A locally enriched space-time finite element method for fluid-structure interaction of thin flexible bodies

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- motivation
- fluid & structure
- coupling conditions
- discretization
- space-time XFEM
- applications
Fluid-structure interaction

- boundary-coupled systems
# Formulation of structure and fluid

## Structure

- **Nonlinear kinematics**
- **Constitutive equations in rate form.**

\[
\nabla_0 \cdot (FS) - \rho_0 \dot{v} + \rho_0 b_0 = 0
\]

\[
C^{-1} : \dot{S} - E = 0
\]

On reference configuration \(Q_0 = \Omega_0 \times I\).

## Fluid

- **Incompressible**
- **Viscous**

\[
\nabla \cdot T - \rho (v_t + v \cdot \nabla v) + \rho b = 0
\]

\[
\nabla \cdot v = 0
\]

On current configuration \(Q = \Omega \times I\).

## Coupling Conditions

\[
\mathbf{v}^S - \mathbf{v}^F = 0
\]

\[
\mathbf{t}^S + \mathbf{t}^F = 0
\]

On interfacial current configuration \(R = \Gamma \times I\).
Weighted residuals & discretization (FEM)

- approximation and method of weighted residuals
  - model equations in the domains,
  - initial and boundary conditions,
  - coupling conditions
- weak form of the equations describing the instationary boundary-coupled multifield system
- unknowns:
  - primal variables,
  - interface tractions
- Galerkin-/least-squares stabilization
- time integration with discontinuous Galerkin-approach
Combining structure and fluid

**thin structure (beam model)**

**flow field**

**requirements**

- huge structural motion and deformations, rotations
- limitations of fitting mesh approaches
- embedding the structure into the flow field
- continuous remeshing, chimera techniques (penalty methods)
Discretization strategy & discontinuities

- (infinite) thin body induces flow discontinuities (velocity & pressure)
- evolution of discontinuities in time

use a level set function \( \phi \) to represent the infinite thin structure

- \( C_1 \)-discontinuities ?
- \( C_0 \)-discontinuities ?
- realization of coupling ?

moving fluid-structure interface

discrete interface capturing
Space-time finite element method

- uniform method for space and time
- time-dependent domain of integration
- efficient use of additional dofs
- improved convergence properties
- increased effort in numerical quadrature
- post-processing
- more complex algorithms

shape functions in \( x \) and \( t \)

\[ \mathcal{V} = \bigcup_{k \in \mathcal{N}} N_k(x, t) \]

modification of approximation space

\[ \mathcal{V}_{\text{ext}} = \mathcal{V} \otimes \bigcup_{j \in \mathcal{N}_{\text{ext}}} N_j(x, t) \psi(x, t) \]

discretization of the space-time domain
Discontinuous solutions

properties

- a priori - knowledge on the solution properties from modeling coupling conditions
- $C_0, C_1, C_0/C_1$ - discontinuities

extended ansatz functions in space and time – XFEM

$$u^h_{\text{ext}}(\mathbf{x}, t) = \sum_{k \in \mathcal{N}^{\text{std}}} N_k(\mathbf{x}, t) \hat{u}_k + \sum_{j \in \mathcal{N}^{\text{ext}}} N_j(\mathbf{x}, t) \psi_j(\mathbf{x}, t) \hat{u}^*_j$$ (1)

enriching function:

$$\psi_j(\mathbf{x}, t) = \frac{1}{2} (1 - \text{sign } \phi(\mathbf{x}, t) \cdot \text{sign } \phi(\mathbf{x}_j, t_j))$$ (2)

advantages

- better convergence
- approximation has $\delta$-property

disadvantages

- more expensive numerical quadrature
- more complex algorithms
Discontinuous ansatz functions

construction 1-D

\[ \phi = 0 \]

\[ N_j \]

\[ N'_j \]

\[ (N'_j \psi_j)' \]

coupling conditions / continuity

- LAGRANGE multipliers

local enrichment

discontinuities in space and time
Coupling conditions – Lagrange multiplier formulation

- enforce interfacial coupling conditions using *perturbed* Lagrange multiplier formulation

\[ \delta(\phi(x, t)) \tilde{\lambda} = \lambda_1(x, t) \quad \forall (x, t) \in Q_1 \]  

(3)

- use weak form of coupling conditions:

\[ \int_{R_n} \delta \tilde{\lambda}^F (v^F - v^S) \, dR - \int_{R_n} \delta v^F \tilde{\lambda}^F \, dR + \int_{R_n} \delta v^S \tilde{\lambda}^F \, dR \]  

(4)

- add perturbation (stabilization) for \( \tilde{\lambda} \) in elements \( e_R \) cut by the interface:

\[ + \sum_{e_R} \int_{Q_n} (\delta \tilde{\lambda}) \tau_C (\tilde{\lambda}) \, dQ \quad \text{with e.g. } \tau_C = 10^{-8} \]  

(5)
Monolithic solution strategy

- use velocity-based formulation of fluid & structure
- use space-time finite element method
- apply local enrichment of fluid’s space-time approximation
- use perturbed \text{LAGRANGE} multiplier approach for F-S-coupling

solution strategy

- results in single (monolithic) system
- describes coupled F-S-system for time slab at once
- use Picard iteration scheme to resolve all non-linearities
- use preconditioned BiCGStab solver
Example: Rotating flap in channel flow

beam model

coupled system

channel flow field
Summary & Outlook

**summary**

- fully and strongly coupled solution procedure
- for FSI problems of thin flexible structures with large deformations
- space-time FEM for fluid and structure
- extension to *locally enriched* space-time FEM for fluid to embed structures
- whole coupled physical system described by 1 linearized algebraic system in each time step / iteration step

**outlook**

- special enrichment of beam tip (singularity)
- special enrichment for boundary layer close to the structure
- multiple structures
- industrial & biomechanical applications
References

- Kölke, A.: Modellierung und Diskretisierung bewegter Diskontinuitäten in randgekoppelten Mehrfeldaufgaben, Technische Universität Braunschweig, 2005, PhD thesis

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