A locally enriched space-time finite element method for fluid-structure interaction of thin flexible bodies

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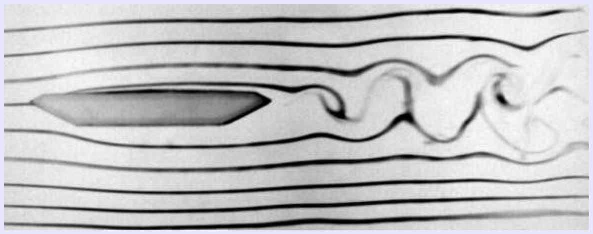
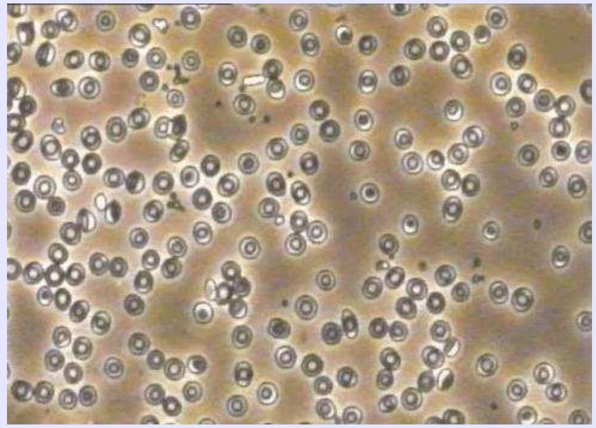
- motivation
- fluid & structure
- coupling conditions
- discretization
- space-time XFEM
- applications

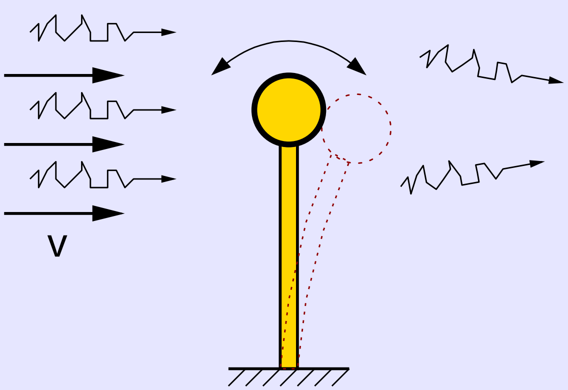
Fluid-structure interaction

- boundary-coupled systems

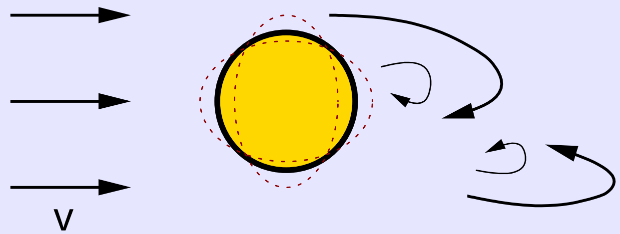




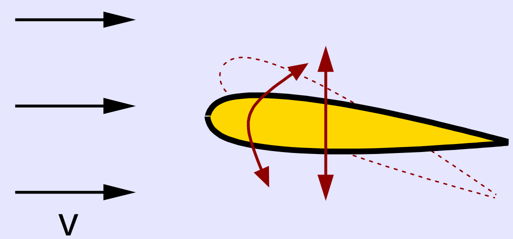
flow-induced



vortex-induced



self-excitation



Formulation of structure and fluid

structure

- nonlinear kinematics
- constitutive equations in rate form.

$$\begin{aligned} \nabla_0 \cdot (\mathbf{FS}) - \rho_0 \dot{\mathbf{v}} + \rho_0 \mathbf{b}_0 &= \mathbf{0} \\ \underline{\underline{\mathbf{C}}}^{-1} : \dot{\mathbf{S}} - \dot{\mathbf{E}} &= \mathbf{0} \end{aligned}$$

on reference configuration $Q_0 = \Omega_0 \times I$.

fluid

- incompressible
- viscous

$$\begin{aligned} \nabla \cdot \mathbf{T} - \rho(\mathbf{v}_{,t} + \mathbf{v} \cdot \nabla \mathbf{v}) + \rho \mathbf{b} &= \mathbf{0} \\ \nabla \cdot \mathbf{v} &= 0 \end{aligned}$$

on current configuration $Q = \Omega \times I$.

coupling conditions

$$\begin{aligned} \mathbf{v}^S - \mathbf{v}^F &= 0 \\ \mathbf{t}^S + \mathbf{t}^F &= 0 \end{aligned}$$

on interfacial current configuration $R = \Gamma \times I$.

Weighted residuals & discretization (FEM)

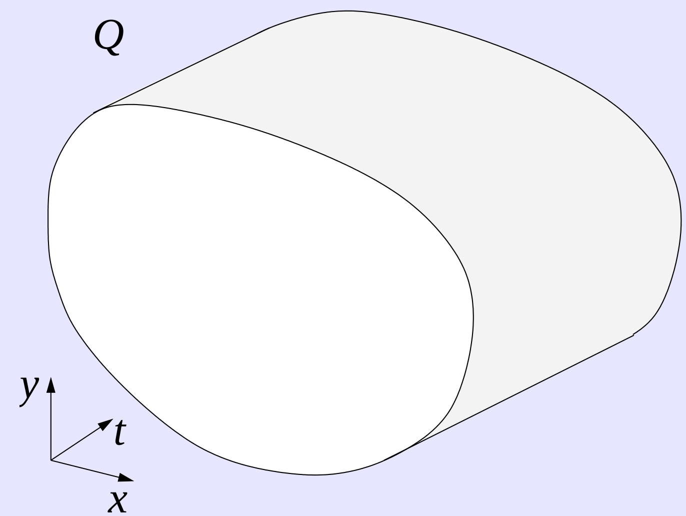
weak form

- approximation and method of weighted residuals
  - model equations in the domains,
  - initial and boundary conditions,
  - coupling conditions
- weak form of the equations describing the instationary boundary-coupled multifield system
- unknowns:

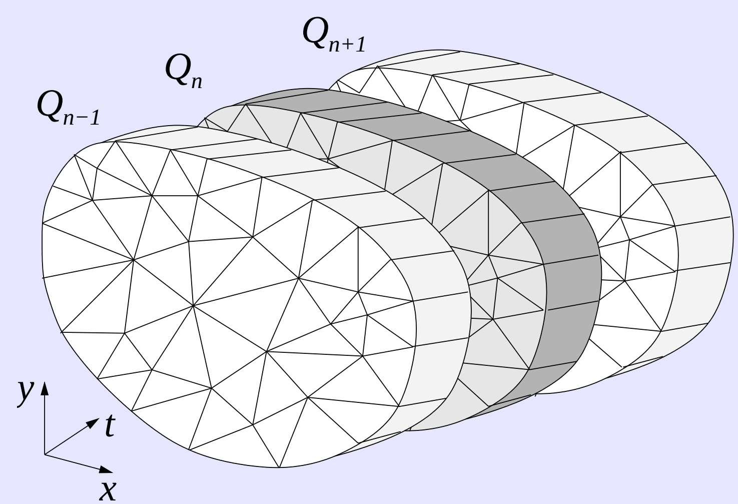
  primal variables,

  interface tractions
- GALERKIN-/least-squares stabilization
- time integration with discontinuous GALERKIN-approach

space-time domain

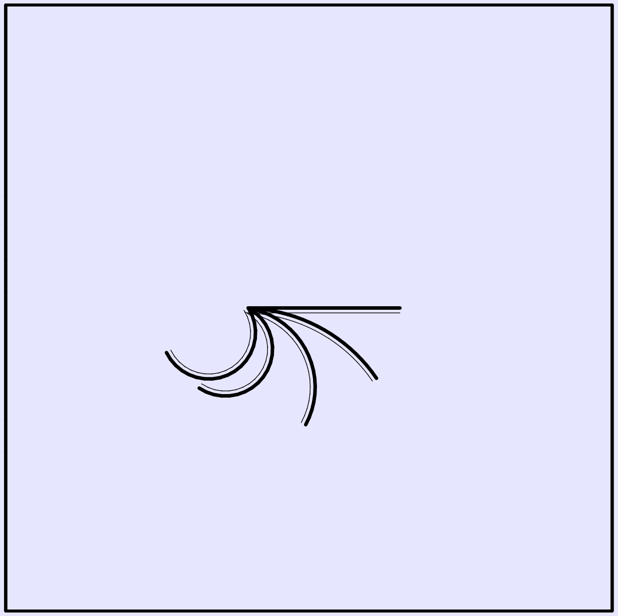


finite space-time elements

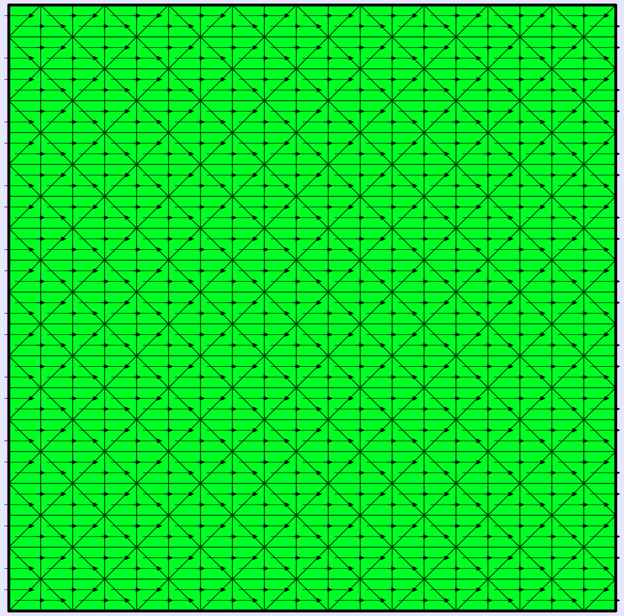


Combining structure and fluid

thin structure (beam model)



flow field

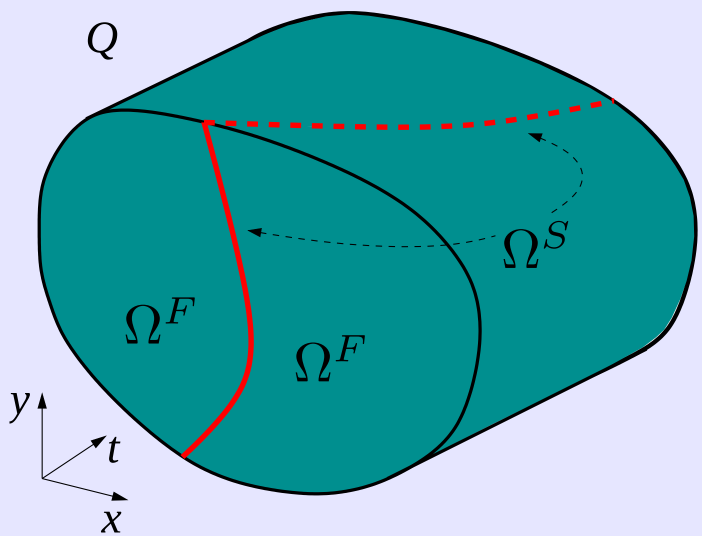


requirements

- huge structural motion and deformations, rotations
- limitations of fitting mesh approaches
- embedding the structure into the flow field
- continuous remeshing, chimera techniques (penalty methods)

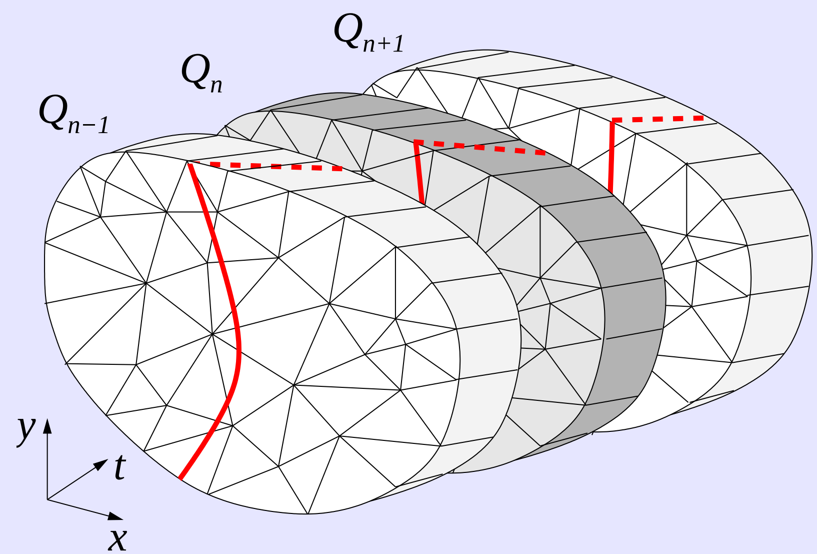
Discretization strategy & discontinuities

moving fluid-structure interface



- (infinite) thin body induces flow discontinuities (velocity & pressure)
- evolution of discontinuities in time

discrete interface capturing

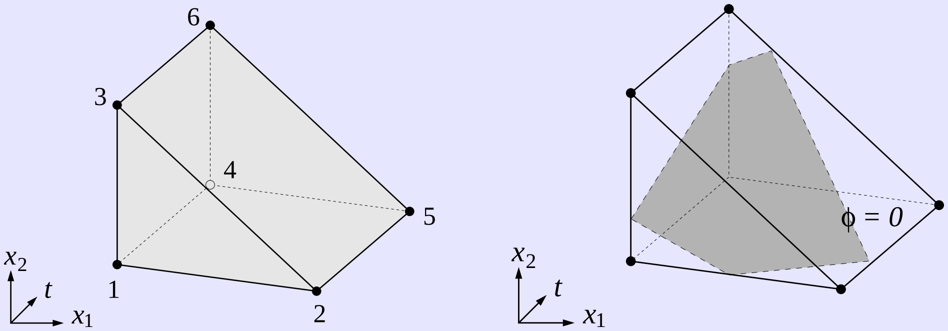


- C_1-discontinuities ?
- C_0-discontinuities ?
- realization of coupling ?

use a level set function ϕ to represent the infinite thin structure

Space-time finite element method

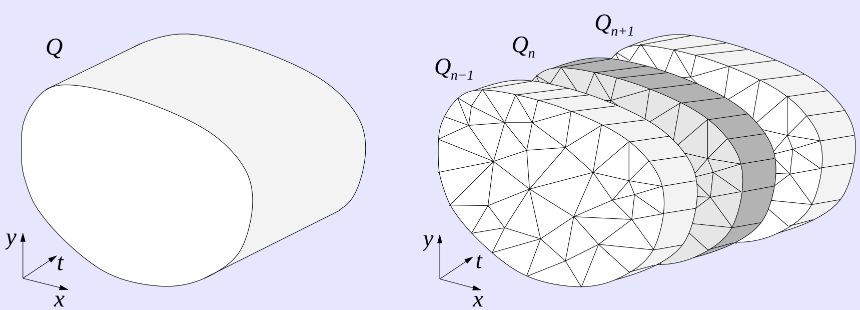
space time finite element



shape functions in x and t

$$\mathcal{V} = \bigcup_{k \in \mathcal{N}} N_k(\mathbf{x}, t)$$

discretization of the space-time domain



modification of approximation space

$$\mathcal{V}_{\text{ext}} = \mathcal{V} \quad \otimes \bigcup_{j \in \mathcal{N}^{\text{ext}}} N_j(\mathbf{x}, t)\psi(\mathbf{x}, t)$$

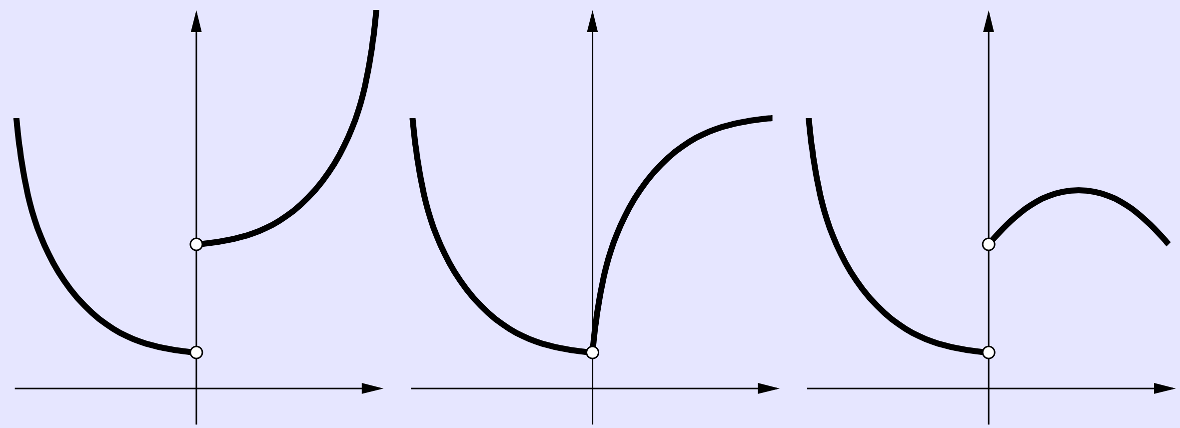
+ uniform method for space and time
+ time-dependent domain of integration
+ efficient use of additional dofs
+ improved convergence properties

– increased effort in numerical quadrature
– post-processing
– more complex algorithms

Discontinuous solutions

properties

- *a priori* - knowledge on the solution properties from modeling coupling conditions
- C_0, C_1, C_0/C_1 - discontinuities



extended ansatz functions in space and time – XFEM

$$u^h_{\text{ext}}(\mathbf{x},t) = \sum_{k\in\mathcal{N}^{\text{std}}} N_k(\mathbf{x},t)\ \hat{u}_k + \sum_{j\in\mathcal{N}^{\text{ext}}} N_j(\mathbf{x},t)\psi_j(\mathbf{x},t)\ \hat{u}^*_j \tag{1}$$

enriching function :

$$\psi_j(\mathbf{x},t) = \frac{1}{2}\left(1 - \text{sign}\ \phi(\mathbf{x},t)\cdot\text{sign}\ \phi(\mathbf{x}_j,t_j)\right) \tag{2}$$

advantages

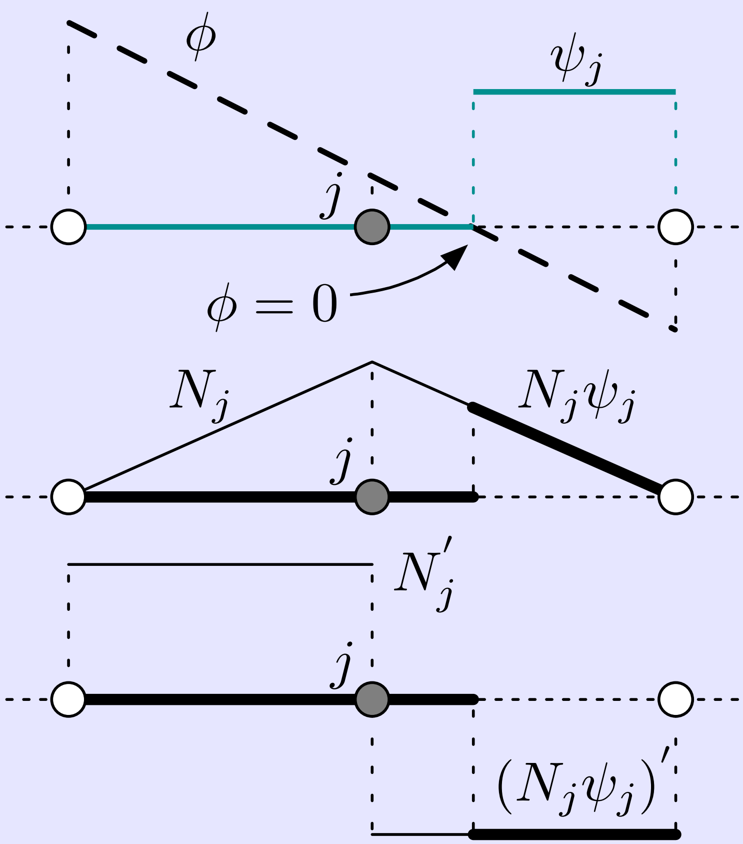
- better convergence
- approximation has δ-property

disadvantages

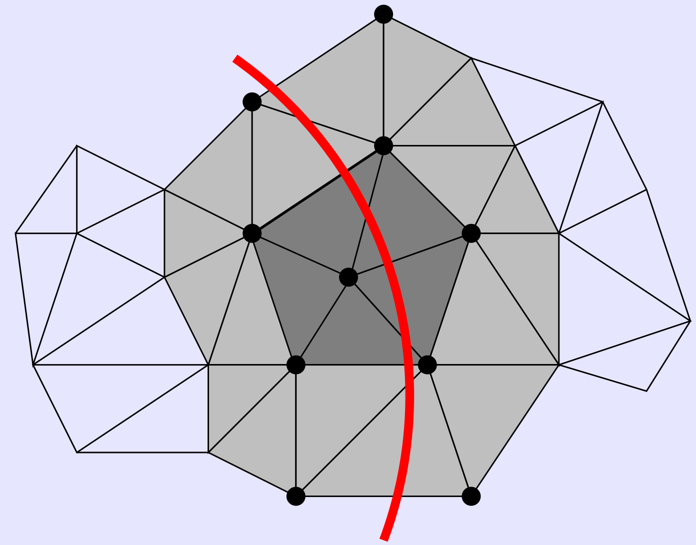
- more expensive numerical quadrature
- more complex algorithms

Discontinuous ansatz functions

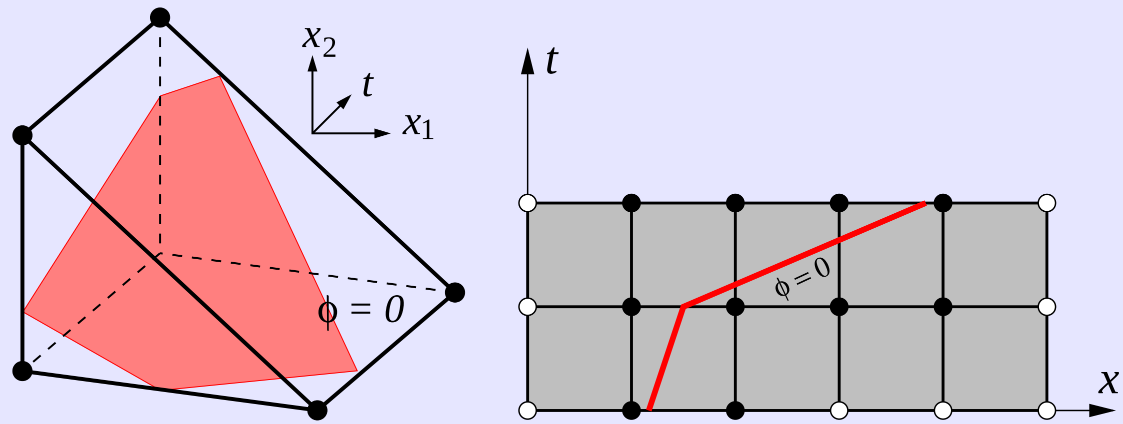
construction 1-D



local enrichment



discontinuities in space and time



coupling conditions / continuity

- LAGRANGE multipliers

Coupling conditions – Lagrange multiplier formulation

- enforce interfacial coupling conditions using *perturbed* Lagrange multiplier formulation

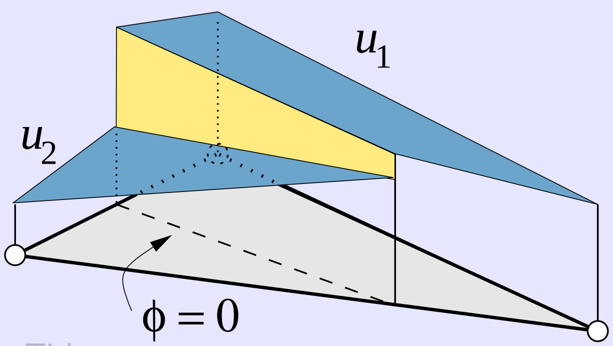
$$\delta(\phi(\mathbf{x},t))\ \tilde{\lambda} = \lambda_1(\mathbf{x},t) \qquad \forall\,(\mathbf{x},t) \in Q_1 \tag{3}$$

- use weak form of coupling conditions:

$$\int_{R_n} \delta\tilde{\lambda}^F \left(\mathbf{v}^F - \mathbf{v}^S\right)\ \mathrm{d}R - \int_{R_n} \delta\mathbf{v}^F \tilde{\lambda}^F \,\mathrm{d}R + \int_{R_n} \delta\mathbf{v}^S \tilde{\lambda}^F\ \mathrm{d}R \tag{4}$$

- add peturbation (stabilization) for $\tilde{\lambda}$ in elements e_R cut by the interface:

$$+\sum_{e_R} \int_{Q_n} (\delta\tilde{\lambda})\,\tau_C\,(\tilde{\lambda})\,\mathrm{d}Q \qquad \text{with e.g.} \quad \tau_C = 10^{-8} \tag{5}$$



Monolithic solution strategy

formulation strategy

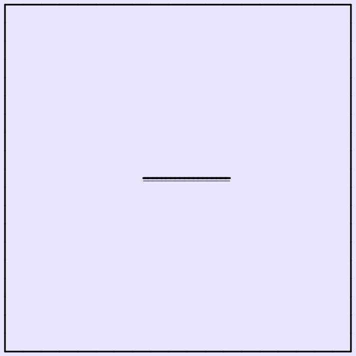
- use velocity-based formulation of fluid & structure
- use space-time finite element method
- apply local enrichment of fluid's space-time approximation
- use perturbed LAGRANGE multiplier approach for F-S-coupling

solution strategy

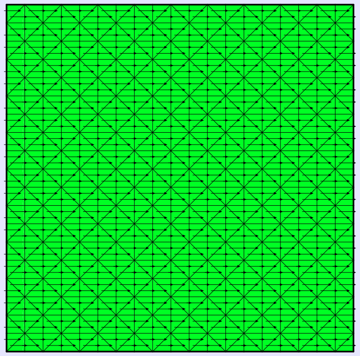
- results in single (monolithic) system
- describes coupled F-S-system for time slab at once
- use Picard iteration scheme to resolve all non-linearities
- use preconditioned BiCGStab solver

Example: Rotating flap in channel flow

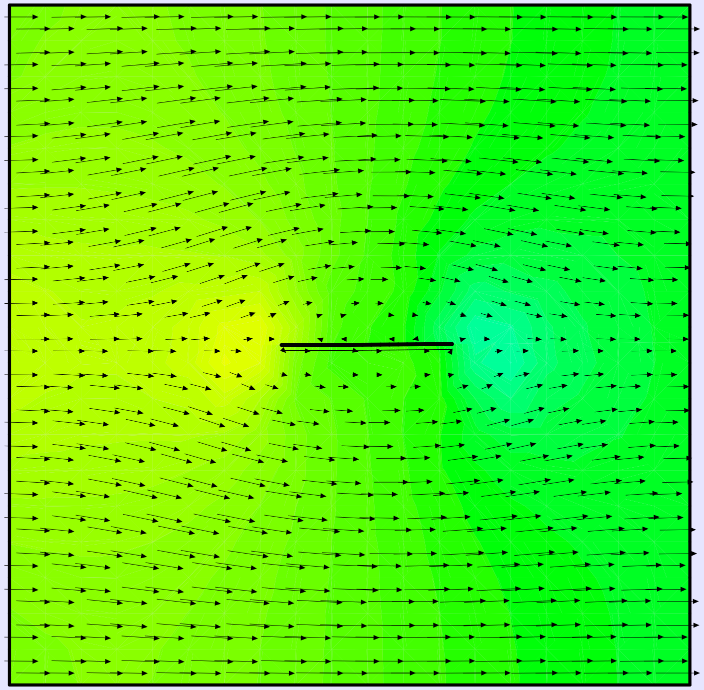
beam model



channel flow field



coupled system



Summary & Outlook

summary

- fully and strongly coupled solution procedure
- for FSI problems of thin flexible structures with large deformations

- space-time FEM for fluid and structure
- extension to *locally enriched* space-time FEM for fluid to embed structures
- whole coupled physical system described by 1 linearized algebraic system in each time step / iteration step

outlook

- special enrichment of beam tip (singularity)
- special enrichment for boundary layer close to the structure

- multiple structures
- industrial & biomechanical applications

References & Contact

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