 2 & 0 & 0 & 0 \\ & & & 1 & 0 & 0 \\ & & & & 1 & 0 \\ symm & & & & & 1 \end{bmatrix}$$

Fibrous wools

Layered, within each layer randomly dispersed fibres

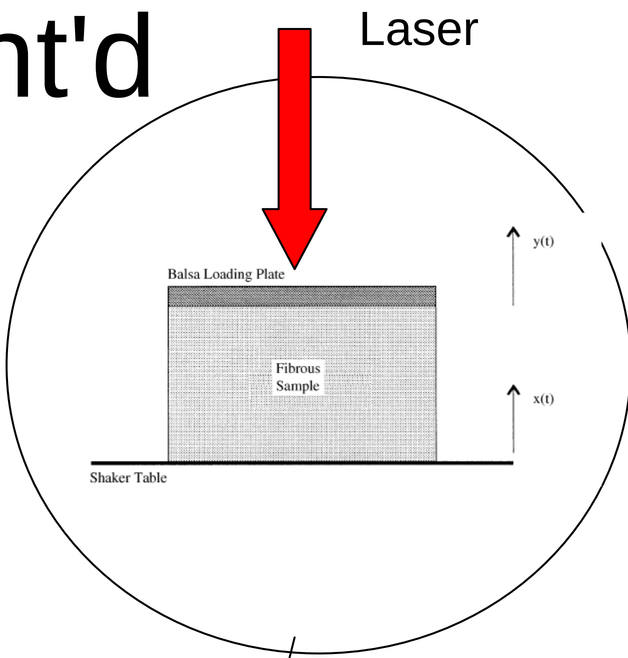
Elasticity assumed transversely isotropic

Poisson's ratios are ~zero, simplifying the process of finding the various moduli.

From static tests (example of fibrous wool used in aircraft):

Moduli	E_1	E_2	G_1	G_2
[Pa]	225	17200	1200	13700

Fibrous wools, cont'd



Viscoelasticity assumed isotropic (!)

Uniform motion of the loading plate

From dynamic tests in vacuum chamber:

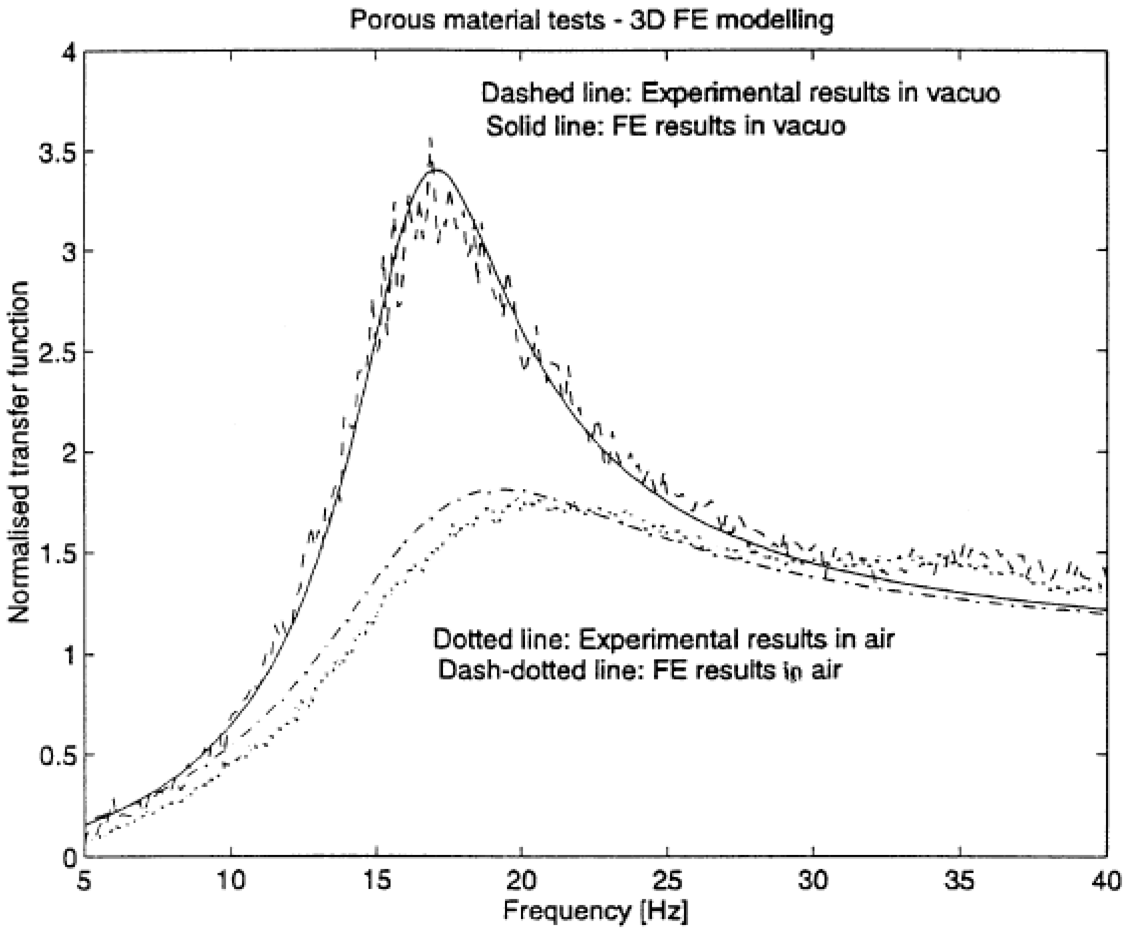
Moduli	β_1	μ_1	δ_1	φ_1
	6.65 Hz	15.4 Pa	1.0 Pa	0 Pa

Note: Only one term needed + only in shear modulus

Equivalent loss factor ~10%

Fibrous wools, cont'd

Results



Open cell foams (PU)

Foaming process induces a directional micro-structure

Properties vary throughout a real foam block

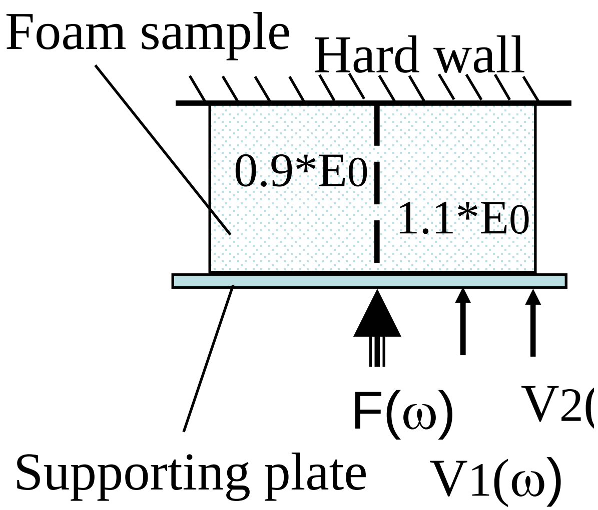
Degree and influence of non-isotropic elasticity?

"Naive" approach, use an isotropic elasticity model, but assume a variation in the in-plane direction.

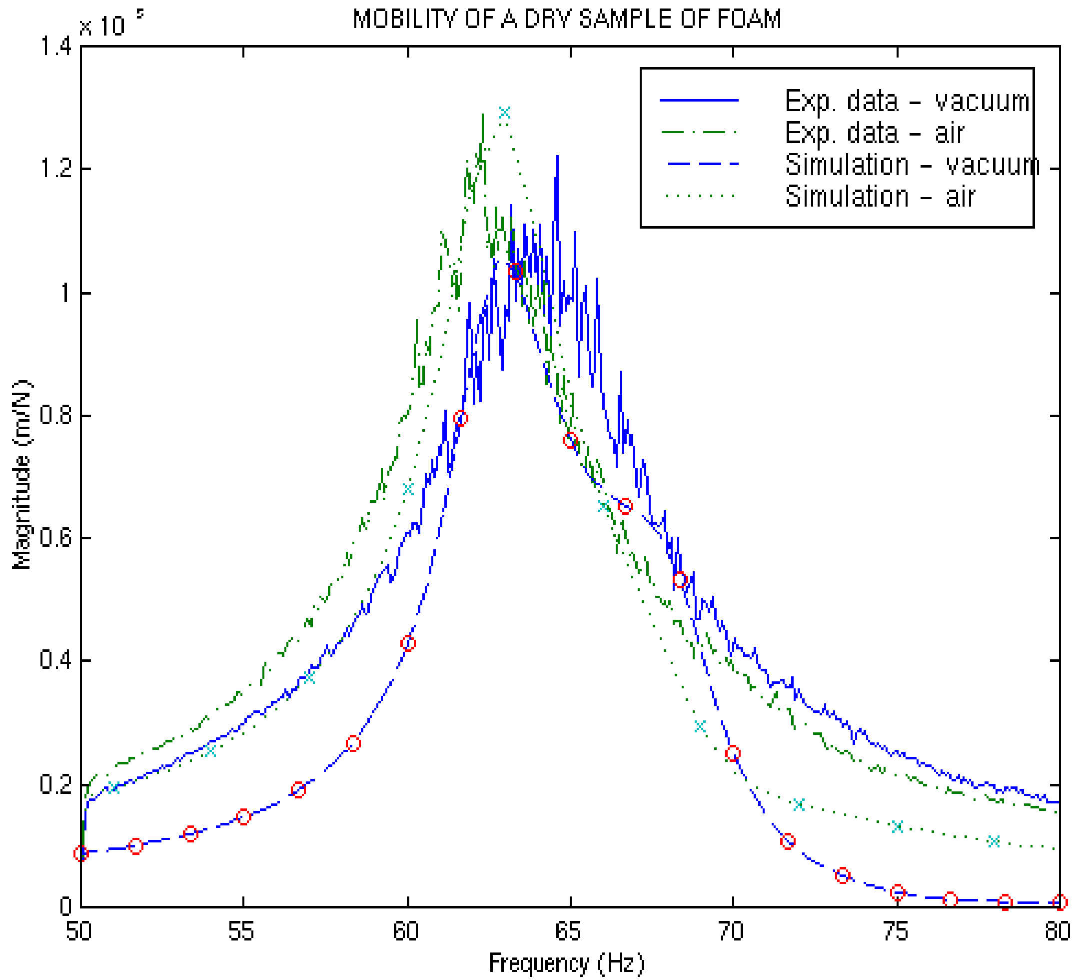
$$E = 75 \text{ kPa}; \quad \nu = 0.40$$

Moduli	β_l	μ_l	δ_l	φ_l
$l=1$	1. Hz	71.30 Pa	1.0 Pa	71.95 Pa
$l=2$	1.E4 Hz	396.7 Pa	1.0 Pa	0 Pa

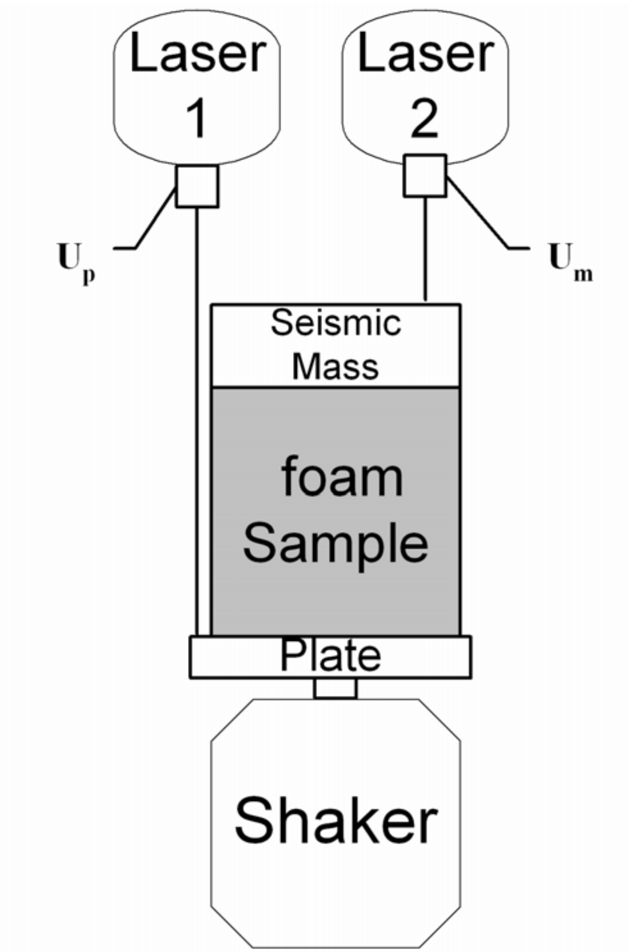
Open cell foams (PU), cont'd

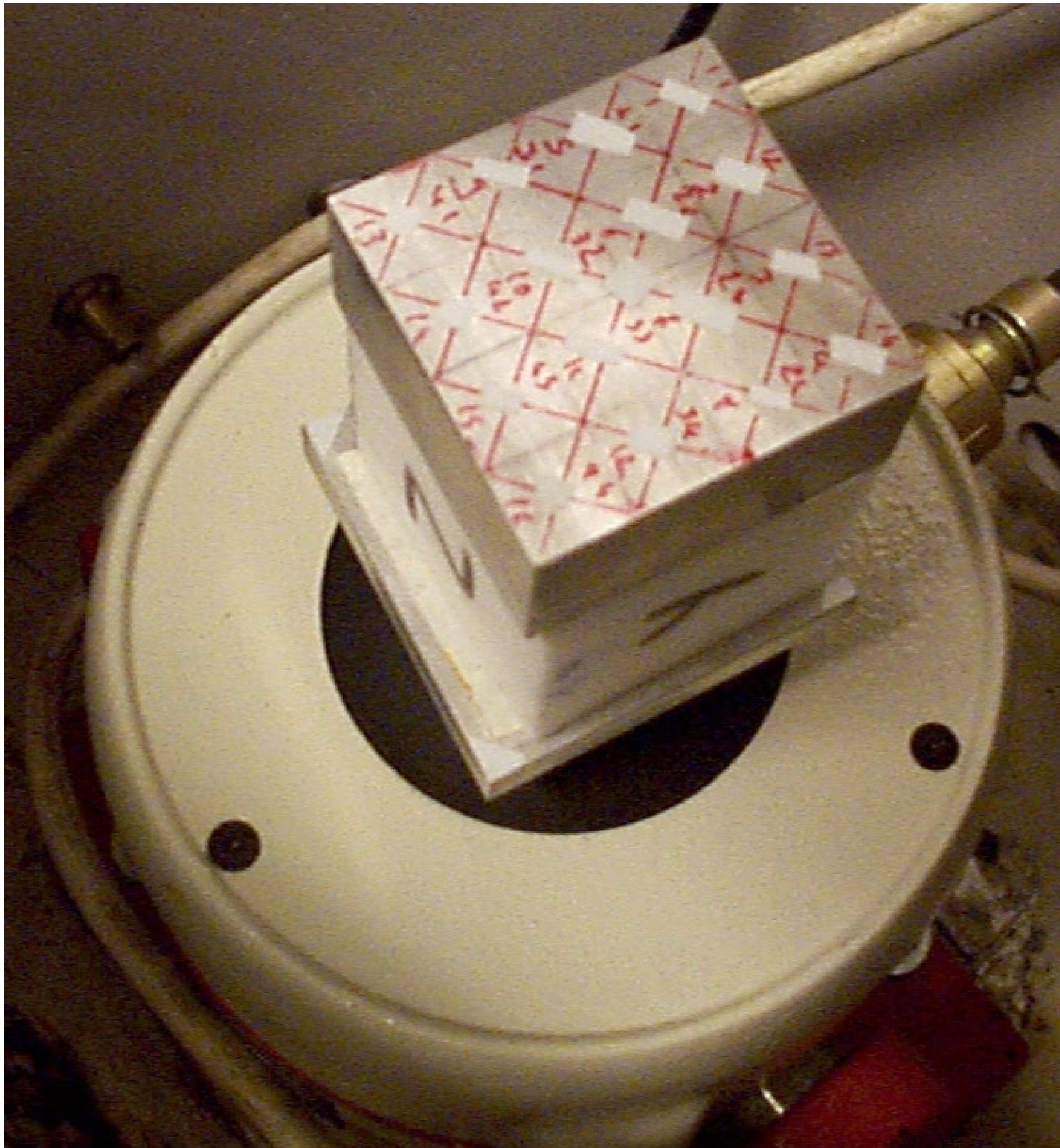


Non-uniform motion of supporting plate

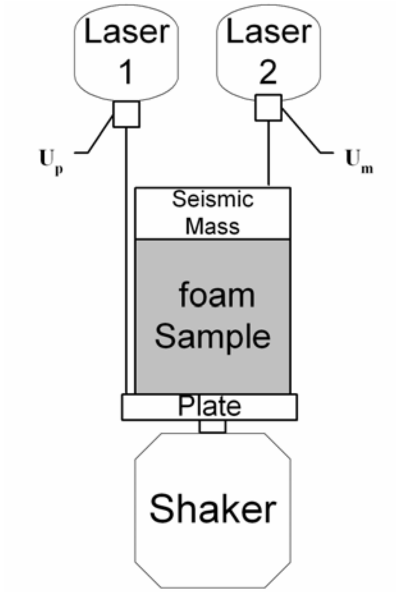


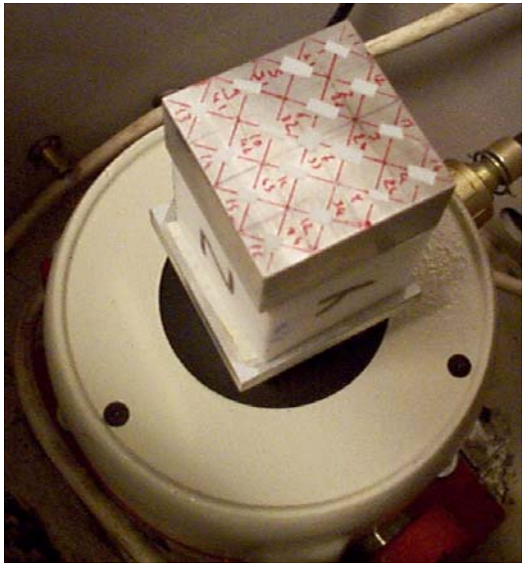
Dynamic experimental Setup

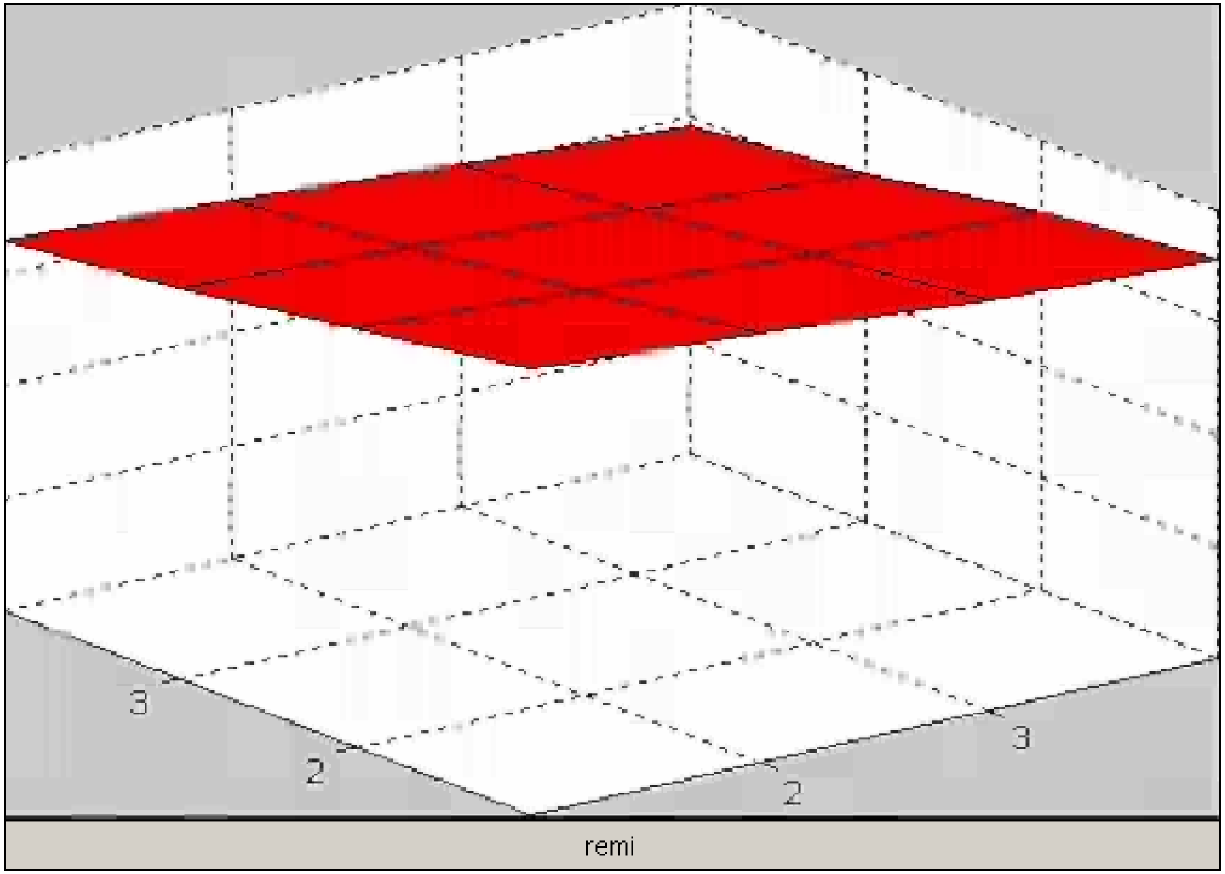




Open cell foams (ME), cont'd

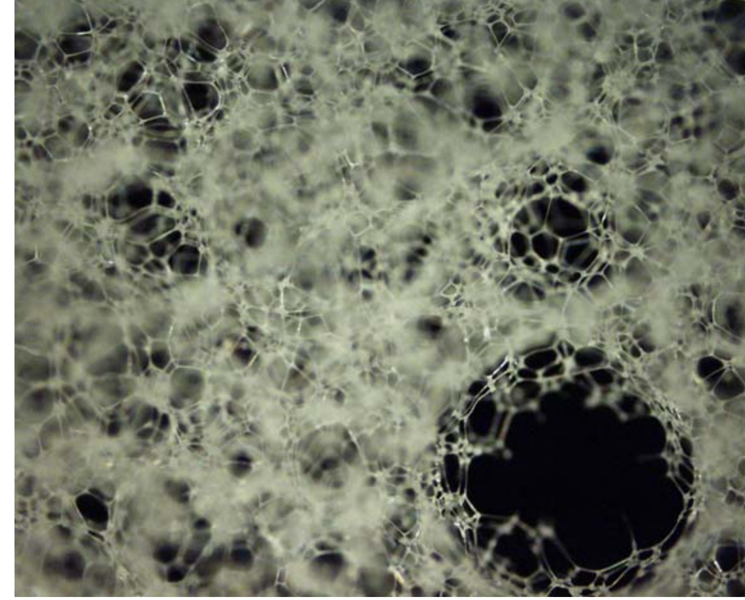


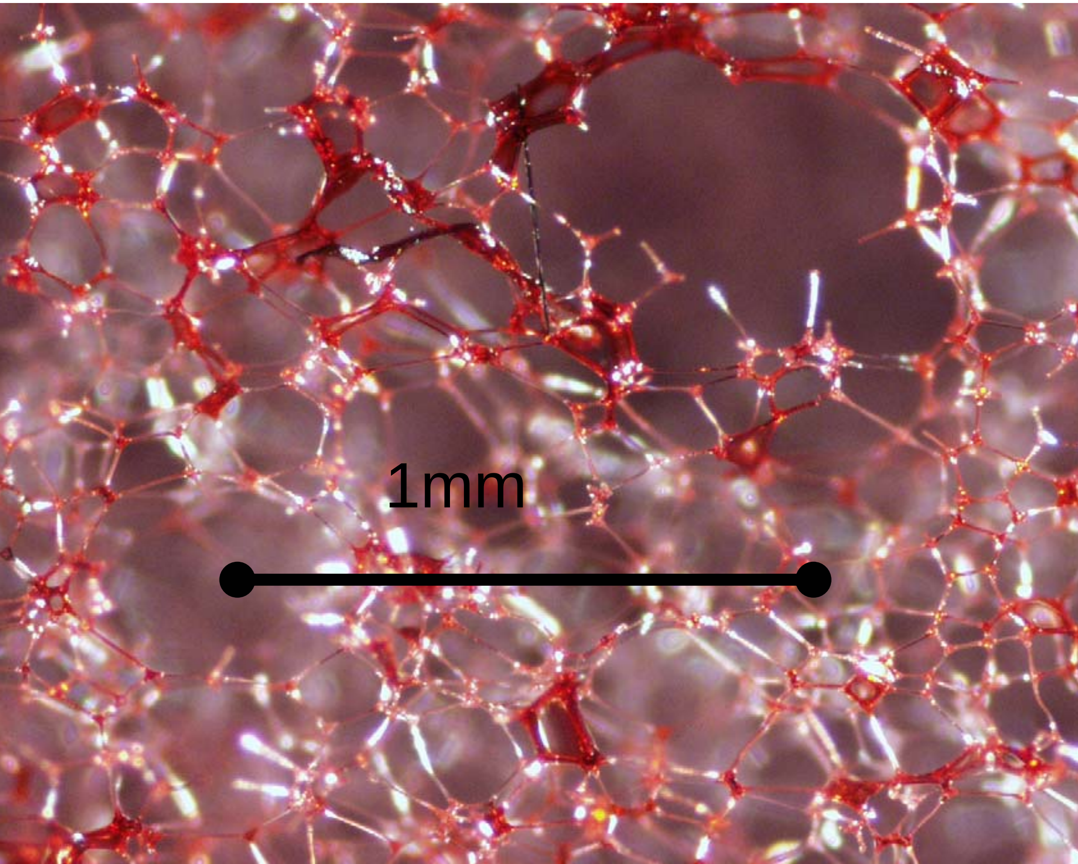


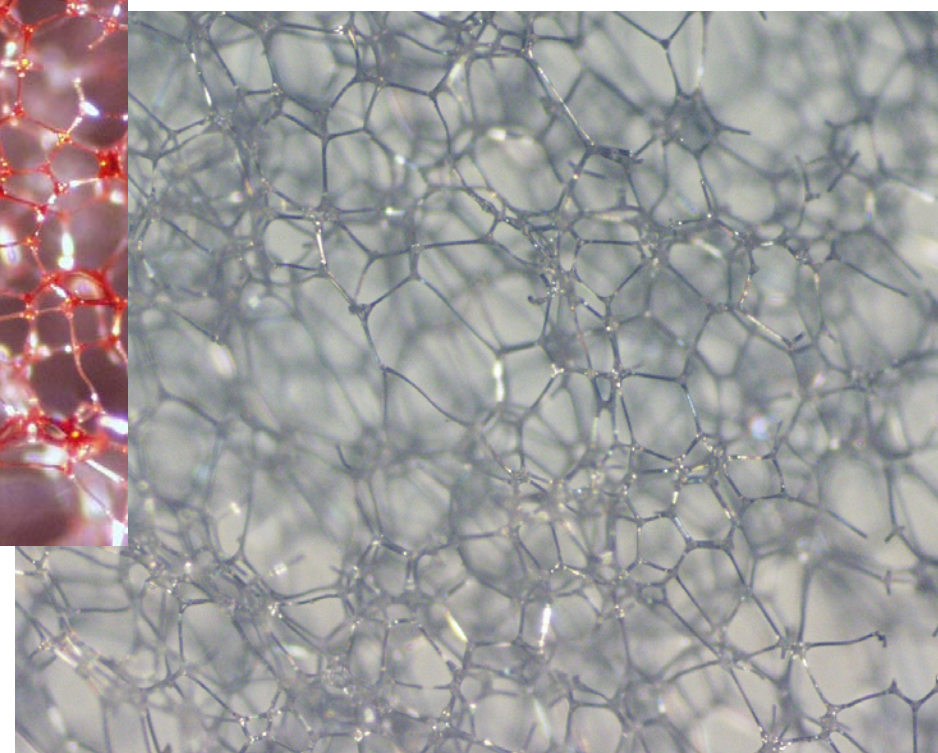


Non-uniform motion of supporting plate

Microstructure
Melamine







Cell orientation determines anisotropy

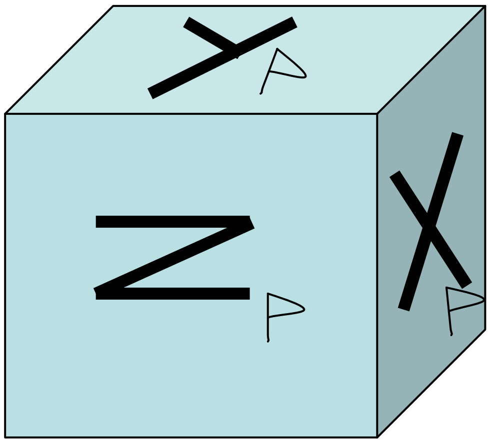
Finding the elastic constants

Combined experimental/numerical

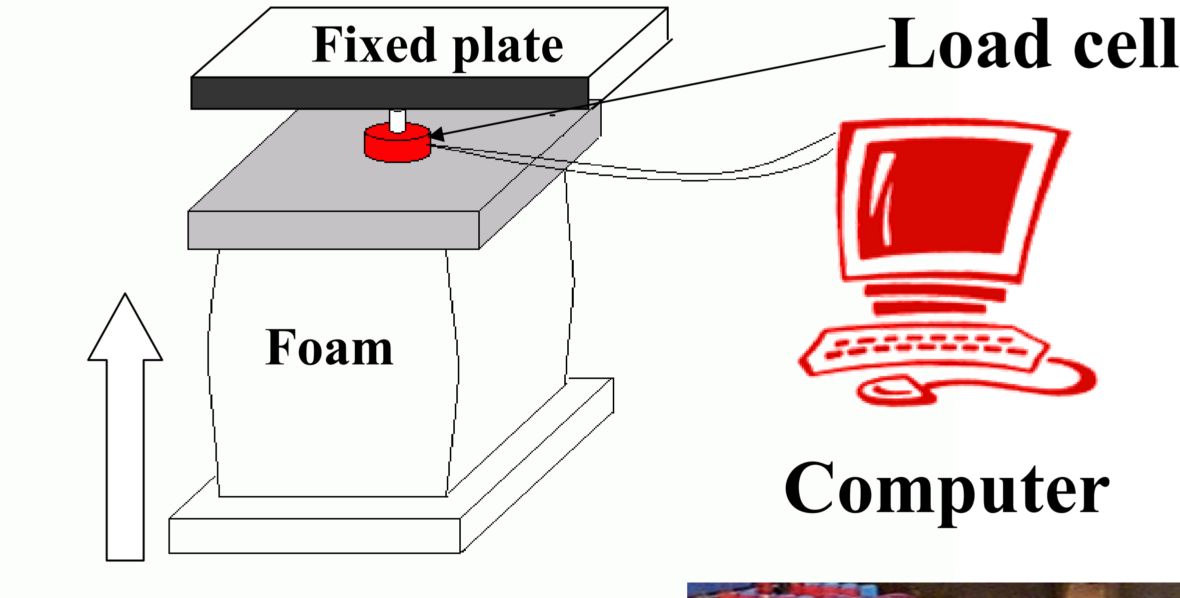
Inverse estimation through a least squares fit

First problem:

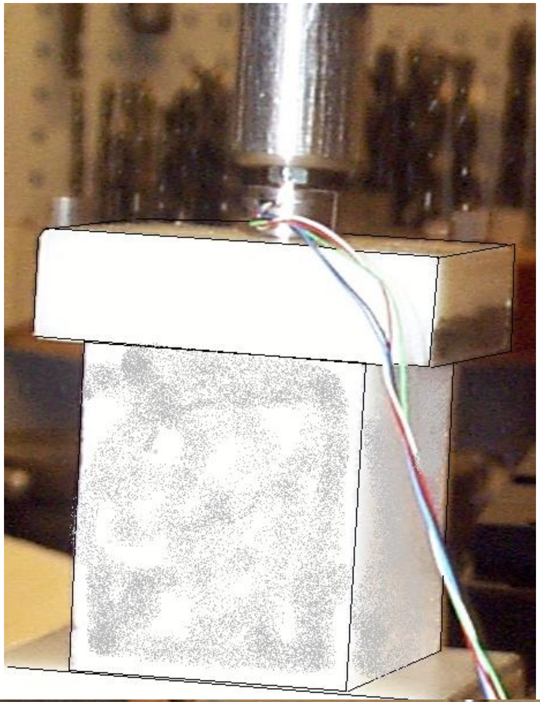
How to measure accurate deformations/forces?



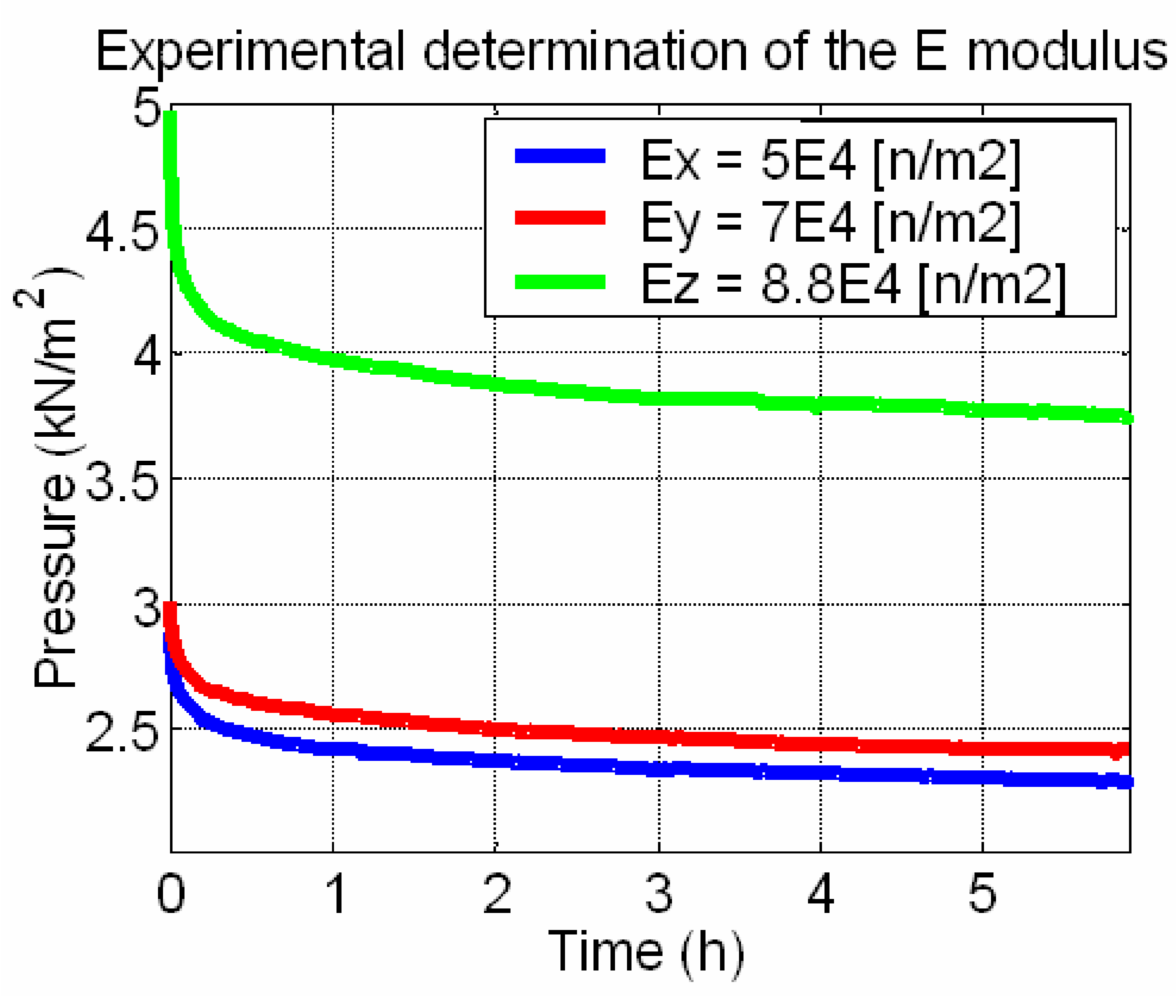
1st static experimental setup /Load

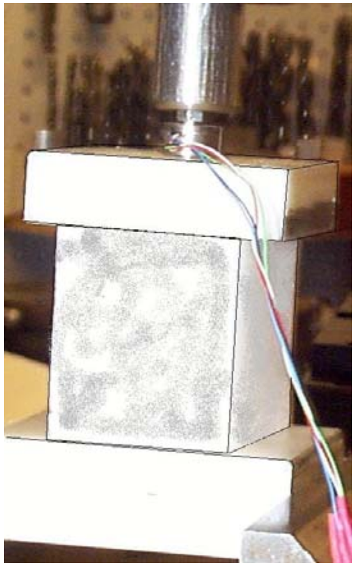


2,5%
deformation



Relaxation curve



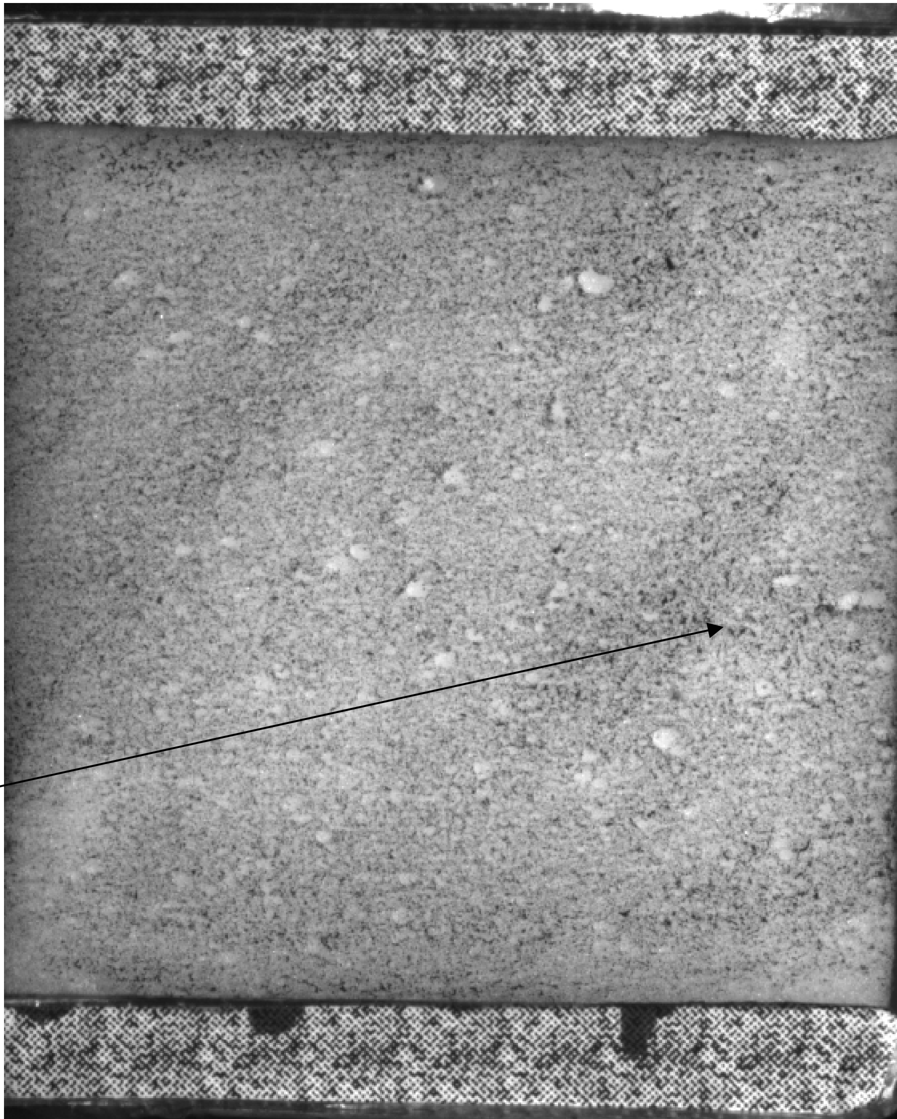


Measuring small forces, weakness of load cell significant problem

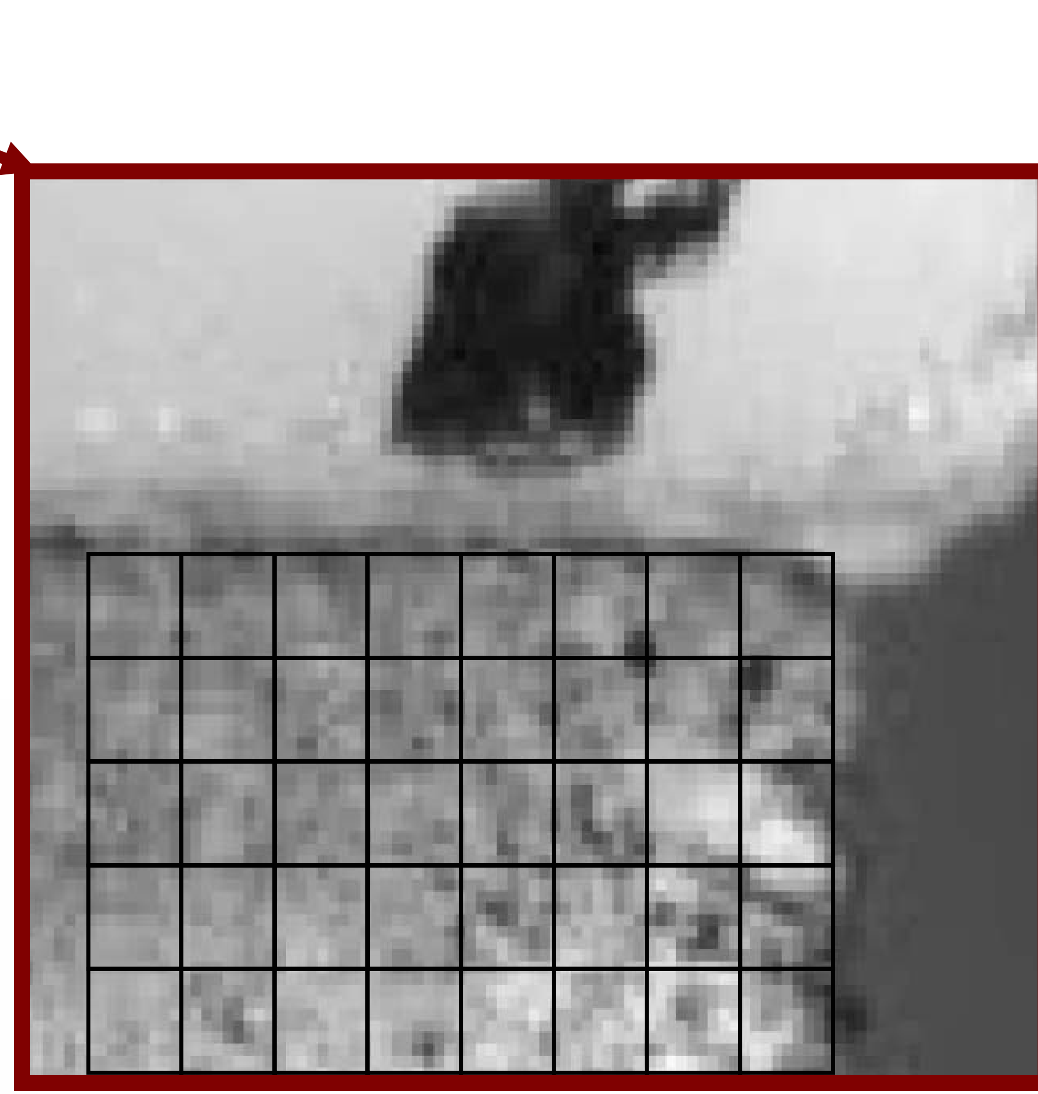
2nd static test set-up
CCD cameras and speckle photogrammetry

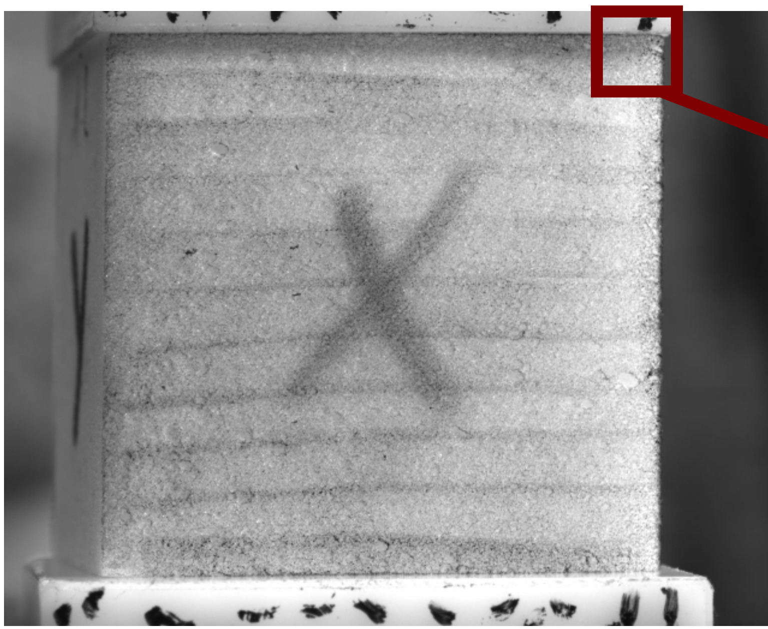
Used deformation level = 2.5 % strain

to avoid nonlinearity



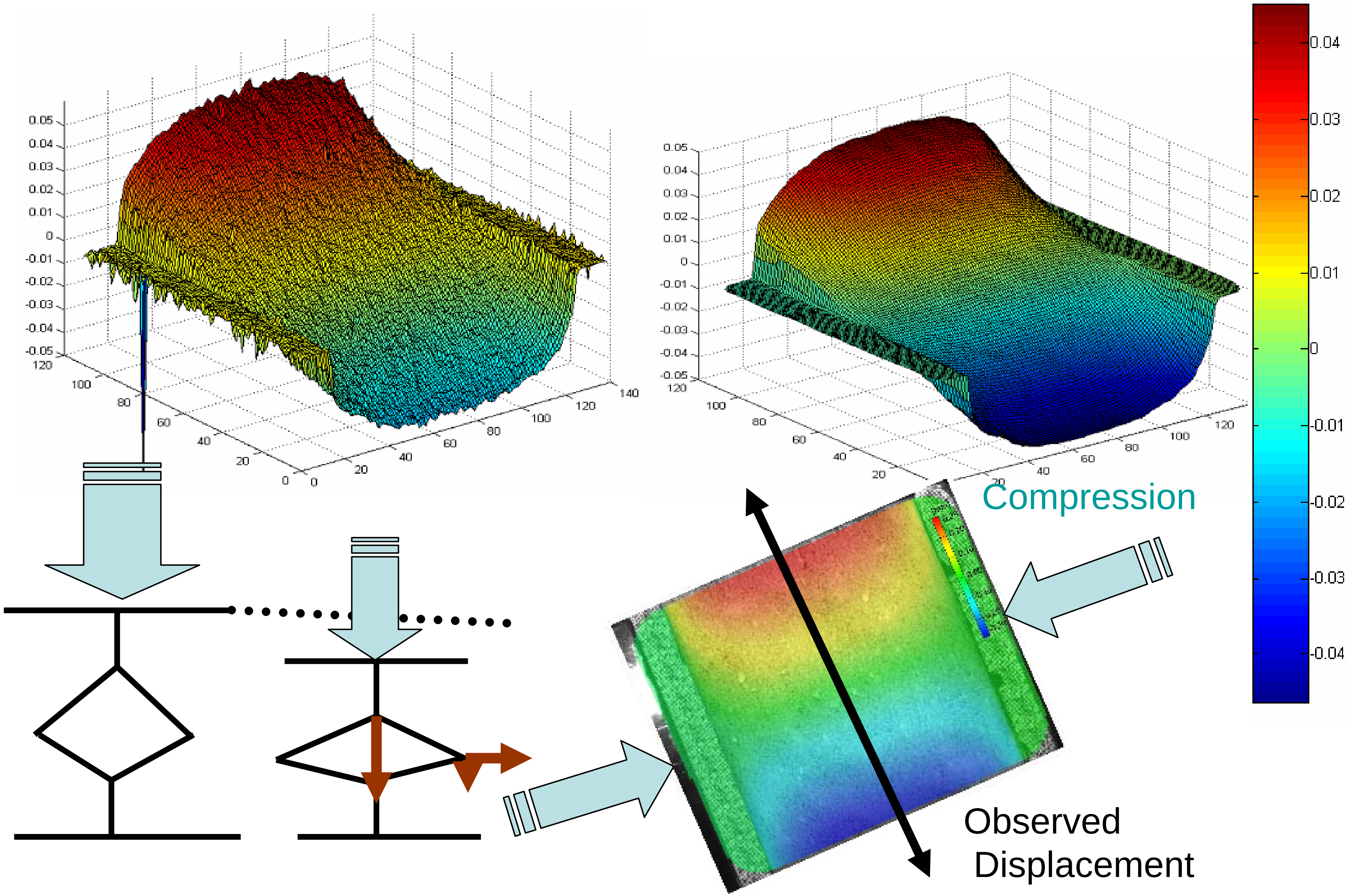
CCD measurement



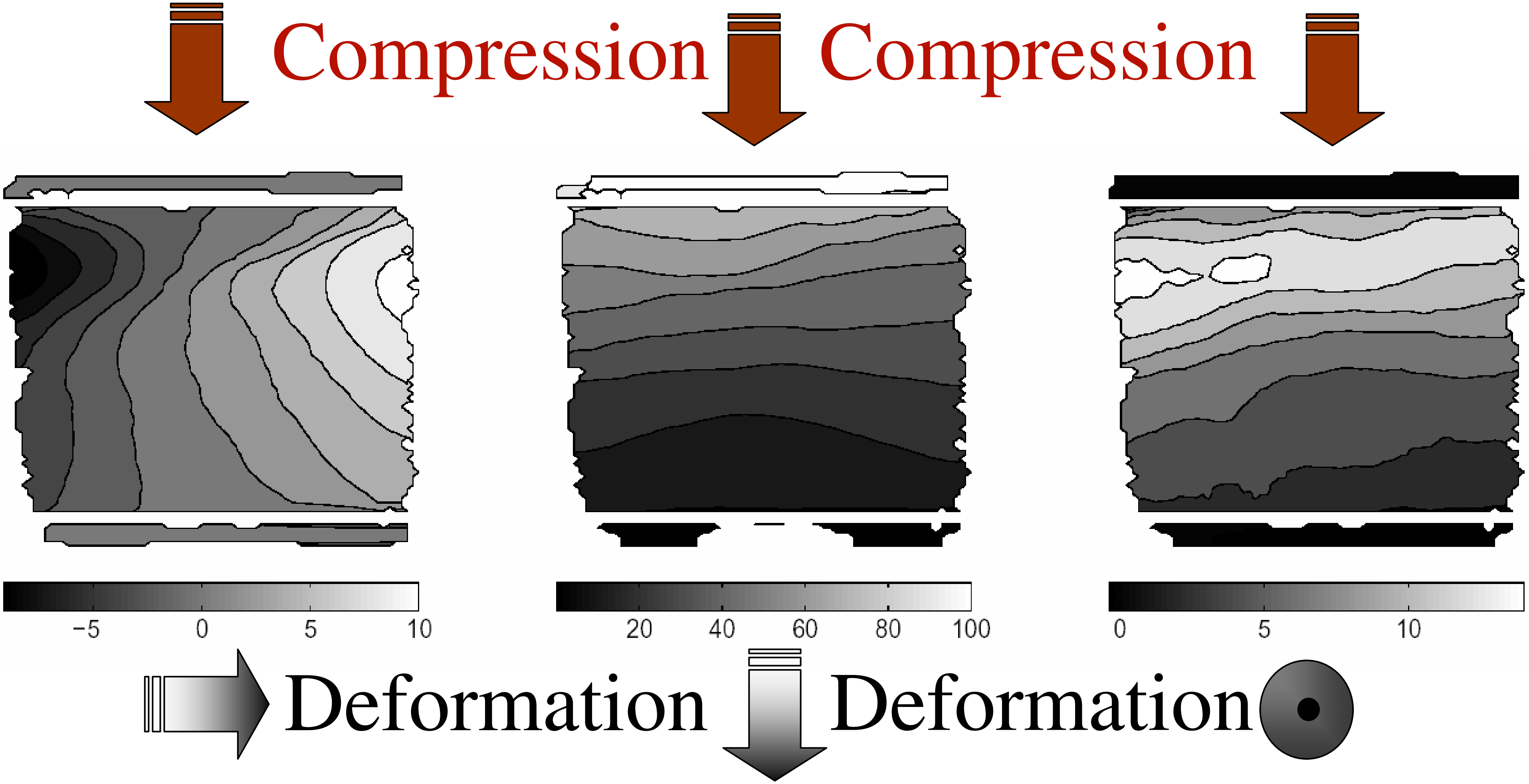


Before deformation During deformation

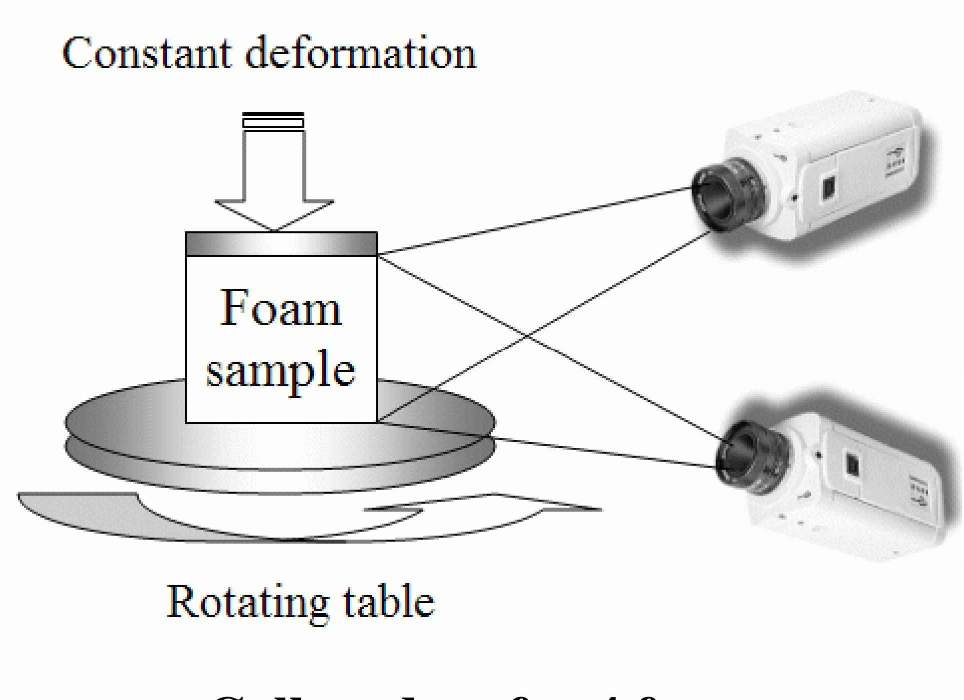
Typical result & smoothing



Optical measurement result

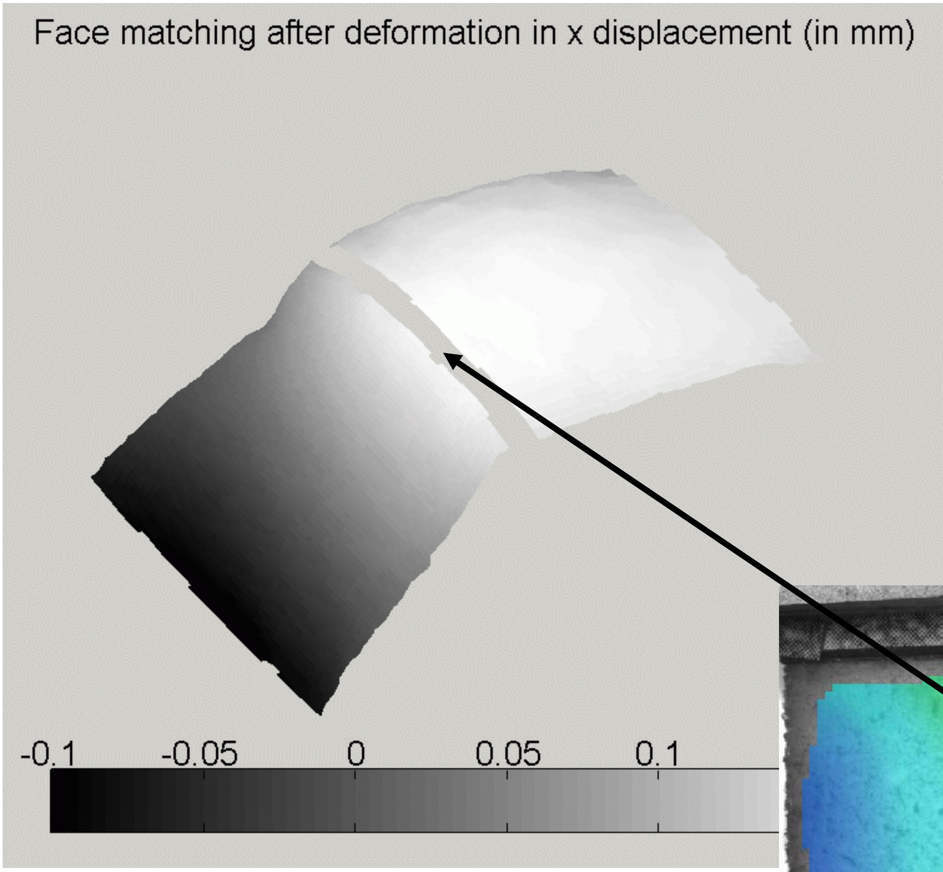


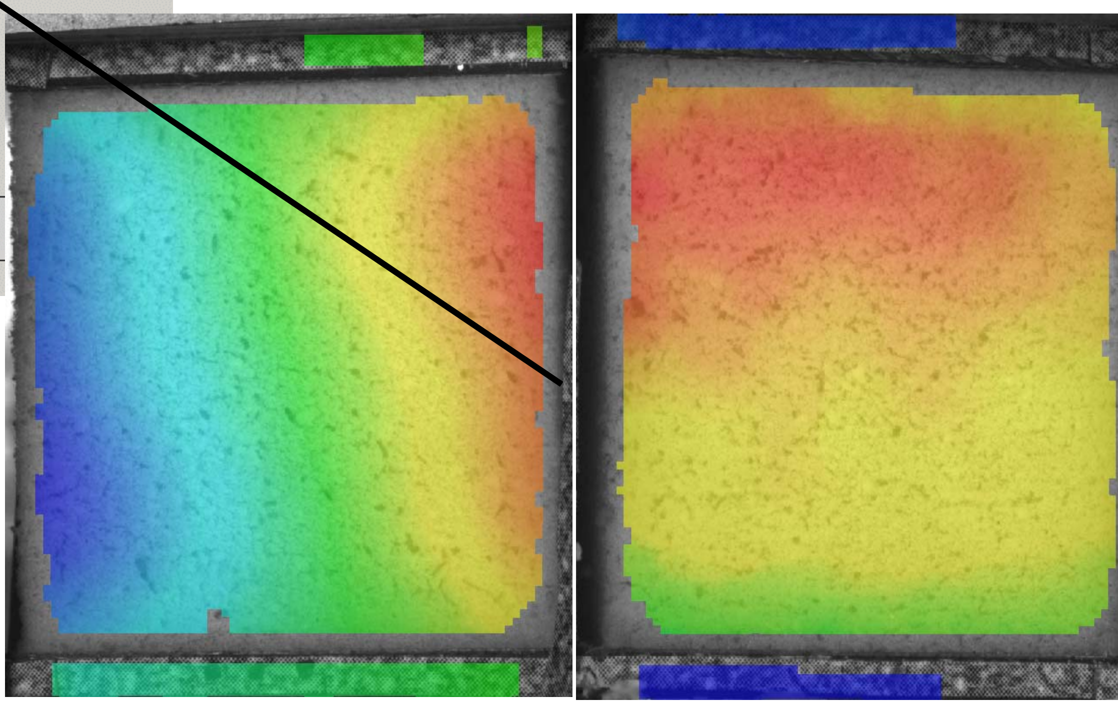
Rotating table

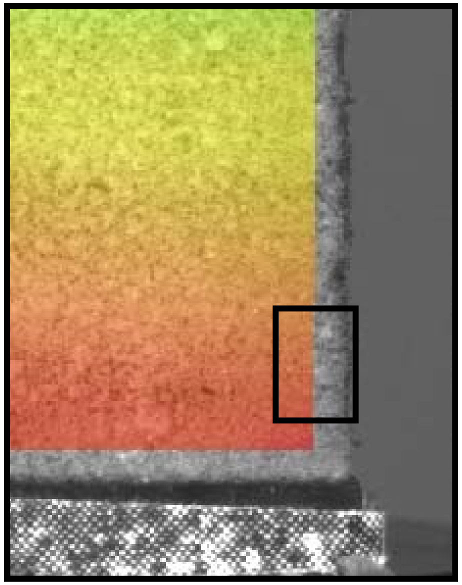


Collect data for 4 faces without dismantling

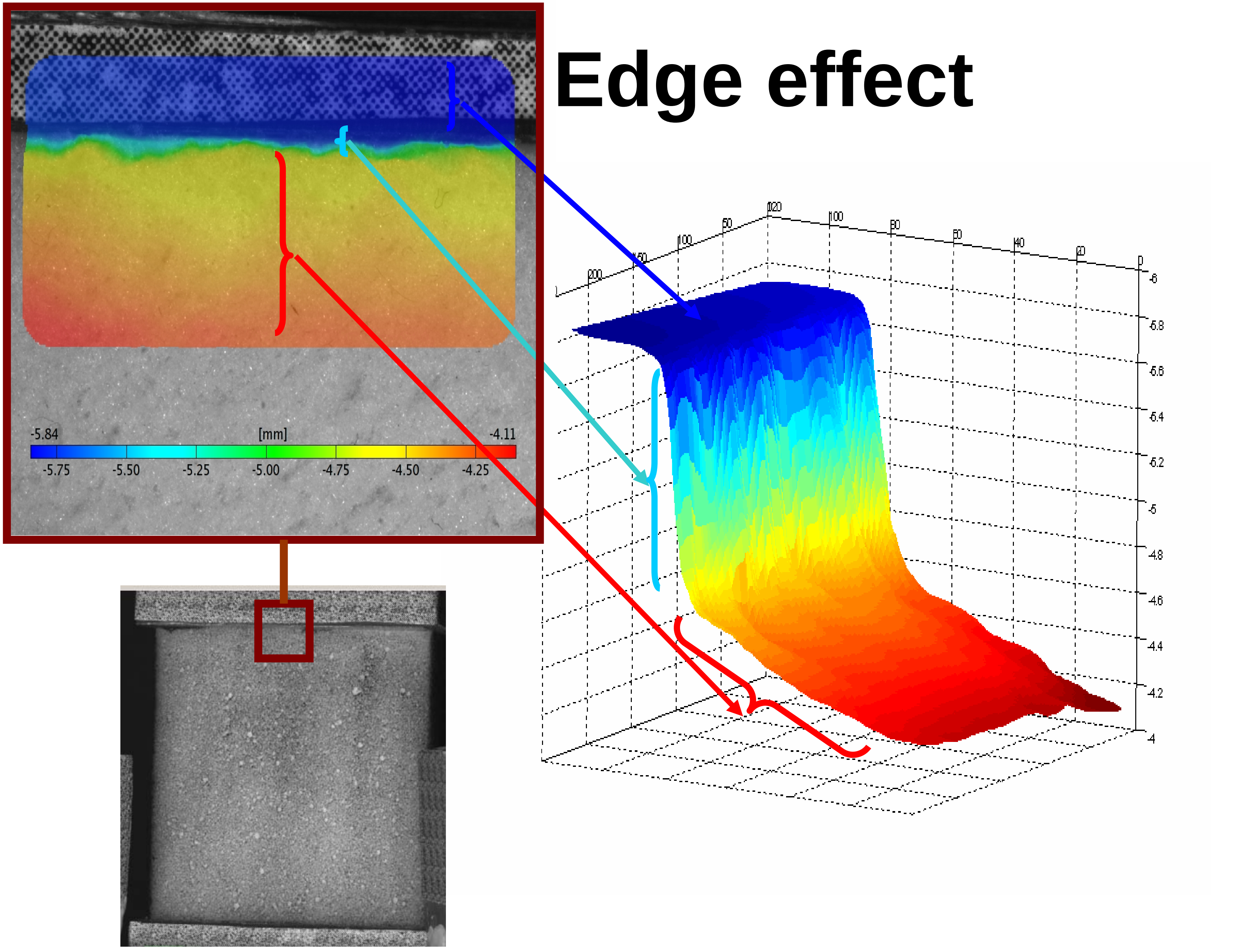
Face matching



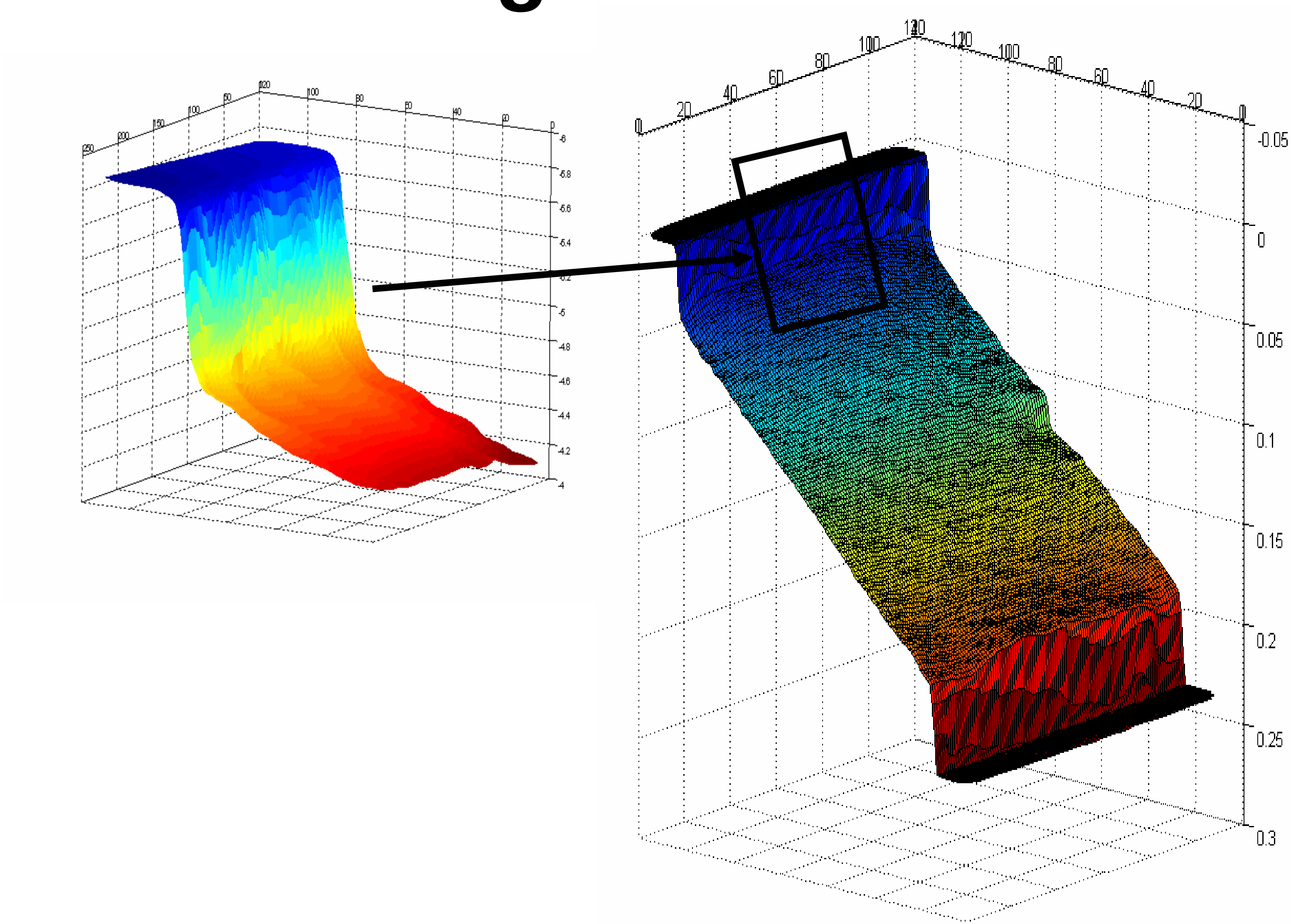




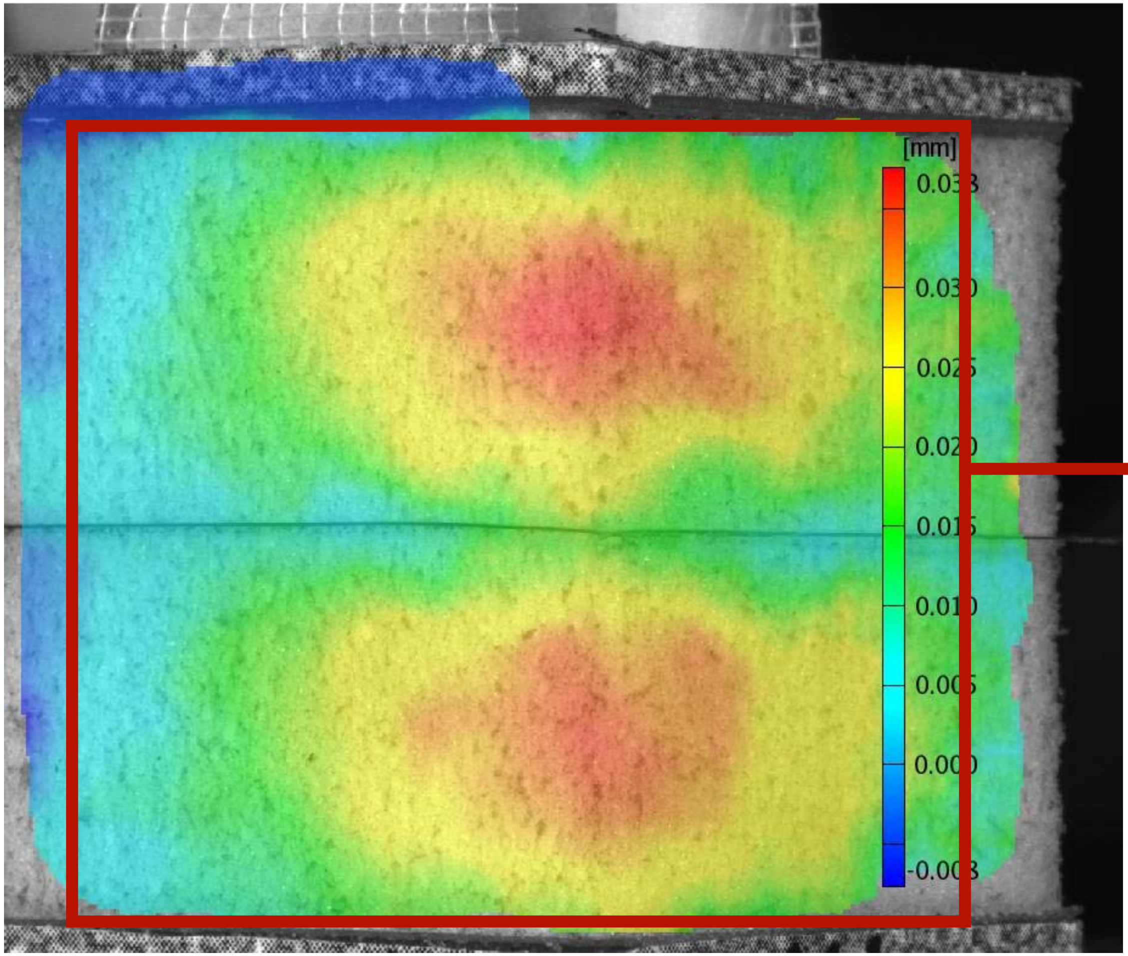
Edge effect



Edge effect



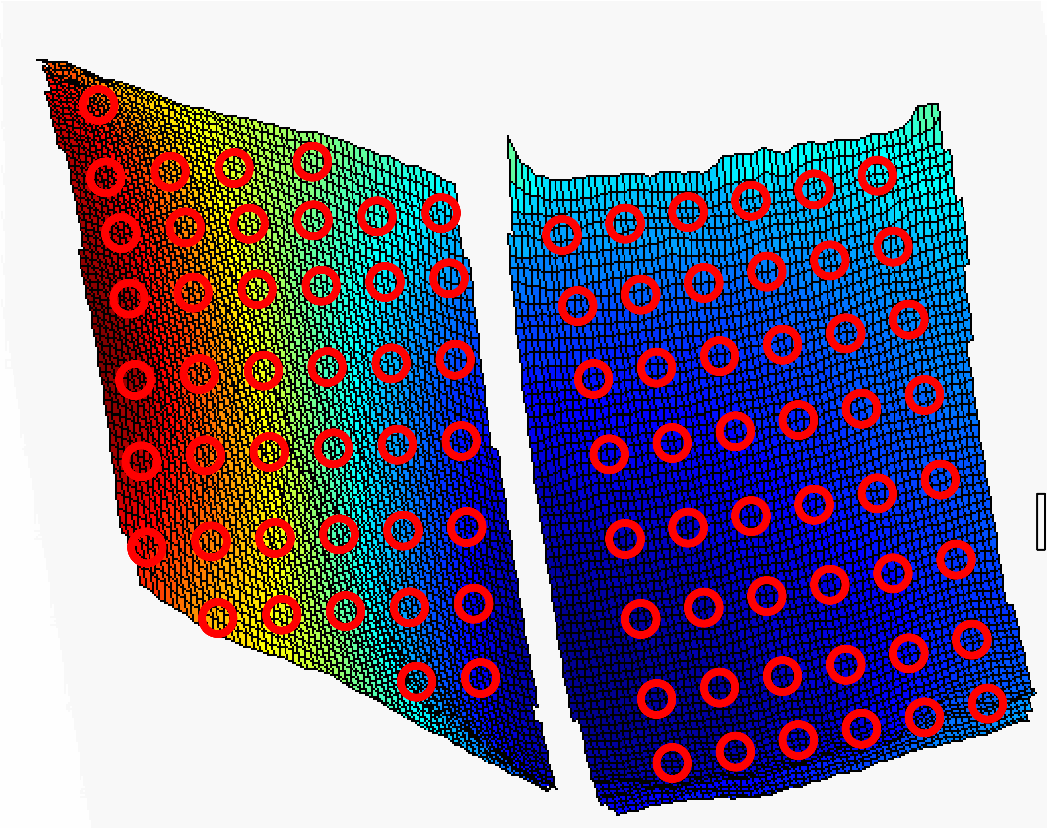
Artificially introduced discontinuity



Valid zone of focus

Out of picture plane displacement

Inverse estimation of elastic properties



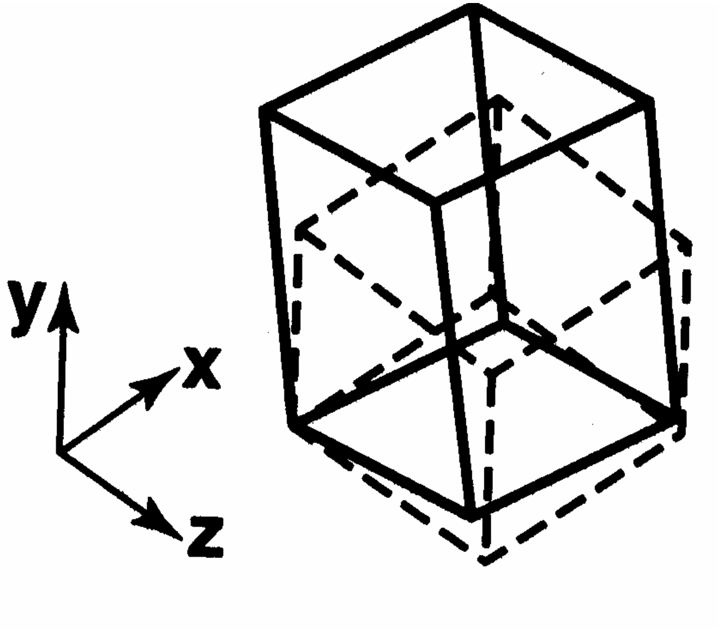
Approx 200 points /cube

F.E model

Loads in 3 directions

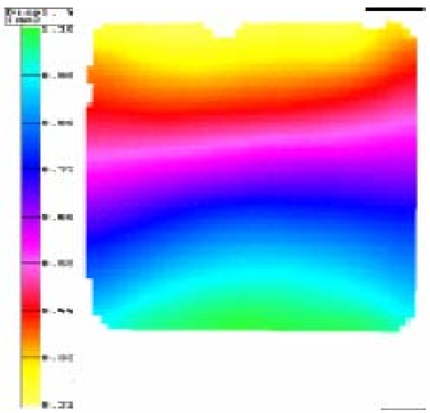
Elastic material parameters

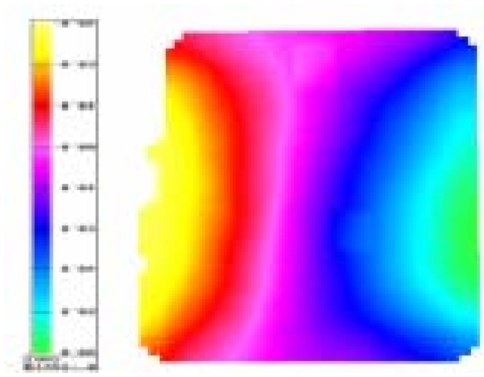
- In general anisotropic requires 21 constants.
- Here a general orthotropic model is assumed.
- This requires 9 independent constants and 3 angles of rotation (assuming body and material coordinate systems are different).

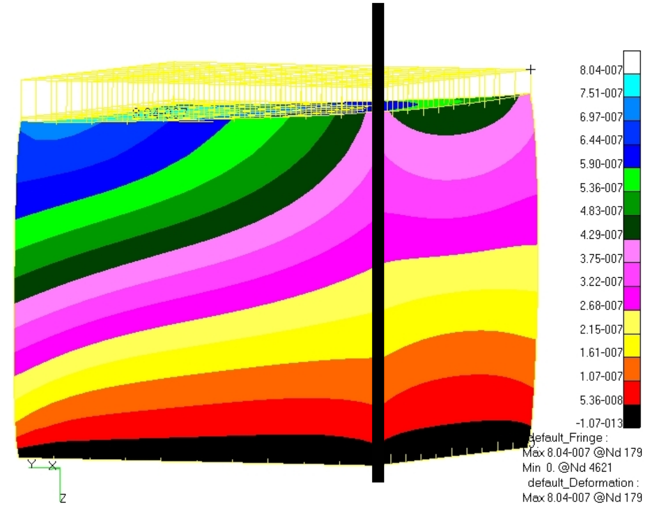


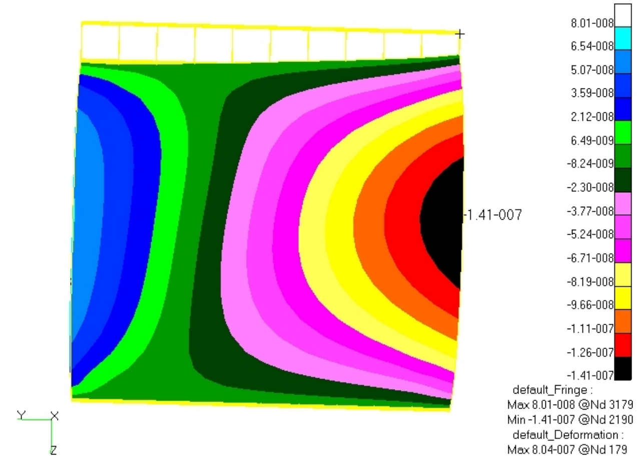
A total of ~600 points to fit a model of 12 parameters

F.E Simulation & experimental









Elastic material parameters

Possible (?) Hooke's matrix for Melamine

7.0734241e+05	4.1745677e+05	6.7939378e+05	2.8622851e+04	-2.2577011e+05	-1.8548001e+05
	6.6604861e+05	6.3696404e+05	8.7890445e+04	-1.3654600e+05	-1.7325758e+05
		1.7495901e+06	2.2418971e+05	-4.5269967e+05	-5.0137319e+05
			2.8930366e+05	-1.6297201e+04	-5.1279601e+04
				3.7323774e+05	2.5116848e+05
					3.5726698e+05

$$\boldsymbol{\sigma}_f^{\,T} = \begin{bmatrix} \sigma_{11} & \sigma_{22} & \sigma_{33} & \sigma_{12} & \sigma_{13} & \sigma_{23} \end{bmatrix}$$

Conclusion

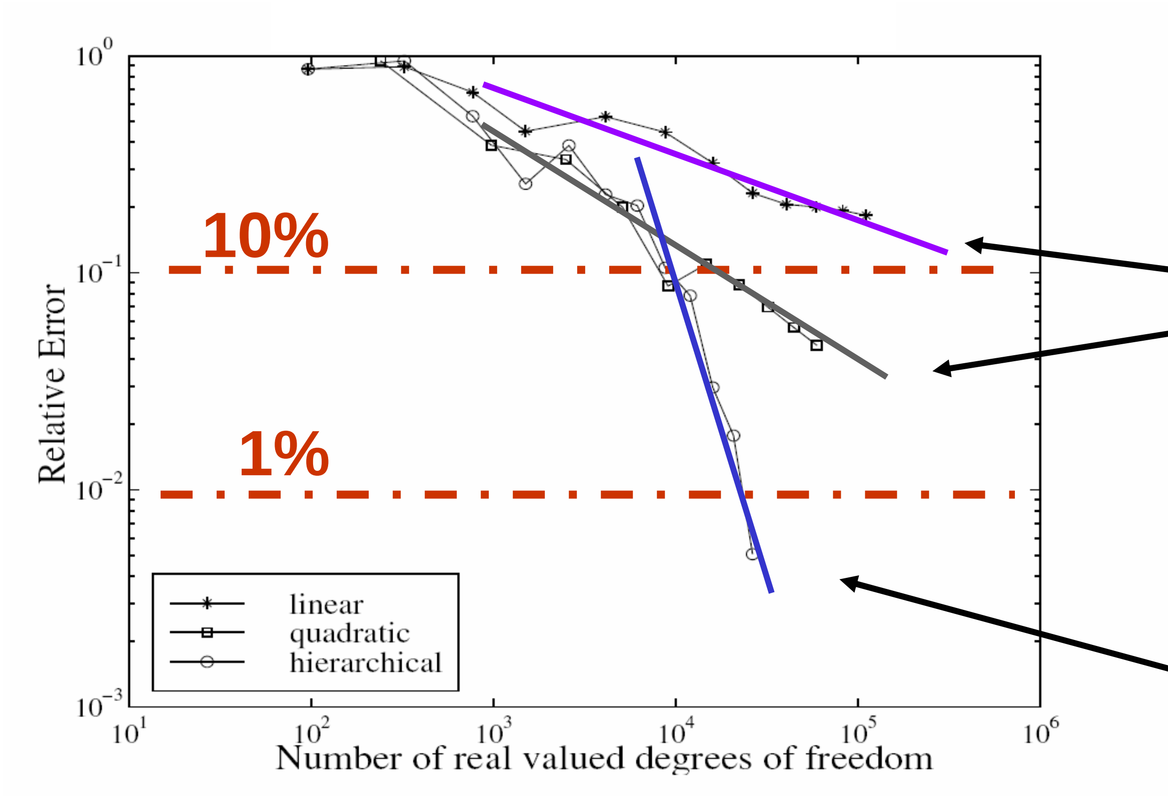
- Inverse estimation requires very high quality experimental data
- A methodology for precise characterization is under development
- Aiming at finding a static anisotropic elastic and fluid model of the foam
- Precise dynamic and acoustic model is under development

Presentation Outline

- **Numerical design of automotive multilayer trim for low-mid frequencies**
- **Higher order FE solutions to Biot's equations with convergent results**
- **Lightweight layered trim components with solid, porous, viscoelastic materials**
- **Convergence for single & multilayer**
- **Influence of boundary conditions**

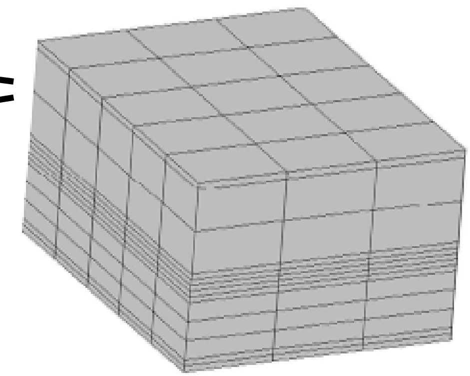
Numerical Simulation Summary

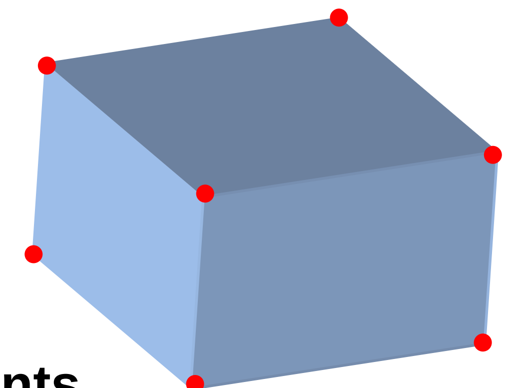
Convergence of 3D Porous FE Solutions (u-U)



Hörlin et al & Rigobert et al

Single Block of Foam @ 190Hz

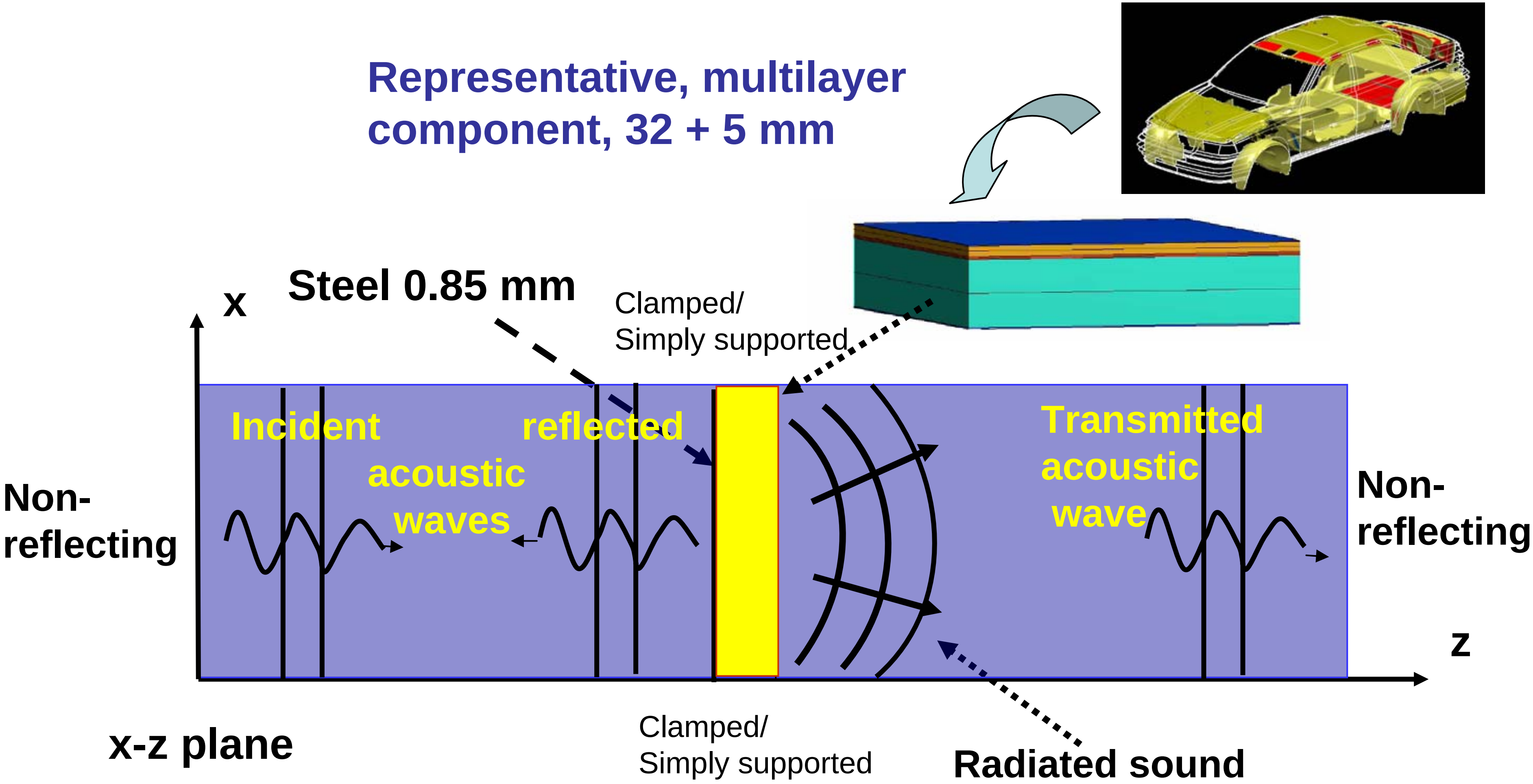




Insufficient accuracy for linear and quadratic elements

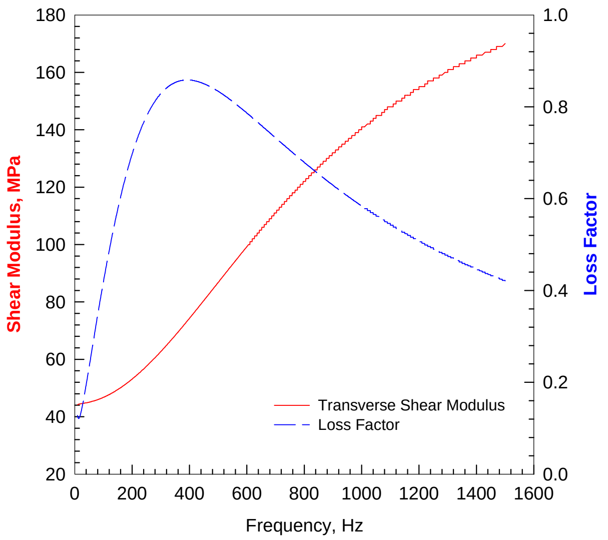
Remedy: Using higher order hierarchical elements

3D Numerical Design Problem

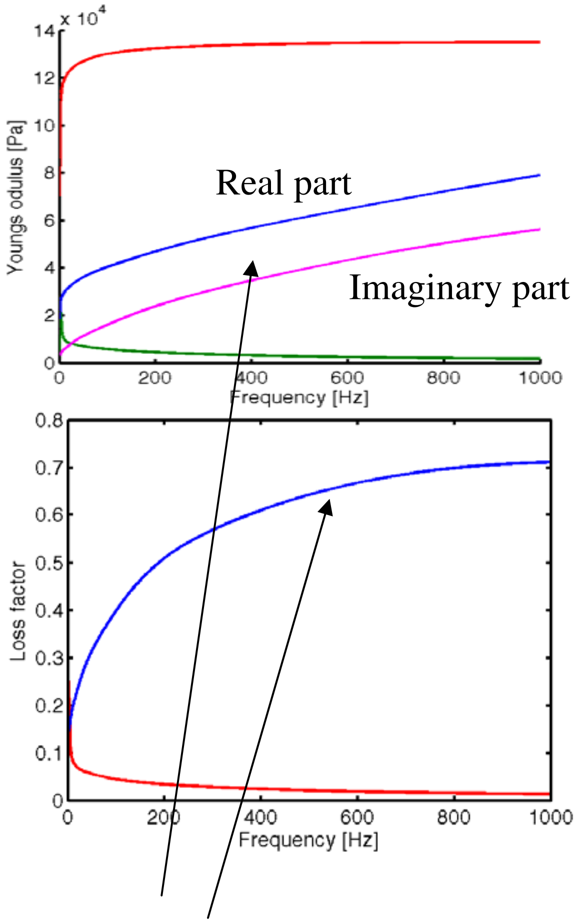


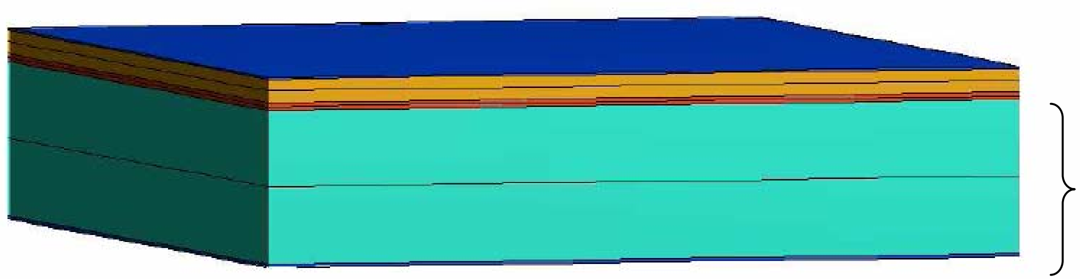
Cross dimensions
400 mm x 500 mm

Multilayer Trim Configuration



Visco-elastic barrier





Visco-elastic PU foam

Multilayer Trim Configuration, cont'd

Evaluate convergence for increasingly complex

- layering
- boundary conditions

- Excitation through an acoustic wave
- Meshing:
  - One element over the surface
  - Two elements over the foam thickness
  - One element over the steel and barrier thicknesses

L_2-norm

$$\sqrt{\int_{\Gamma} uu^{*} d\Gamma}$$

Sequence of convergence assessments @ 500 Hz

- Visco-elastic layer (rel. fast convergence)
- **Visco-elastic foam**
- **Visco-elastic foam + barrier on steel plate**

Fixed &
Simply supported

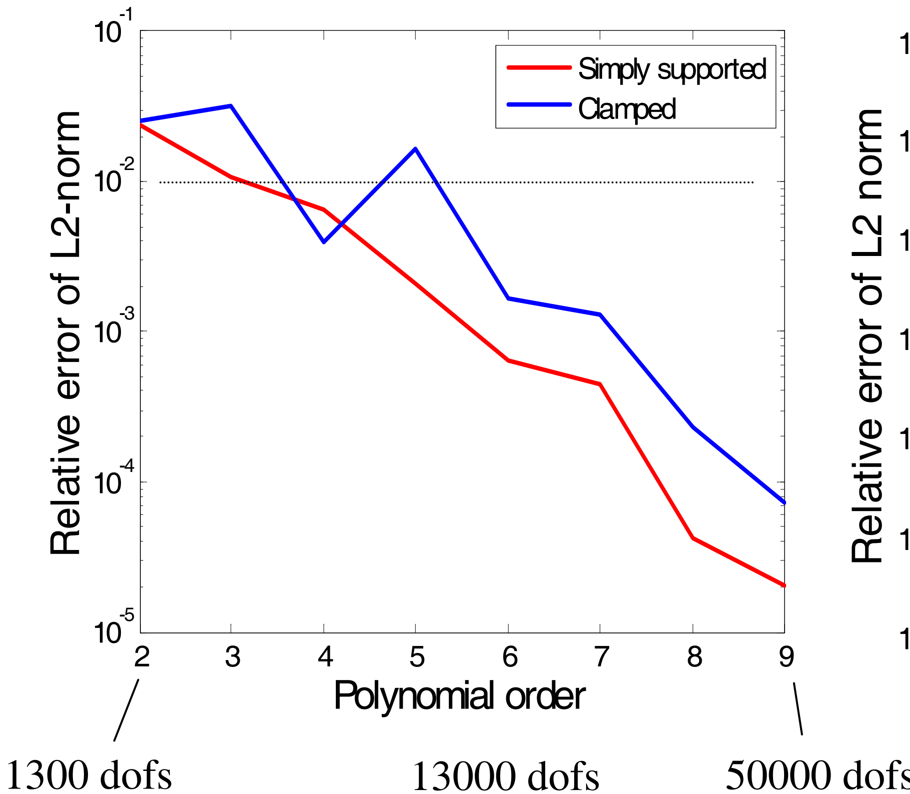
Convergence in L_2-norm (u-p)

L_2-norm

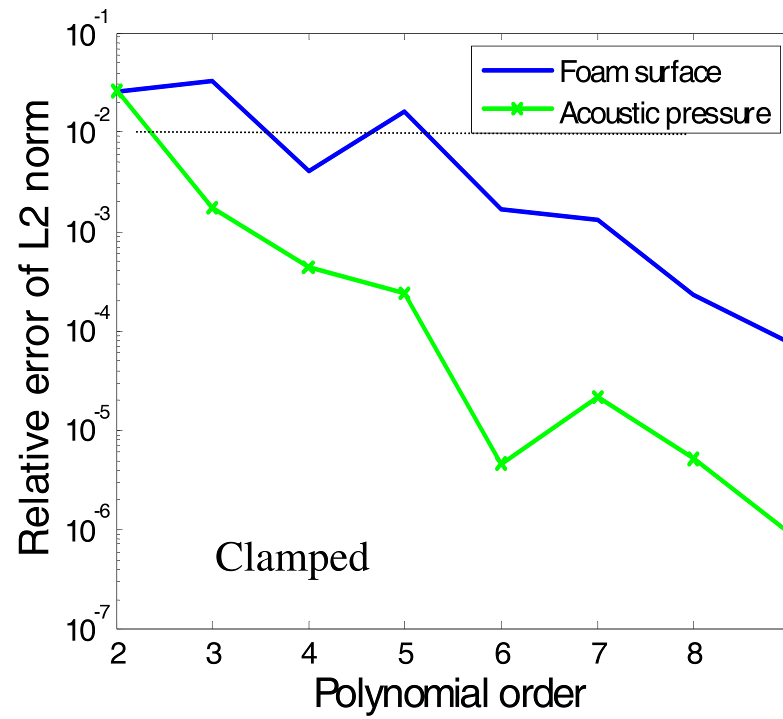
$$\sqrt{\int_{\Gamma} u u^{*} d\Gamma}$$

Visco-elastic foam

Influence of boundary conditions

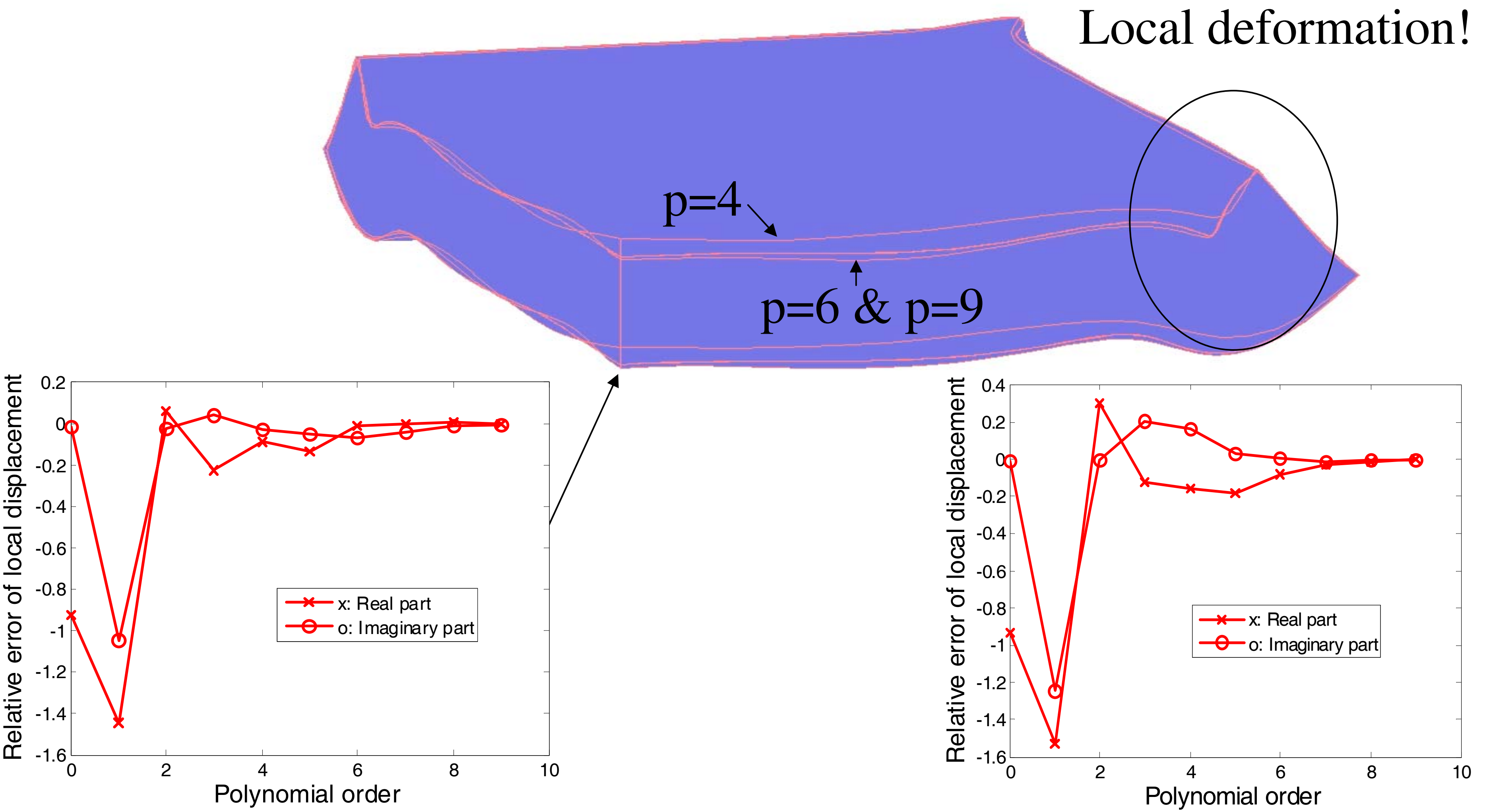


Deformation & pressure in far-field



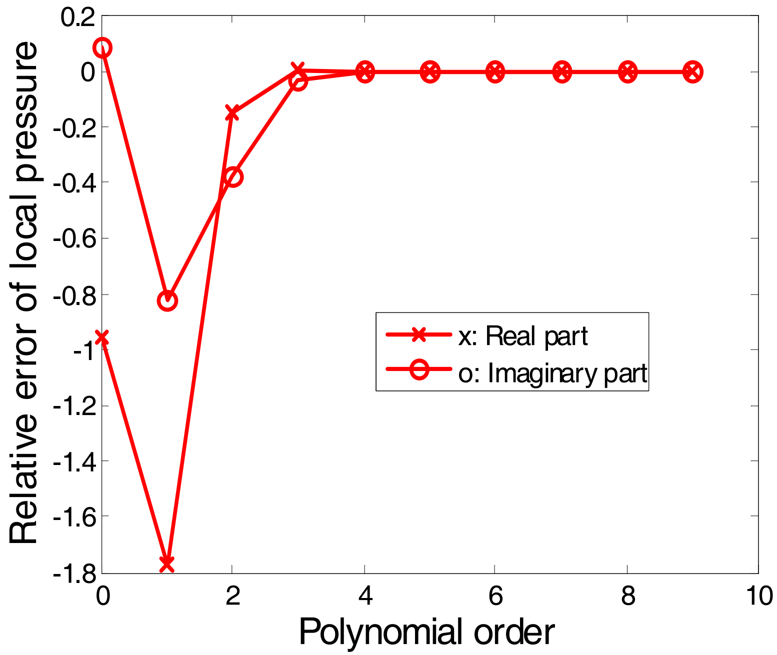
Visco-elastic foam, simply supported

Convergence in deformation (u-p)

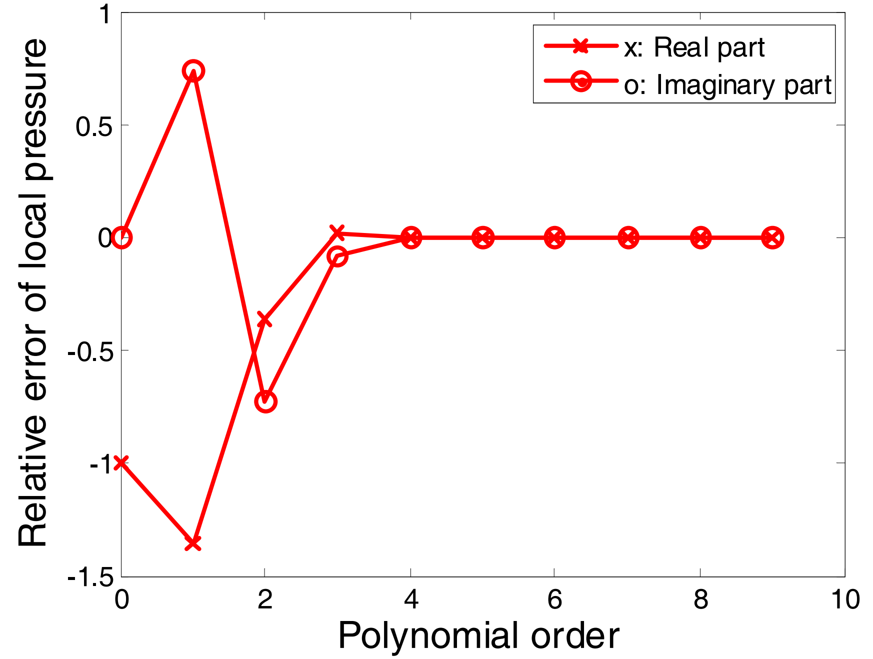


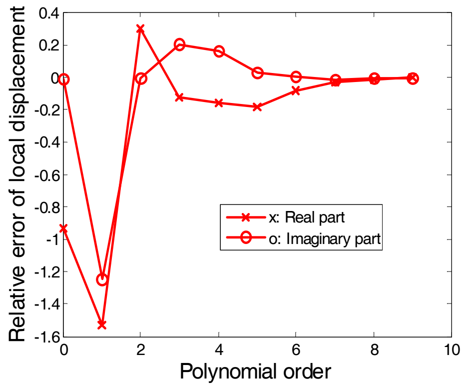
Convergence in pressure (u-p)

Pressure close to foam surface



Pressure in far-field

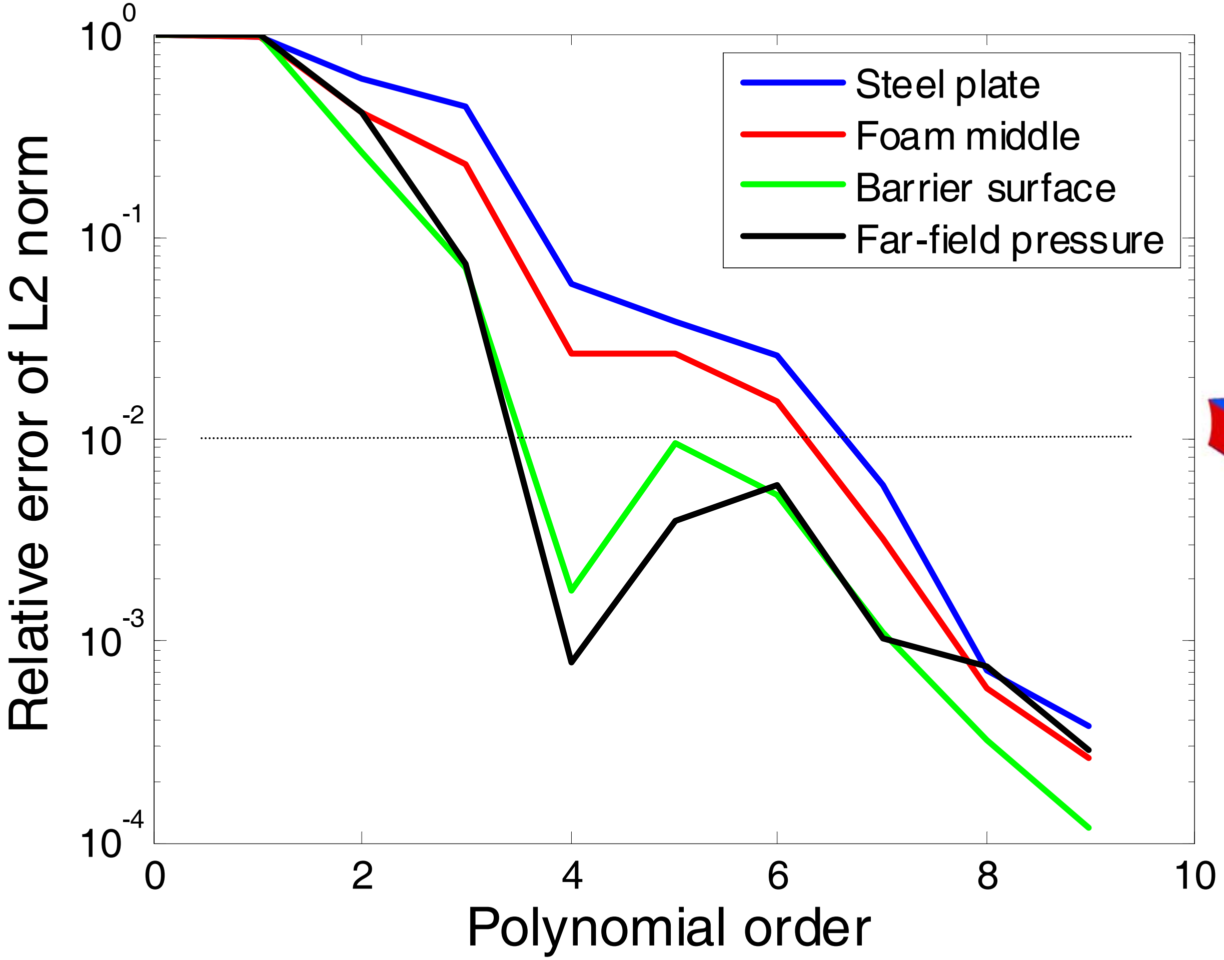




Visco-elastic foam, simply supported

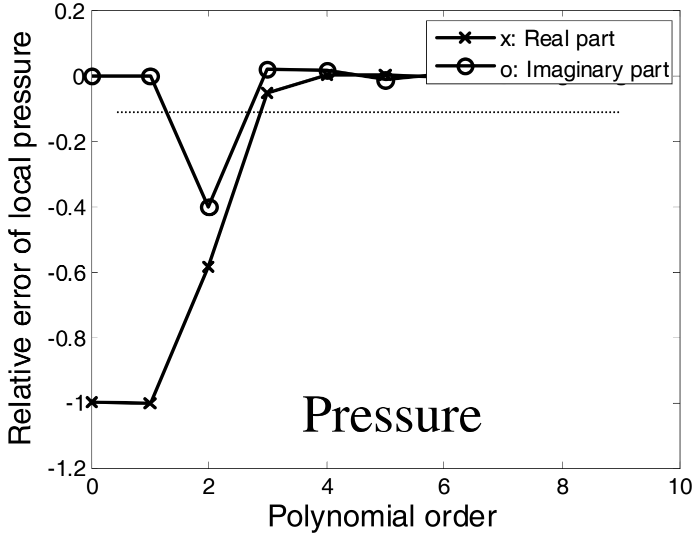
Convergence in L_2-norm (u-p)

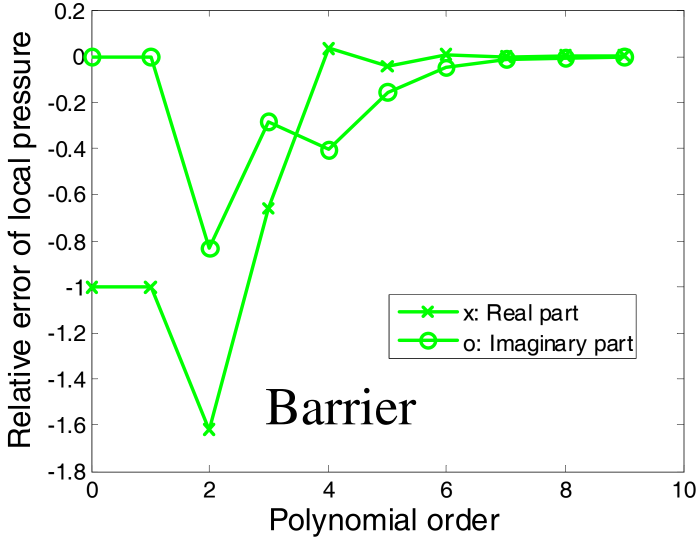
Visco-elastic foam + barrier on steel, clamped

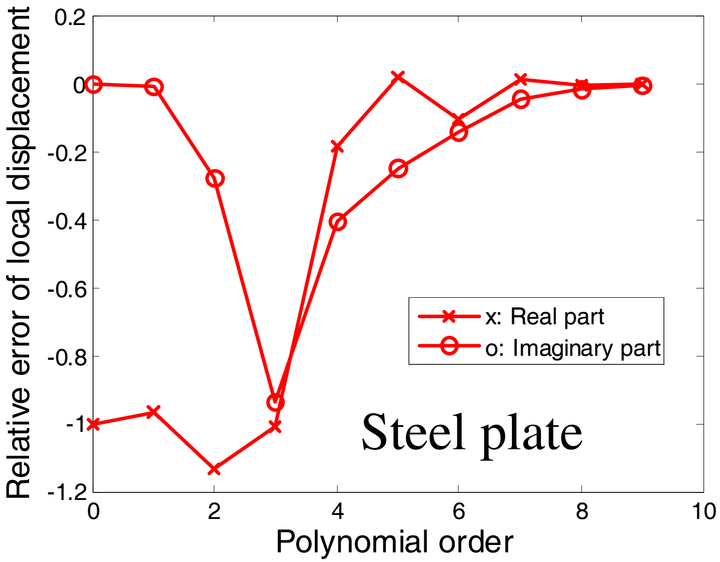


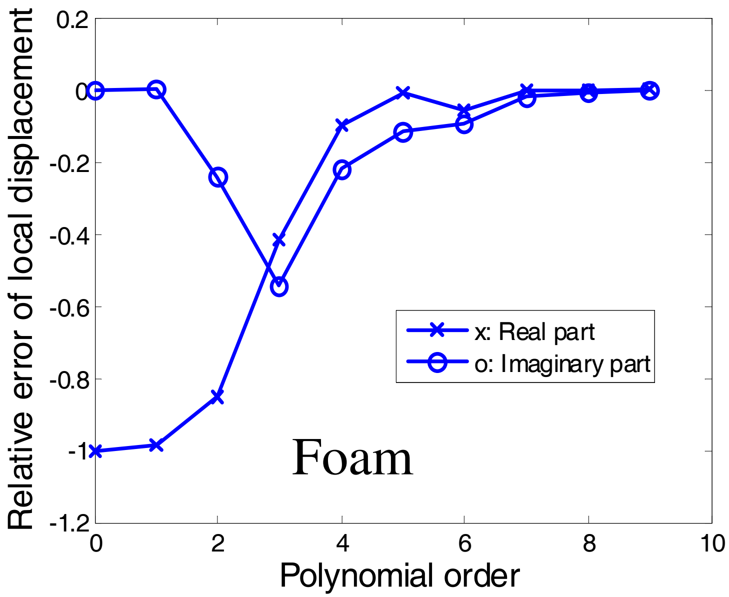
Steel+Visco-elastic foam + Barrier, clamped

Convergence in deformation (u-p)





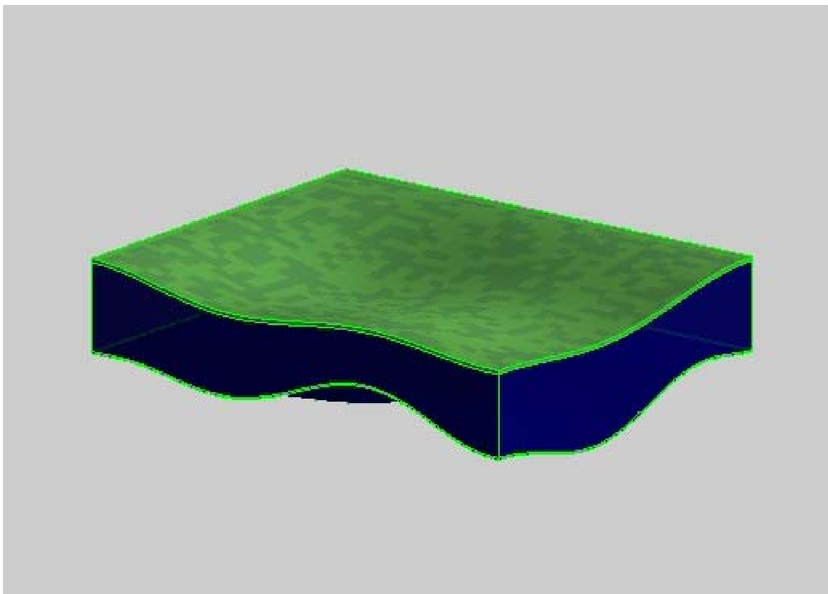


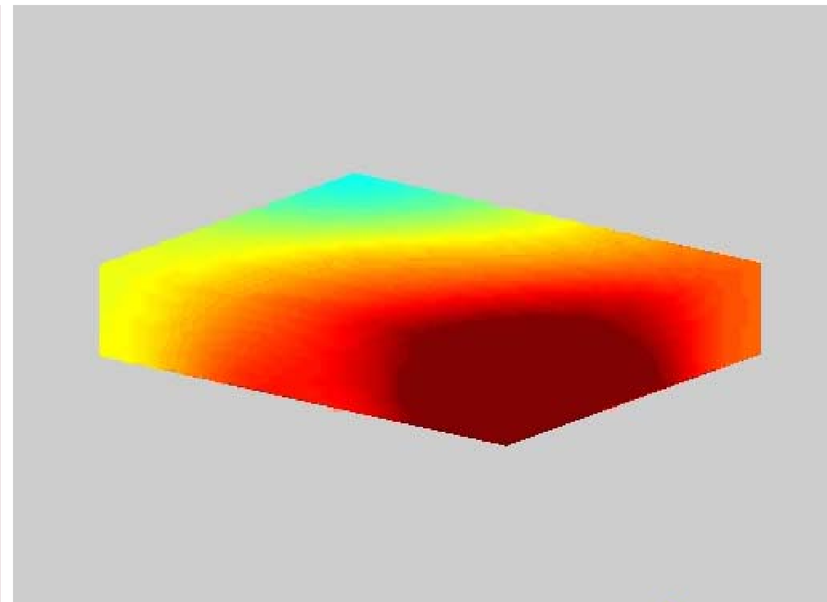


Steel+Visco-elastic foam + Barrier, clamped

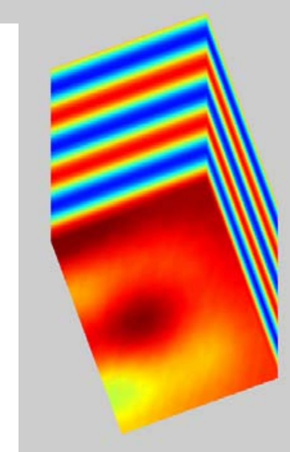
Deformations and Internal Pressures (u-p)

Deformation 500 Hz Pressure in foam





Transmitted pressure



Concluding Remarks

- Boundary conditions tends to slow down convergence for a single layer
- No substantial change in the condition number for multilayer configurations

- Pressure solution converges in general at lower p-levels (~2-3 lower)

- To reach an accuracy better than 1% of the surface average we need:
  - for a single foam layer p=6
  - for a multilayer sandwich p=7

  } for the foam

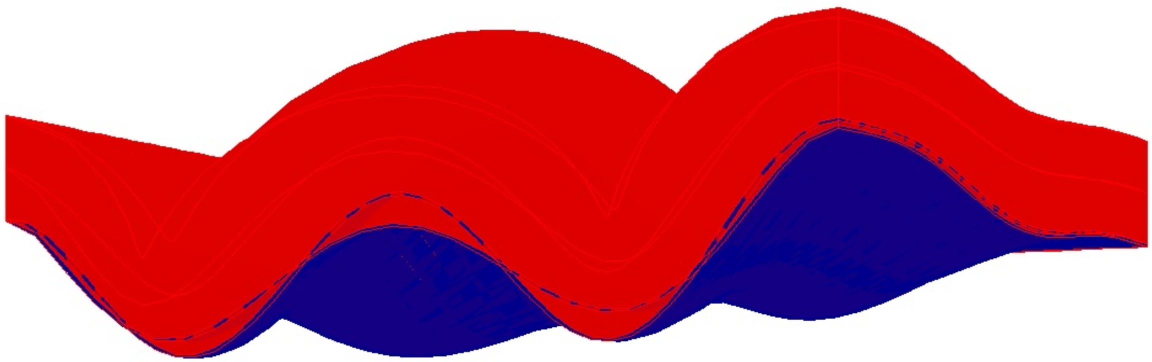
- To reach an accuracy better than 10% of point wise quantities we need:
  - for a single foam layer p=6-7
  - for a multilayer sandwich p=5-6

  } for the foam

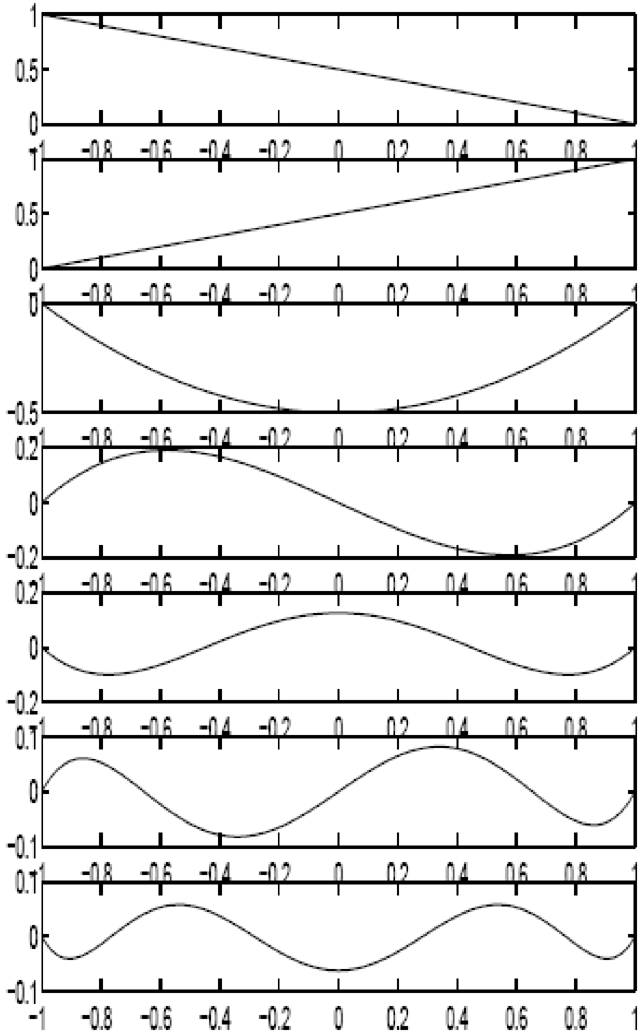
- N.B. @ 500 Hz

Convergence in deformation (u-p)

Visco-elastic foam on steel plate, clamped



Hierarchical basis functions



$$w(\xi_1, \xi_2, \xi_3) = \sum_{l_1=1}^{p_1+2} \sum_{l_2=1}^{p_2+2} \sum_{l_3=1}^{p_3+2} c_{\{l_1,l_2,l_3\}} \phi_{\{l_1,l_2,l_3\}}(\xi_1, \xi_2, \xi_3)$$

$$\xi_1, \xi_2, \xi_3 \in [-1, 1]$$

$$\phi_{\{l_1,l_2,l_3\}}(\xi_1, \xi_2, \xi_3) = f_{l_1}(\xi_1) f_{l_2}(\xi_2) f_{l_3}(\xi_3)$$

(2)

$$f_l(\chi) \stackrel{def}{=} \begin{cases} \frac{1}{2}(1-\chi) & l = 1 \\ \frac{1}{2}(1+\chi) & l = 2 \\ \sum_{s=0}^{\text{int}(l/2)} \frac{(-1)^s (2l-2s-5)!!}{2^s s!(l-2s-1)!} \chi^{l-2s-1} & l \geq 3. \end{cases}$$

Convergence in L_2-norm

Visco-elastic layer