Advanced (composite) Beam Theory and 1D-3D-computations

Central solution and end effects

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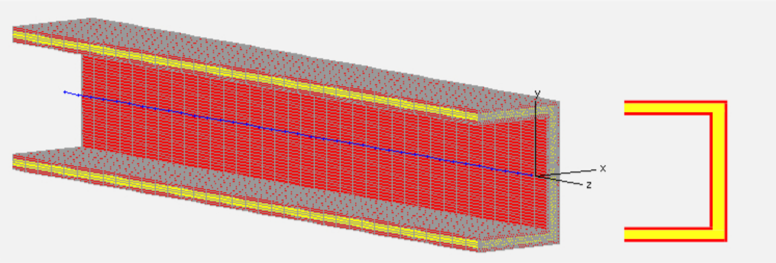
LMSSC, CNAM, Paris, 2013


y
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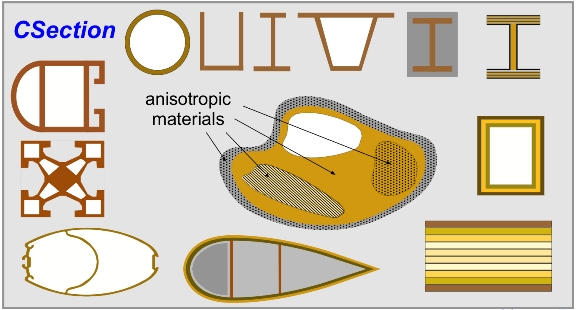
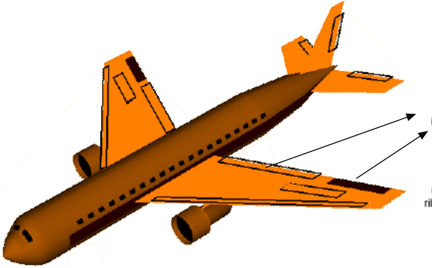
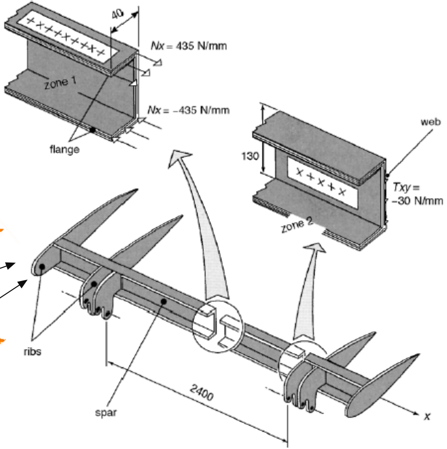

CSection
anisotropic
materials

Aerocraft industry

Longeron et nervures d'une gouverne d'avion



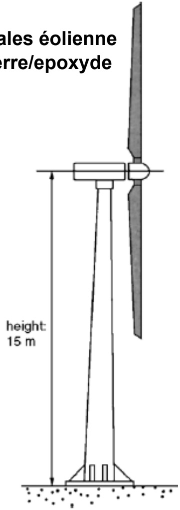
http://home.nordnet.fr/dmorieux/gouvernes0003.htm



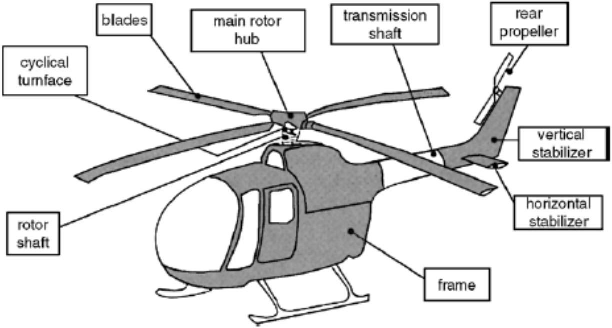
From: *Composite Material*. D. Gay et al. CRC Press, 20

Blades

pales éolienne verre/epoxyde



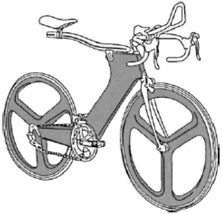
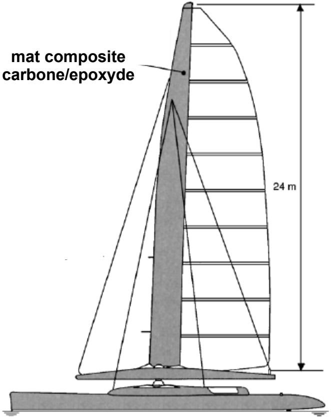
pales hélicopter



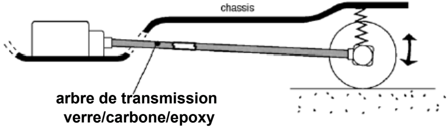
From: *Composite Material*. D. Gay et al. CRC Press, 20

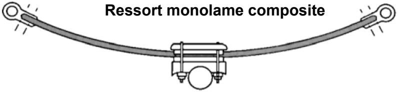
Sport equipment / competition

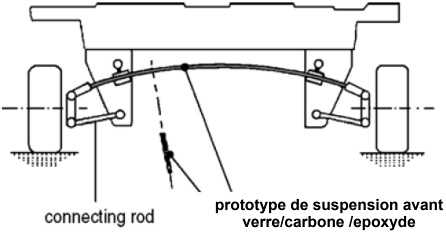


Mechanical industry, automotive industry

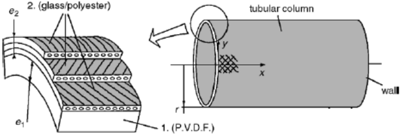




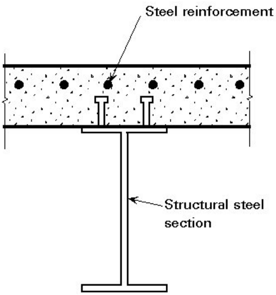


Tuyaux composites

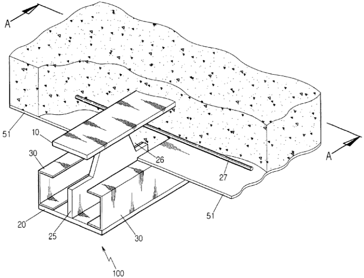
verre/epoxyde

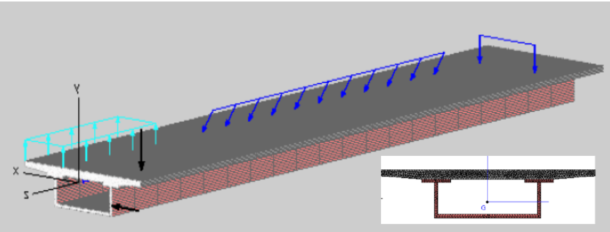


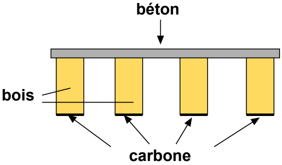
Civil Engineering



sections mixtes
acier béton







Pultrusion → composite profiles

(Elastic and linearized solution)

Objective

- Good description of the structural 1D behavior
- **3D stress** field in each **material** ,
  … in the major **interior** part of the beam
- Common beam slenderness, … and even relatively small

Two difficulties

- **Couplings** extension-bending-twist
  composite anisotropic section
- **Edge effects** / Boundary conditions

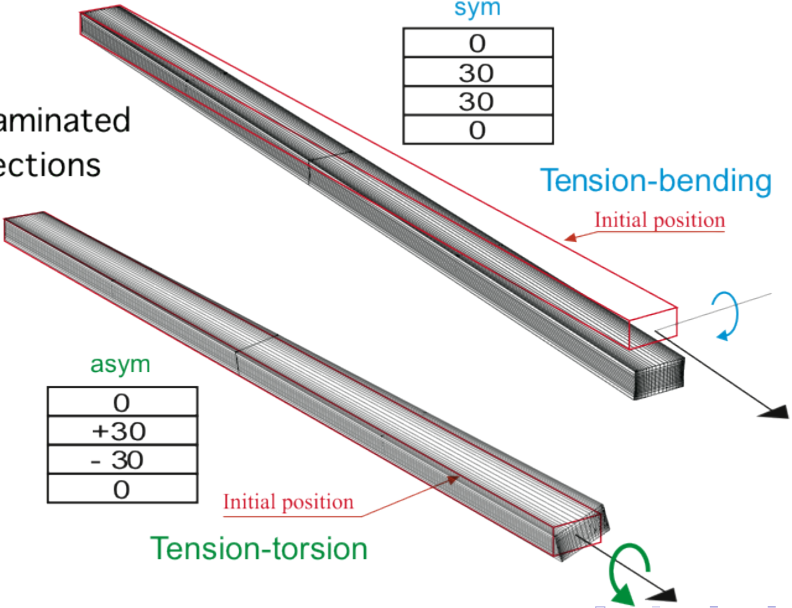
Elasting couplings

Laminated sections

sym

0
30
30
0

Tension-bending

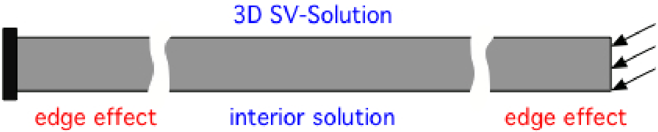


asym

0
+30
- 30
0

Tension-torsion

St Venant Principle, St Venant Solution and edge effects



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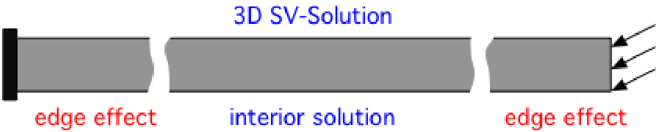
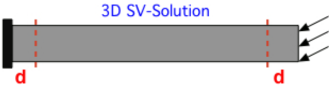
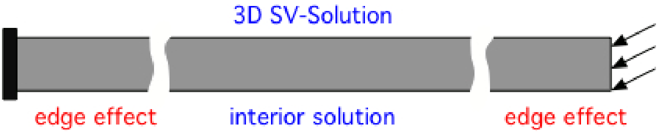


SV-Pr: $\approx$ the end effects are confined close to the ends!





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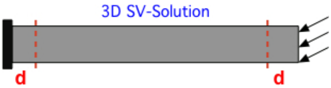

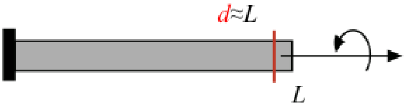

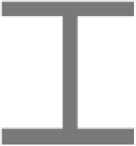

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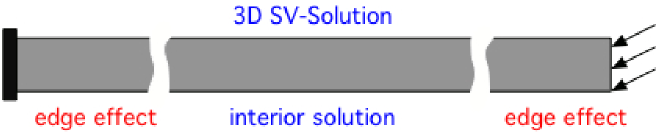

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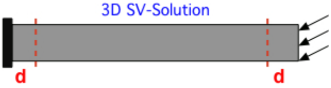

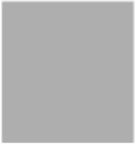

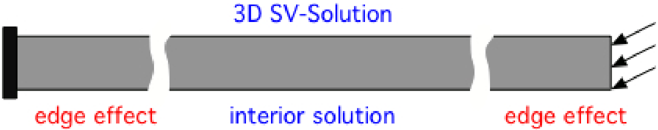

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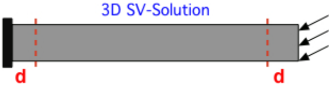

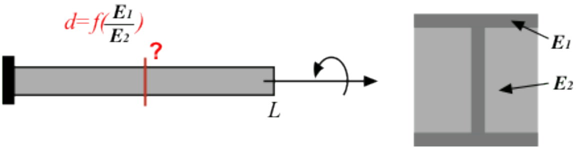

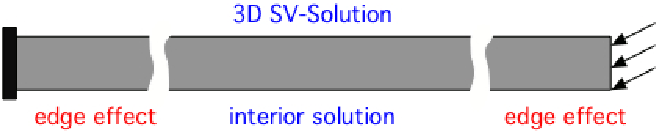

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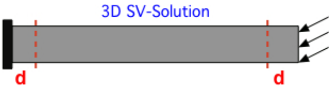
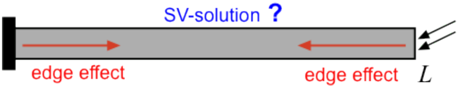
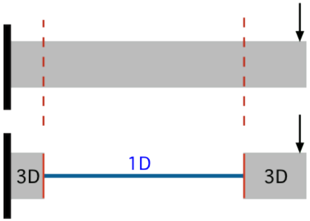


- Composite section=[shape **and** materials] not separable
- The edge effects are **not always confined** close to the **ends** and can dominate the behavior of the beam



Computation Strategy

3D-1D-partition
3D-1D-FEM computations

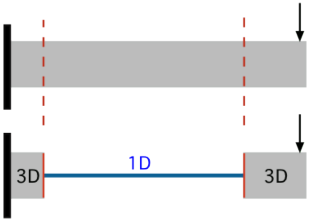


- the 3D/1D-connections: where and how ?
- which beam theory will support the 1D area ?

Computation Strategy

3D-1D-partition

3D-1D-FEM computations



- the 3D/1D-connections: where and how ?
- which beam theory will support the 1D area ?

A | General (composite) beam theory |

able to **catch a significant part** of the edge effects

$\rightarrow$ size(1D-area) $\gg$ size(3D-area) !!

- **The beam theory**

1. Mechanical characteristics of the section: **MCS**
2. → Beam theory
   - Displacement model
   - Equations & Numerical developments
   - 1D-3D connections

- **Applications**: Significant examples

- **The beam theory**

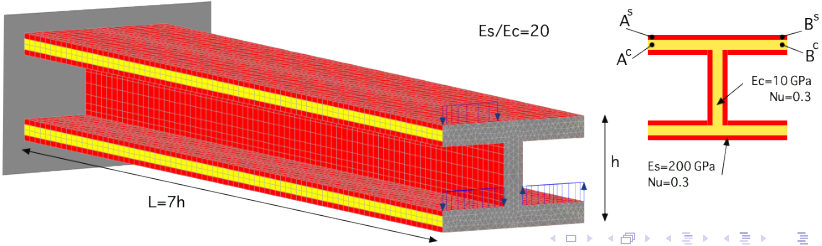
1 Mechanical characteristics of the section: **MCS**

2 → Beam theory

- Displacement model
- Equations & Numerical developments
- 1D-3D connections

- **Applications**: Significant examples

Bending-torsion of a **Short** cantilever **open composite** profile



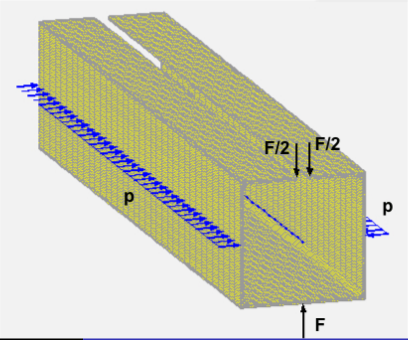
- **The beam theory**

1 Mechanical characteristics of the section: **MCS**

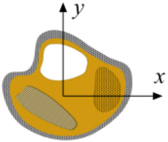
2 → Beam theory
   - Displacement model
   - Equations & Numerical developments
   - 1D-3D connections

- **Applications**: Significant examples

Beam under a loading for which the **resultants are zero**.



1 **MCS** $\rightarrow$ deformability modes of the section $\boldsymbol{\mathcal{M}}^k(x,y)$



2 **Displacement model**

$$\boldsymbol{\xi}(\boldsymbol{u},\boldsymbol{\omega},\boldsymbol{\beta}) \;=\; \overbrace{\boldsymbol{u}(z)+\boldsymbol{\omega}(z)\wedge \boldsymbol{GM}}^{\text{rigid motion of the section}} \;+\overbrace{\beta_k(z)\boldsymbol{\mathcal{M}}^k(x,y)}^{\text{enrichment}}$$

.... the correspondent **Beam Theory**

Beam Theory: two steps

1 **MCS** $\rightarrow$ deformability modes of the section $\boldsymbol{\mathcal{M}}^k(x,y)$

ϕ_i:	Warpings	out-of-plane
$\mathbf{\Pi}_i$:	Poisson's effects	in-plane
$\boldsymbol{D}_j$:	Distorsions	in-plane

2 **Displacement model**

.... the correspondent **Beam Theory**

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ϕ_i:	Warpings	out-of-plane
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$\Leftarrow$ 3D-SV-Solution

2 **Displacement model**

.... the correspondent **Beam Theory**

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ϕ_i:	Warpings	out-of-plane
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2 **Displacement model** built with these **SV-modes**

$$\boldsymbol{M}^k = [\boldsymbol{\Pi}^i + \boldsymbol{D}^j]_{in} \text{ and } [\phi^i]_{out}$$

$$\boldsymbol{\xi}(\boldsymbol{u},\boldsymbol{\omega},\boldsymbol{\eta},\boldsymbol{\alpha}) = \overbrace{\boldsymbol{u}(z)+\boldsymbol{\omega}(z)\wedge \boldsymbol{GM}}^{\text{rigid motion of the section}}$$

$$+\overbrace{\eta_i(z)\phi^i(x,y)+\alpha_k(z)\boldsymbol{M}^k(x,y)}^{\text{enrichment}}$$

.... the correspondent **Beam Theory**

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⇐ 3D-SV-Solution

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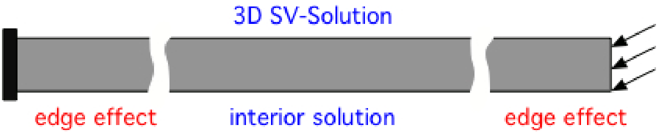
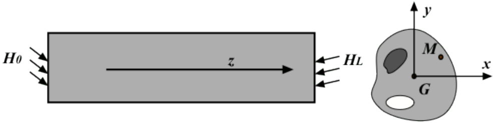
.... the correspondent **Beam Theory**



3D SV-solution: reference solution for beam

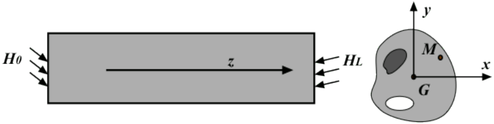
- $\approx$ the central solution (when $L \gg d$)
- contains **a lot of information** on the mechanical behavior of the section
- homg. isotropic $\rightarrow$ composite anisotropic (Iesan, 76)

SV-Problem (extended to composite section Eisan(1976))



$$
\left.\begin{array}{rcll}
\text{div}\, \boldsymbol{\sigma} & = & \mathbf{0} & \text{in} \quad V \\
\boldsymbol{\varepsilon}(\boldsymbol{\xi}) & = & \frac{1}{2}(\nabla^t \boldsymbol{\xi} + \nabla \boldsymbol{\xi}) & \text{in} \quad V \\
\boldsymbol{\sigma} & = & \boldsymbol{K} : \boldsymbol{\varepsilon}(\boldsymbol{\xi}) & \text{in} \quad V \\
\boldsymbol{\sigma}(\boldsymbol{n}) & = & \mathbf{0} & \text{on} \quad S_{lat}
\end{array}\right\} \quad (1)
$$

$$
\left.\begin{array}{rcll}
\boldsymbol{\sigma}(-\boldsymbol{z}) & = & \boldsymbol{H}_0 & \text{on} \quad S_0 \\
\boldsymbol{\sigma}(\boldsymbol{z}) & = & \boldsymbol{H}_L & \text{on} \quad S_L
\end{array}\right\} \quad (2)
$$



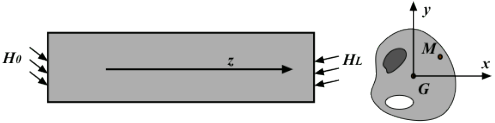
$$\left.\begin{array}{rcll} \text{div}\ \boldsymbol{\sigma} & = & \mathbf{0} & \text{in} \quad V \\ \boldsymbol{\varepsilon}(\boldsymbol{\xi}) & = & \frac{1}{2}(\nabla^t\boldsymbol{\xi} + \nabla\boldsymbol{\xi}) & \text{in} \quad V \\ \boldsymbol{\sigma} & = & \boldsymbol{K} : \boldsymbol{\varepsilon}(\boldsymbol{\xi}) & \text{in} \quad V \\ \boldsymbol{\sigma}(\boldsymbol{n}) & = & \mathbf{0} & \text{on} \quad S_{lat} \end{array}\right\} \qquad (1)$$

$$\left.\begin{array}{rcll} \boldsymbol{\sigma}(-\boldsymbol{z}) & = & \boldsymbol{H}_0 & \text{on} \quad S_0 \\ \boldsymbol{\sigma}(\boldsymbol{z}) & = & \boldsymbol{H}_L & \text{on} \quad S_L \end{array}\right\} \qquad (2)$$

3D SV'solution exactly satisfies (1) **but** (2) ....

$$\int_{S_0} \boldsymbol{\sigma}(\text{-}\boldsymbol{z})\, dS = \int_{S_0} \boldsymbol{H}_0\, dS \qquad \int_{S_0} \boldsymbol{GM} \wedge \boldsymbol{\sigma}(\text{-}\boldsymbol{z})\, dS = \int_{S_0} \boldsymbol{GM} \wedge \boldsymbol{H}_0\, dS$$

$$\int_{S_L} \boldsymbol{\sigma}(\boldsymbol{z})\, dS = \int_{S_L} \boldsymbol{H}_L\, dS \qquad \int_{S_L} \boldsymbol{GM} \wedge \boldsymbol{\sigma}(\boldsymbol{z})\, dS = \int_{S_L} \boldsymbol{GM} \wedge \boldsymbol{H}_L\, dS$$



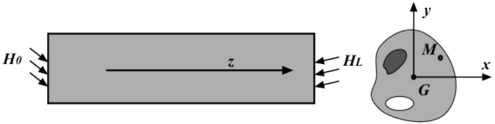
3D SV-solution

$$\left(F_i \;\middle|\; \begin{array}{l} \boldsymbol{R}=[T_x, T_y, N] \\ \boldsymbol{M}=[M_x, M_y, M_t] \end{array}\right)$$

$$\boldsymbol{\sigma}^{sv}(x,y,z) = \sum_{i=1}^{6} F_i(z)\, \boldsymbol{\sigma}^i(x,y)$$

$$\boldsymbol{\xi}^{sv}(x,y,z) = \boldsymbol{u}(z) + \boldsymbol{\omega}(z) \wedge \boldsymbol{GM} + \sum_{i=1}^{6} F_i(z) \boldsymbol{U}^i(x,y)$$

$$\left\{ \begin{array}{rcl} \boldsymbol{R}' & = & \boldsymbol{0} \\ \boldsymbol{M}' + \boldsymbol{z} \wedge \boldsymbol{R} & = & \boldsymbol{0} \\ \begin{bmatrix} \gamma = \boldsymbol{u}' + \boldsymbol{z} \wedge \boldsymbol{\omega} \\ \chi = \boldsymbol{\omega}' \end{bmatrix} & = & \boldsymbol{\Lambda} \begin{bmatrix} \boldsymbol{R} \\ \boldsymbol{M} \end{bmatrix} \\ & + & [\text{B. C.}]_{0,L} \end{array} \right.$$



3D SV-solution

$$\boldsymbol{\sigma}^{sv}(x,y,z) = \sum_{i=1}^{6} F_i(z)\, \boldsymbol{\sigma}^i(x,y) \quad \left(F_i \,\middle|\, \begin{array}{l} \boldsymbol{R}=[T_x, T_y, N] \\ \boldsymbol{M}=[M_x, M_y, M_t] \end{array} \right)$$

$$\boldsymbol{\xi}^{sv}(x,y,z) = \boldsymbol{u}(z) + \boldsymbol{\omega}(z) \wedge \boldsymbol{GM} + \sum_{i=1}^{6} F_i(z) \boldsymbol{U}^i(x,y)$$

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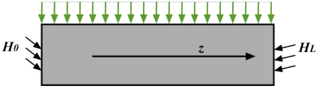
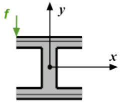
$$\boldsymbol{\Lambda};\ \boldsymbol{\sigma}^i; \boldsymbol{U}^i = \begin{bmatrix} \boldsymbol{\Pi}^i \\ \phi^i \end{bmatrix}$$

sectional characteristics

shape and materials





$$\begin{pmatrix} \boldsymbol{R}=[T_x, T_y, N] \\ \boldsymbol{M}=[M_x, M_y, M_t] \end{pmatrix}$$

$$\boldsymbol{\sigma}^{sv}(x,y,z) = \sum_{i=1}^{6} F_i(z)\ \boldsymbol{\sigma}^i(x,y) + \mathbf{f}\ \boldsymbol{\sigma}(x,y)$$

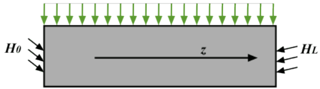
$$\boldsymbol{\xi}^{sv}(x,y,z) = \boldsymbol{u}(z) + \boldsymbol{\omega}(z) \wedge \boldsymbol{GM} + \sum_{i=1}^{6} F_i(z) \begin{bmatrix} \boldsymbol{\Pi}^i(x,y) \\ \phi^i(x,y) \end{bmatrix} + \mathbf{f}\ \boldsymbol{D}(x,y)$$

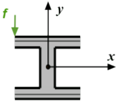
sectional charact. $\rightarrow$ $\boxed{\boldsymbol{\sigma}; \boldsymbol{D}}$

/ shape, mater. and load.

Distorsion = in-plane part of $\boldsymbol{D}$

SV-Problem (extended to comp. section and lat. loading Eisan(76))





$$\left(\begin{array}{l} \boldsymbol{R}=[T_x, T_y, N] \\ \boldsymbol{M}=[M_x, M_y, M_t] \end{array} \right)$$

$$\boldsymbol{\sigma}^{sv}(x,y,z) = \sum_{i=1}^{6} F_i(z)\, \boldsymbol{\sigma}^i(x,y) + \mathbf{f}\, \boldsymbol{\sigma}(x,y)$$

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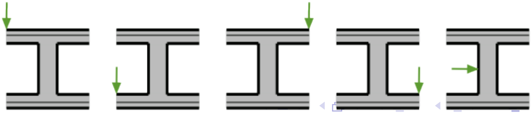
sectional charact. $\rightarrow$ $\boxed{\boldsymbol{\sigma}; \boldsymbol{D}}$

/ shape, mater. and load.

Distorsion = in-plane part of $\boldsymbol{D}$

$\mathbf{f}^j \rightarrow \boldsymbol{D}^j \; (j = 1, \ldots n)$



- **Mechanical Characteristics of a Composite Section**

$\boldsymbol{\Lambda}$	(6x6)-matrix of the 1D behavior
$\boldsymbol{\sigma}^i$	sectional stresses
$\boldsymbol{\Pi}^i, \phi^i$	Poisson's effects and warpings
$\boldsymbol{D}^j$	Distorsions

The numerical Method

- El Fatmi & Zenzri, 2002 (Computers & Structures)
- solving, by **2D FEM**, a set of elastic pbs defined on the section

(6) pbs to deduce $\boldsymbol{\Lambda}$, $\boldsymbol{\sigma}^i, \boldsymbol{\Pi}^i, \phi^i$
(k) pbs to deduce $\boldsymbol{\sigma}^j, \boldsymbol{D}^j$, (j=1,..,k)

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$\Longleftarrow$ *CSection* Matlab Tool

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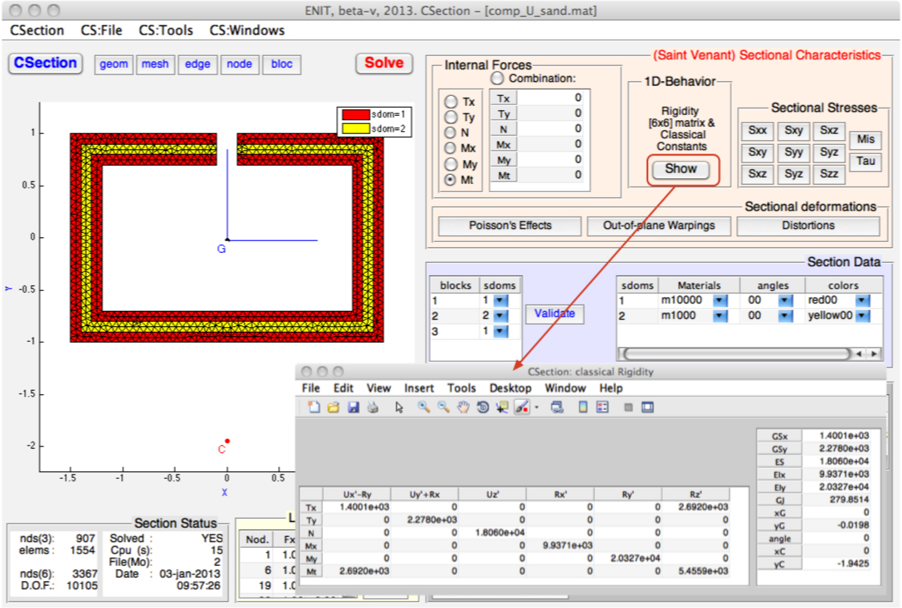
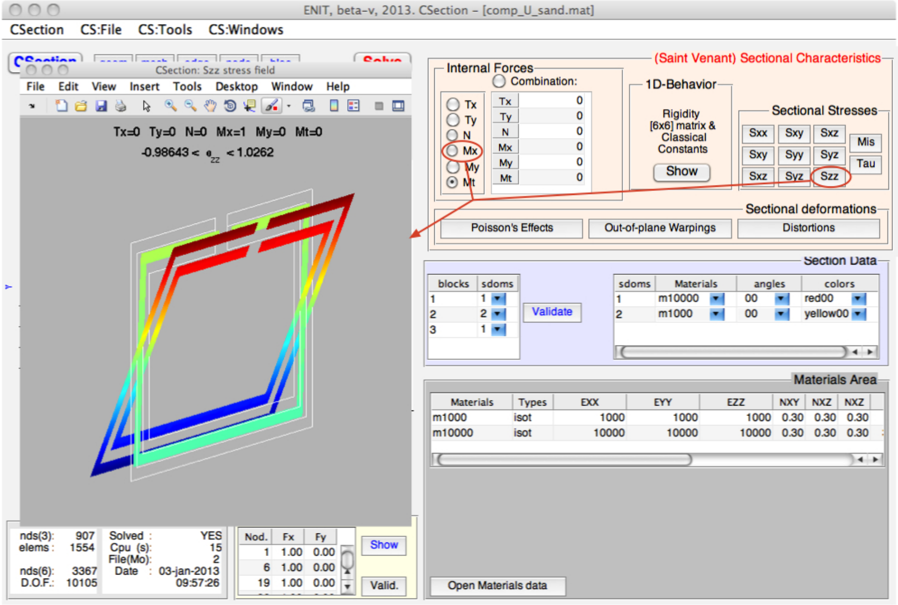

ENIT, beta-v, 2013. CSection – [comp_U_sand.mat]
CSection CS:File CS:Tools CS:Windows
CSection geom mesh edge node bloc Solve
sdom=1
sdom=2
G
C
x
y
(Saint Venant) Sectional Characteristics
Internal Forces
Combination:
Tx Ty N Mx My Mt
1D-Behavior
Rigidity [6x6] matrix & Classical Constants
Show
Sectional Stresses
Sxx Sxy Sxz Sxy Syy Syz Sxz Syz Szz Mis Tau
Sectional deformations
Poisson's Effects
Out-of-plane Warpings
Distortions
Section Data
blocks sdoms
Validate
sdoms Materials angles colors
1 m10000 00 red00
2 m1000 00 yellow00
Section Status
nds(3): 907
elems: 1554
nds(6): 3367
D.O.F.: 10105
Solved : YES
Cpu (s): 15
File(Mo): 2
Date : 03-jan-2013 09:57:26
CSection: classical Rigidity
File Edit View Insert Tools Desktop Window Help
Ux'-Ry Uy'+Rx Uz' Rx' Ry' Rz'
Tx 1.4001e+03 0 0 0 0 2.6920e+03
Ty 0 2.2780e+03 0 0 0 0
N 0 0 1.8060e+04 0 0 0
Mx 0 0 0 9.9371e+03 0 0
My 0 0 0 0 2.0327e+04 0
Mt 2.6920e+03 0 0 0 0 5.4559e+03
GSx 1.4001e+03
GSy 2.2780e+03
ES 1.8060e+04
EIx 9.9371e+03
EIy 2.0327e+04
GJ 279.8514
xG 0
yG -0.0198
angle 0
xC 0
yC -1.9425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CSection CS:File CS:Tools CS:Windows
CSection: Szz stress field
File Edit View Insert Tools Desktop Window Help
Tx=0 Ty=0 N=0 Mx=1 My=0 Mt=0
-0.98643 < σzz < 1.0262
(Saint Venant) Sectional Characteristics
Internal Forces
Combination:
Tx Ty N Mx My Mt
1D-Behavior
Rigidity [6x6] matrix & Classical Constants
Show
Sectional Stresses
Sxx Sxy Sxz Syx Syy Syz Szx Szy Szz Mis Tau
Sectional deformations
Poisson's Effects
Out-of-plane Warpings
Distortions
Section Data
blocks sdoms
Validate
sdoms Materials angles colors
m10000 m1000 red00 yellow00
Materials Area
Materials Types EXX EYY EZZ NXY NXZ NXZ
m1000 isot 1000 1000 1000 0.30 0.30 0.30
m10000 isot 10000 10000 10000 0.30 0.30 0.30
Open Materials data
nds(3): 907
elems : 1554
nds(6): 3367
D.O.F.: 10105
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Cpu (s): 15
File(Mo): 2
Date : 03-jan-2013 09:57:26
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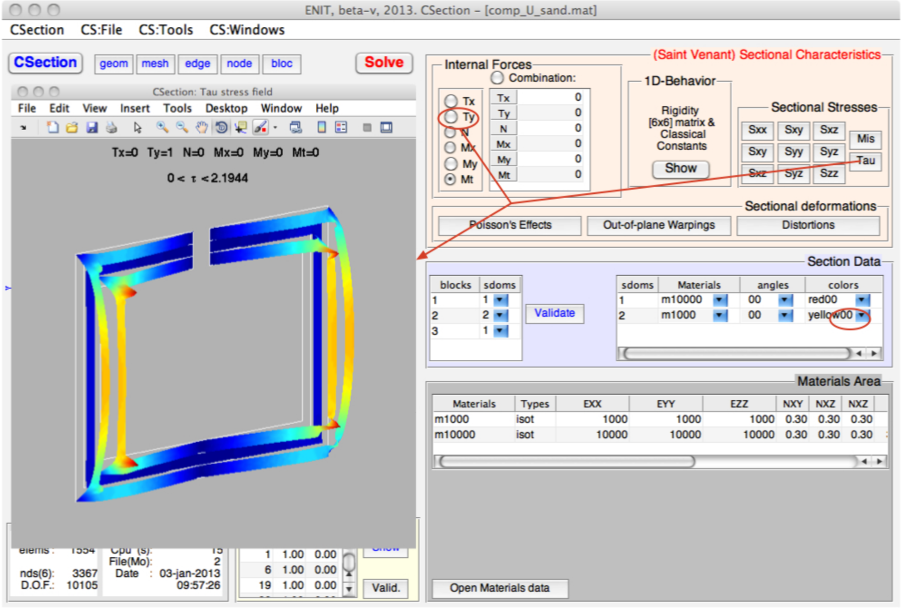
Show
Valid.


ENIT, beta-v, 2013. CSection - [comp_U_sand.mat]
CSection CS:File CS:Tools CS:Windows
CSection geom mesh edge node bloc Solve
CSection: Tau stress field
File Edit View Insert Tools Desktop Window Help
Tx=0 Ty=1 N=0 Mx=0 My=0 Mt=0
0 < τ < 2.1944
(Saint Venant) Sectional Characteristics
Internal Forces
Combination:
Tx Ty N Mx My Mt
1D-Behavior
Rigidity [6x6] matrix & Classical Constants
Show
Sectional Stresses
Sxx Sxy Sxz Sxy Syy Syz Sxz Syz Szz Mis Tau
Sectional deformations
Poisson's Effects
Out-of-plane Warpings
Distortions
Section Data
blocks sdoms
Validate
sdoms Materials angles colors
m10000 m1000 red00 yellow00
Materials Area
Materials Types EXX EYY EZZ NXY NXZ NXZ
m1000 isot 1000 1000 1000 0.30 0.30 0.30
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nds(6): 3367
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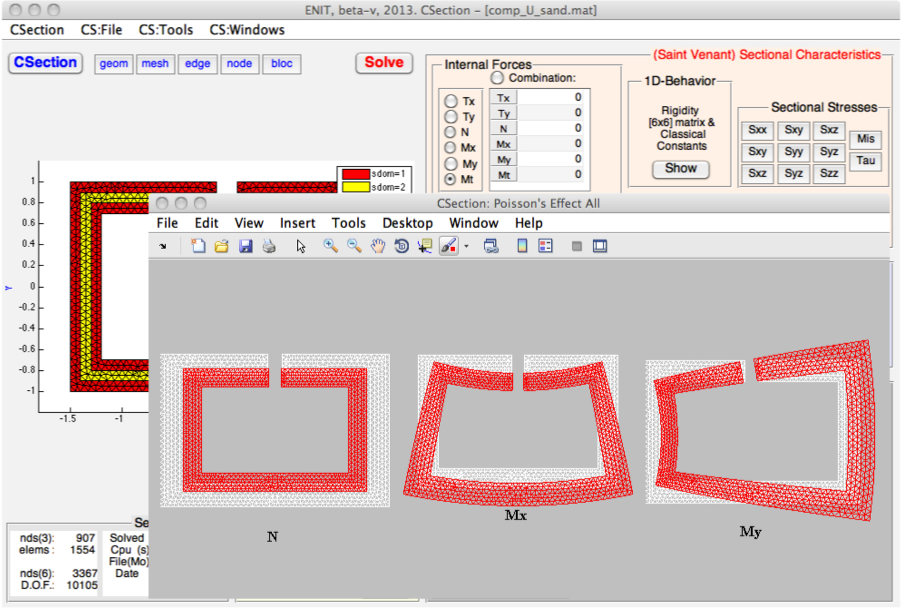
Valid.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CSection CS:File CS:Tools CS:Windows
CSection geom mesh edge node bloc Solve
(Saint Venant) Sectional Characteristics
Internal Forces
Combination:
Tx Ty N Mx My Mt
Tx 0
Ty 0
N 0
Mx 0
My 0
Mt 0
1D-Behavior
Rigidity [6x6] matrix & Classical Constants
Show
Sectional Stresses
Sxx Sxy Sxz
Sxy Syy Syz
Sxz Syz Szz
Mis
Tau
sdom=1
sdom=2
CSection: Poisson's Effect All
File Edit View Insert Tools Desktop Window Help
N
Mx
My
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elems : 1554
nds(6): 3367
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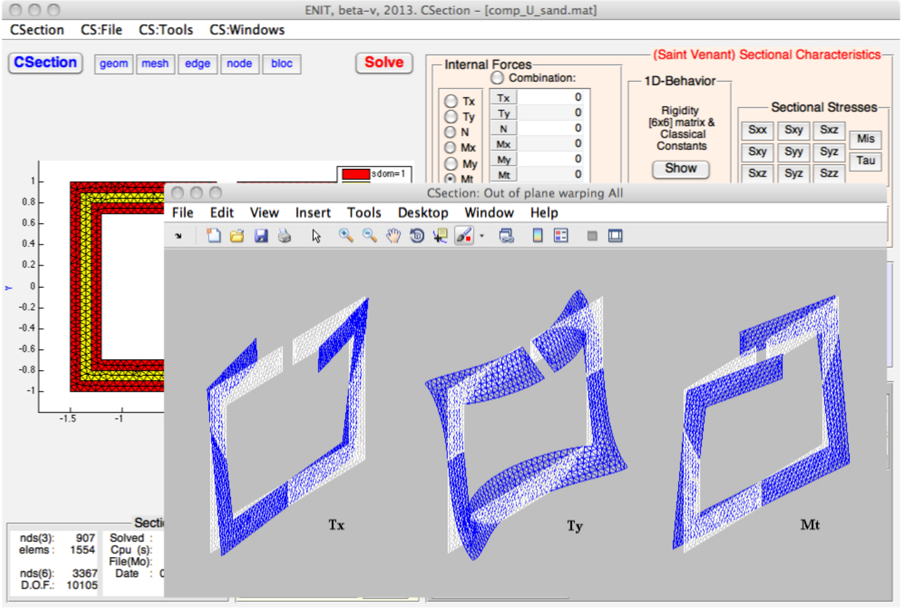
Solved
Cpu (s)
File(Mo)
Date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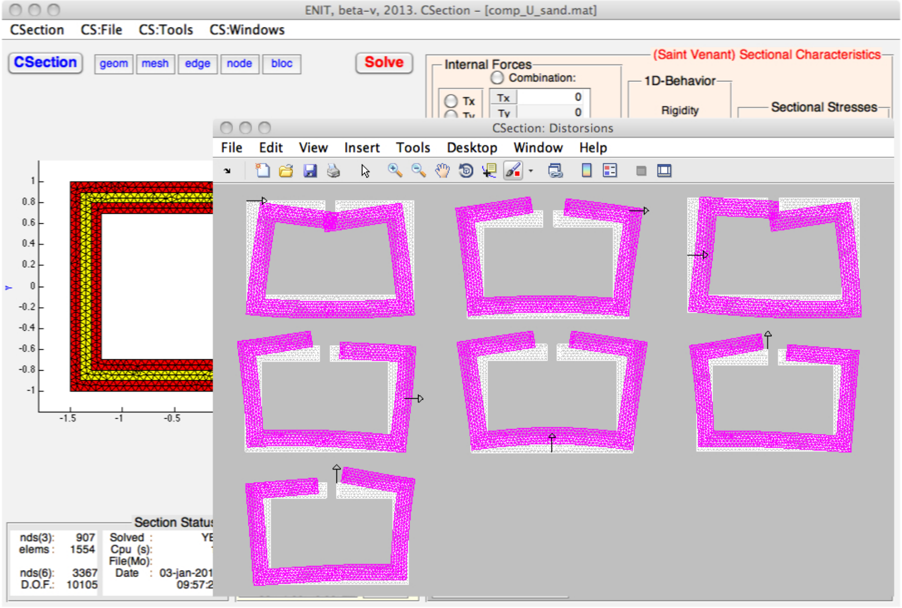
CSection CS:File CS:Tools CS:Windows
CSection geom mesh edge node bloc Solve
Internal Forces
Combination:
Tx Ty N Mx My Mt
Tx 0
Ty 0
N 0
Mx 0
My 0
Mt 0
(Saint Venant) Sectional Characteristics
1D-Behavior
Rigidity [6x6] matrix & Classical Constants
Show
Sectional Stresses
Sxx Sxy Sxz
Sxy Syy Syz
Sxz Syz Szz
Mis
Tau
sdom=1
nds(3): 907
elems: 1554
nds(6): 3367
D.O.F.: 10105
Solved :
Cpu (s):
File(Mo):
Date :
CSection: Out of plane warping All
File Edit View Insert Tools Desktop Window Help
Tx
Ty
Mt


ENIT, beta–v, 2013. CSection – [comp_U_sand.mat]
CSection CS:File CS:Tools CS:Windows
CSection
geom
mesh
edge
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Solve
(Saint Venant) Sectional Characteristics
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Tx
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1D-Behavior
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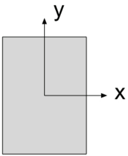
CSection: Distorsions
File Edit View Insert Tools Desktop Window Help
Section Status
nds(3): 907
elems: 1554
nds(6): 3367
D.O.F.: 10105
Solved :
Cpu (s):
File(Mo):
Date : 03-jan-201
09:57:2

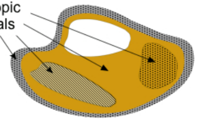
Arbitrary composite section



$$\begin{bmatrix} T_x \\ T_y \\ N \\ M_x \\ M_y \\ M_t \end{bmatrix} = \overbrace{\begin{bmatrix} GA_x & 0 & 0 & 0 & 0 & 0 \\ 0 & GA_y & 0 & 0 & 0 & 0 \\ 0 & 0 & EA & 0 & 0 & 0 \\ 0 & 0 & 0 & EI_x & 0 & 0 \\ 0 & 0 & 0 & 0 & EI_y & 0 \\ 0 & 0 & 0 & 0 & 0 & GJ \end{bmatrix}}^{\boldsymbol{\Gamma}_{[6\times6]}} \times \begin{bmatrix} u'_x - \omega_y \\ u'_y + \omega_x \\ u'_z \\ \omega'_x \\ \omega'_y \\ \omega'_z \end{bmatrix}$$

$$\boldsymbol{\sigma}^{sv}(x, y, z) = \begin{bmatrix} 0 & 0 & \sigma_{xz} \\ 0 & 0 & \sigma_{yz} \\ \sigma_{xy} & \sigma_{yz} & \sigma_{zz} \end{bmatrix}$$

Arbitrary composite section

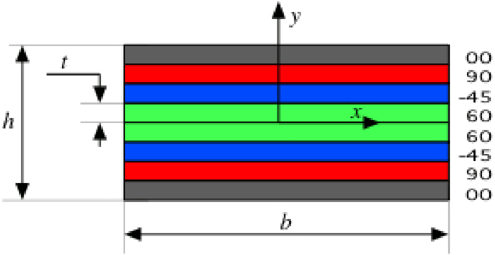


Several elastic couplings !

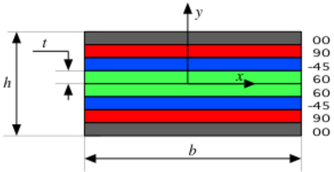
$$
\begin{bmatrix} T_x \\ T_y \\ N \\ M_x \\ M_y \\ M_t \end{bmatrix}
=
\begin{bmatrix}
\widetilde{GA_x} & \bullet & {\color{blue}\times} & \bullet & {\color{green}\times} & \bullet \\
\bullet & \widetilde{GA_y} & \bullet & {\color{brown}\times} & \bullet & \bullet \\
{\color{blue}\times} & \bullet & \widetilde{EA} & \bullet & \bullet & \bullet \\
\bullet & {\color{brown}\times} & \bullet & \widetilde{EI_x} & \bullet & {\color{red}\times} \\
{\color{green}\times} & \bullet & \bullet & \bullet & \widetilde{EI_y} & \bullet \\
\bullet & \bullet & \bullet & {\color{red}\times} & \bullet & \widetilde{GJ}
\end{bmatrix}
\times
\begin{bmatrix} u'_x - \omega_y \\ u'_y + \omega_x \\ u'_z \\ \omega'_x \\ \omega'_y \\ \omega'_z \end{bmatrix}
$$

$$
\boldsymbol{\sigma}^{sv}(x, y, z) =
\begin{bmatrix}
{\color{red}\sigma_{xx}} & {\color{red}\sigma_{xy}} & \sigma_{xz} \\
{\color{red}\sigma_{xy}} & {\color{red}\sigma_{yy}} & \sigma_{yz} \\
\sigma_{xy} & \sigma_{yz} & \sigma_{zz}
\end{bmatrix}
$$

Laminated [0,90,-45,60]$_s$ section



- $\boldsymbol{\Gamma}$, the 1D structural behavior
- $\boldsymbol{\sigma}^i$: Two critical points
  - The shear through the layers
  - Interlaminar stresses / free edge effect)
- $\boldsymbol{\mathcal{M}}^k$: the sectional modes

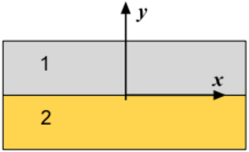


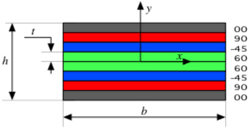
(6×6)-matrix 1D stiffness

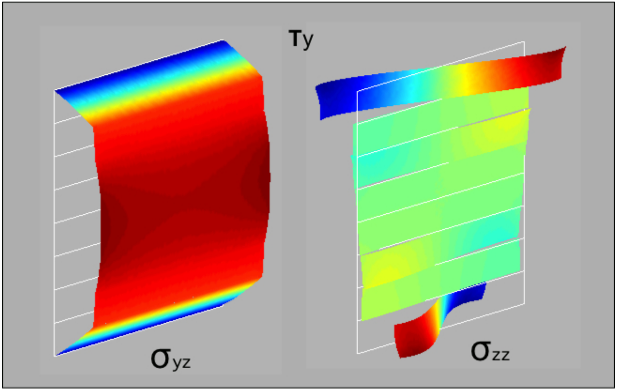
$$\boldsymbol{\Gamma} = 10^9 \times \begin{bmatrix} 6.1657 & 0.0000 & -1.8607 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 4.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ -1.8607 & 0.0000 & 38.3708 & 0.0000 & 0.0000 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 3.7226 & 0.0000 & 0.0645 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 3.0925 & 0.0000 \\ 0.0000 & 0.0000 & 0.0000 & 0.0645 & 0.0000 & 0.5722 \end{bmatrix}$$

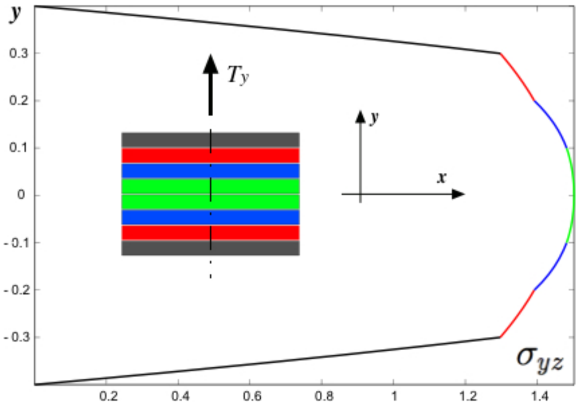
Elastic couplings: **extension-bending** and **twist-bending**

Laminated $[0,90,-45,30]_s$ section

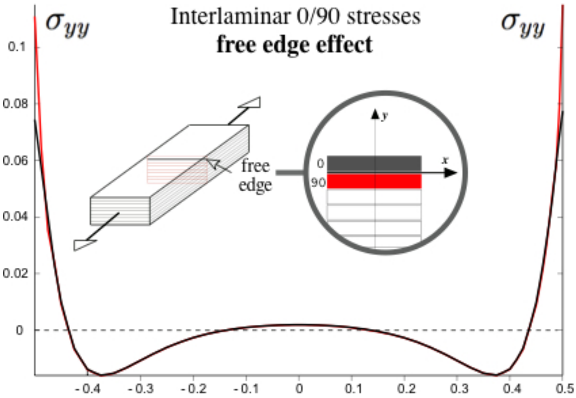








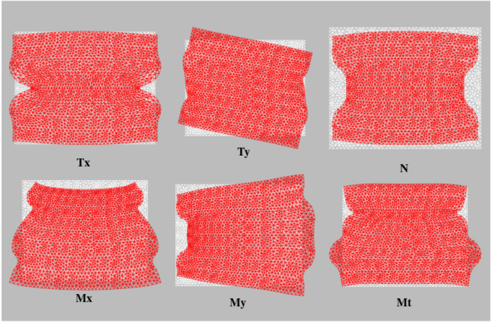
The shear σ_{yz} due to shear force T_y through the layers



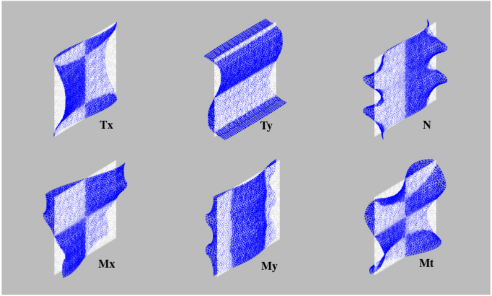
Interlaminar stresses σ_{yy} on $0/90$-interface
due to an axial force N
(free edge efffect)

Laminated $[0,90,-45,30]_s$ section

Poisson's Effects



Warpings



1 **Composite section** $\rightarrow$ CSection $\rightarrow$ $[M^j, \phi^i]$

SV-sectional modes

that contains the physics

of the section

1 **Composite section** → $CSection$ → $[M^j, \phi^i]$
SV-sectional modes
that contains the physics
of the section

2 **ABT** (Advanced Beam Theory)= *A Higher order beam theory*
built on **SV-sectional results**

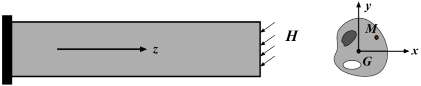
$$\boldsymbol{\xi}(\boldsymbol{u}, \boldsymbol{\omega}, \boldsymbol{\eta}, \boldsymbol{\alpha}) = \overbrace{\boldsymbol{u}(z) + \boldsymbol{\omega}(z) \wedge \boldsymbol{GM}}^{\text{rigid motion of the section}}$$

$$+ \overbrace{\eta_i(z)\phi^i(x,y) + \alpha_k(z)\boldsymbol{M}^k(x,y)}^{\text{enrichment}}$$

.... the correspondent **Beam Theory**

El Fatmi & Ghazouani

Composite structures, 2011



- Displacement model $\boldsymbol{\xi} = \boldsymbol{\xi}(\boldsymbol{u}, \boldsymbol{\omega}, \eta_i, \alpha_j) \Longrightarrow$ ......BT equations

$$\left\{\begin{array}{rcll} \boldsymbol{R}' &=& \boldsymbol{0} & \\ \boldsymbol{M}' + \boldsymbol{z} \wedge \boldsymbol{R} &=& \boldsymbol{0} & \\ (M^i_\phi)' - M^i_s &=& \boldsymbol{0} & \forall i \\ (A^j_\nu)' - A^j_s &=& \boldsymbol{0} & \forall j \end{array}\right. \quad \left\{\begin{array}{ll} 0 & (\boldsymbol{u}, \boldsymbol{\omega}, \eta^i, \alpha^j) = (\boldsymbol{0}, \boldsymbol{0}, 0, 0) \\ L & (\boldsymbol{R}, \boldsymbol{M}, M^i_\psi, A^j_\nu) = (\boldsymbol{P}_L, \boldsymbol{C}_L, Q^i_L, S^j_L) \end{array}\right.$$

+ 1D behavior (Constitutive law)

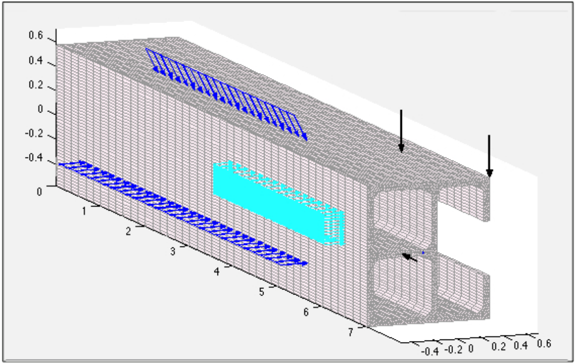
- Beam-Problem. → 1D-Sol. $(\boldsymbol{u}^e, \boldsymbol{\omega}^e, \eta^e_i, \alpha^e_j)$

$$\rightarrow \boldsymbol{\xi}^e = \boldsymbol{\xi}(\boldsymbol{u}^e, \boldsymbol{\omega}^e, \eta^e_i, \alpha^e_j) \rightarrow \boldsymbol{\sigma}^{1D \rightarrow 3D} = \mathbf{K}\boldsymbol{\varepsilon}(\boldsymbol{\xi}^e)$$

Loading and Boundary conditions

- 1D BC: Several DOF: $[u_x, u_y, u_z]$; $[\omega_x, \omega_y, \omega_z]$; $[\alpha_i]_1^k$; $[\eta_j]_1^6$

- 3D loading $\rightarrow$ generalized 1D external forces ?



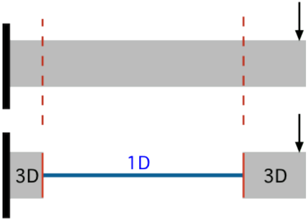
•Numerical **1D-FEM** developement ... $\rightarrow$ ...

$\forall q^i \approx$ Hermite

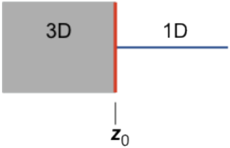
CBeam

(Matlab Tool)

1D-3D connections for the 1D-3D-FEM computation



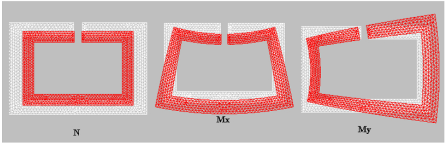
•3D-1D-connection to ensure the **continuity** of the displacement

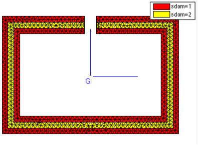


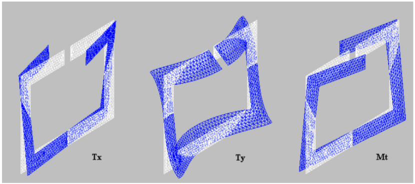
$$\boldsymbol{\xi}^{3D}(x, y, z_0) = \boldsymbol{\xi}^{ABT}(\boldsymbol{u}(z_0), \boldsymbol{\omega}(z_0), \alpha_j(z_0), \eta_i(z_0))$$

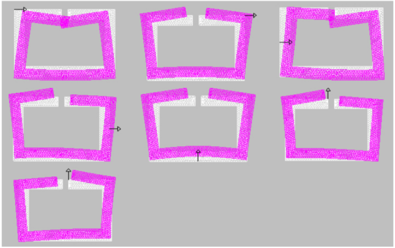
•**1D-3D-FEM** computation of the **whole** problem !

Example

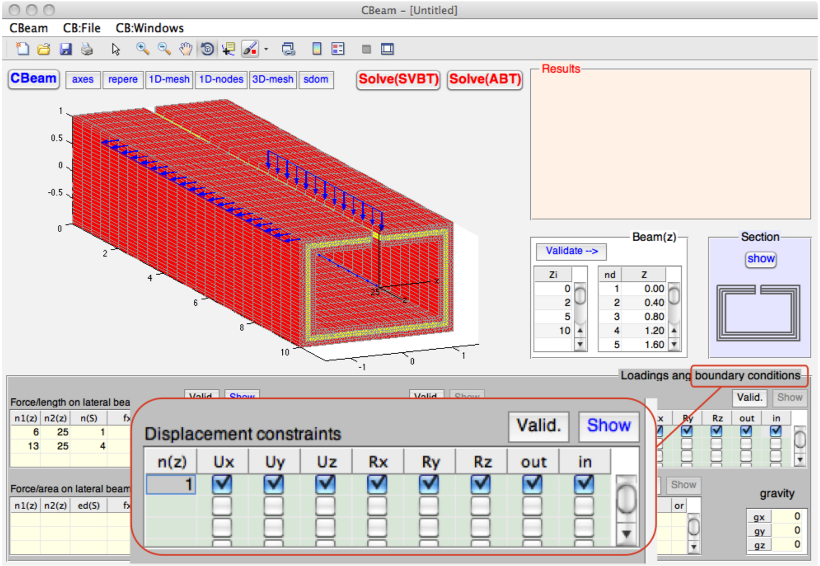




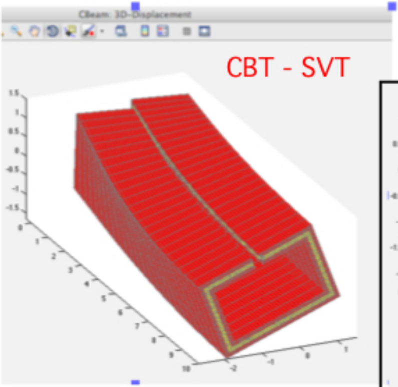


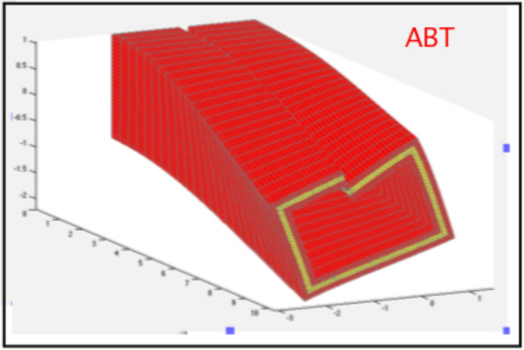


Loads and Boundary Conditions

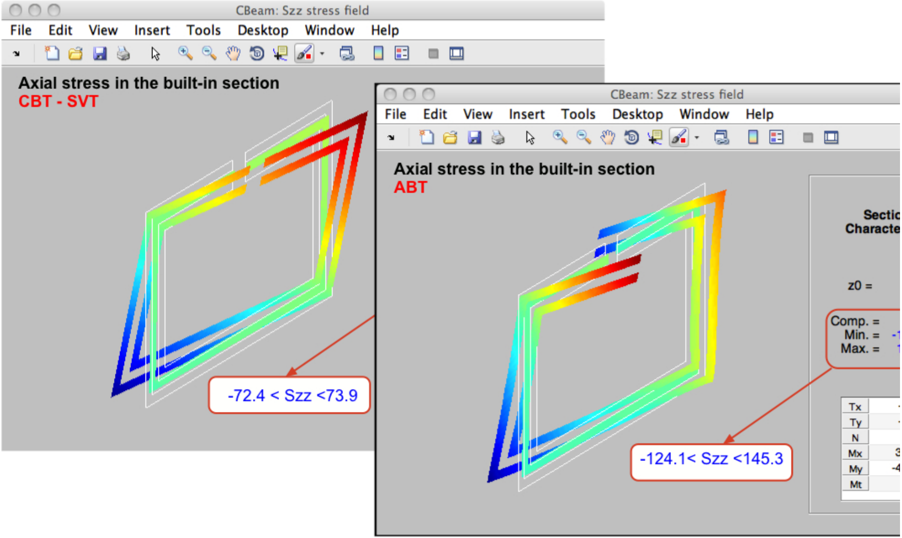


3D deformed beam

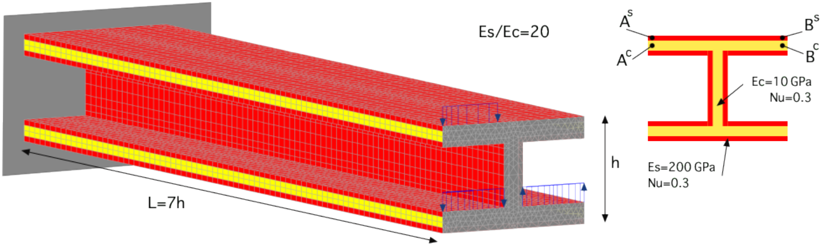




Sectional stresses



Bending-torsion of a **Short** cantilever **open composite** profile

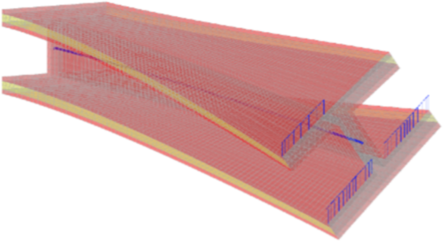


3D-Results

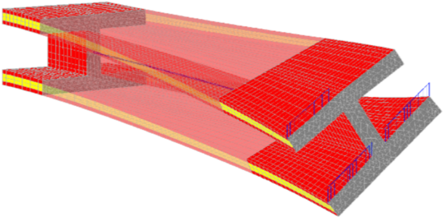
- Displacement
- Axial-Stresses (z): $\boldsymbol{\sigma}_{zz}^{SV}$, $\boldsymbol{\sigma}_{zz}^{3D}$, $\boldsymbol{\sigma}_{zz}^{1D}$, $\boldsymbol{\sigma}_{zz}^{3D-1D}$

in the **skin** and in the **core**

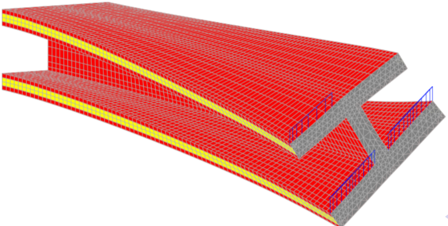
1D



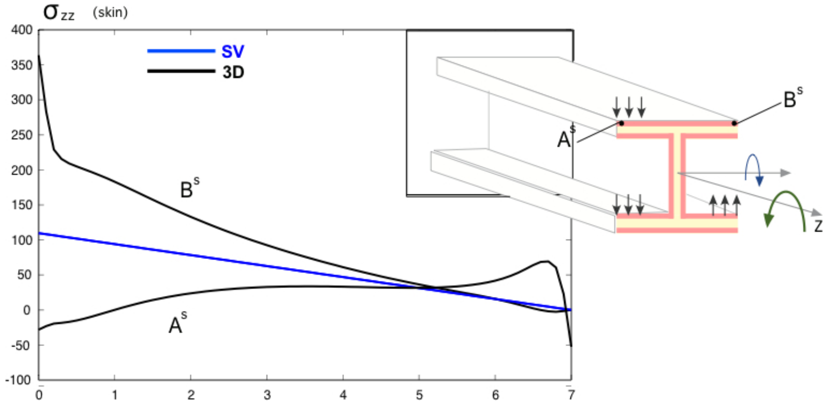
3D-1D

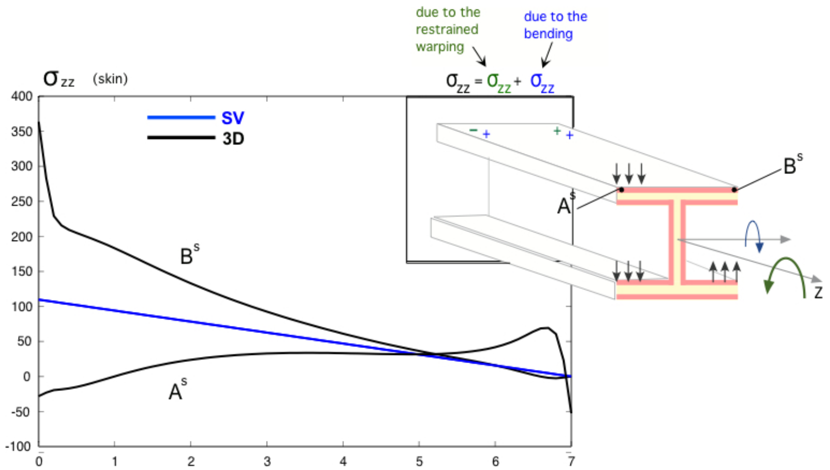


3D



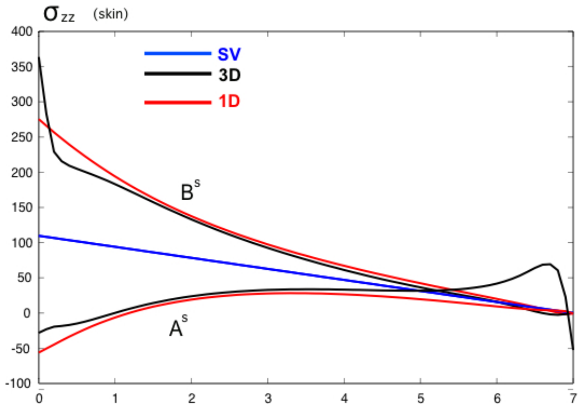
z-variation of σ_{zz} for points in the skin and in the core

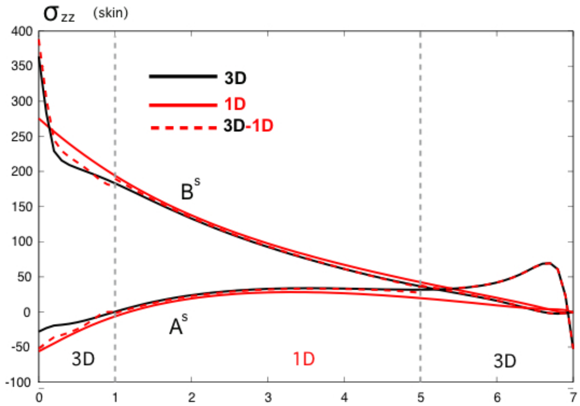




σzz (skin)
400
350
300
250
200
150
100
50
0
-50
-100
0
1
2
3
4
5
6
7
SV
3D
B^s
A^s
due to the restrained warping
due to the bending
σzz = σzz + σzz
− +
+ +
B^s
A^s
z

z-variation of σ_{zz} for points in the skin and in the core




σzz (skin)
3D
1D
3D-1D
B^s
A^s
3D
1D
3D
400
350
300
250
200
150
100
50
0
-50
-100
0
1
2
3
4
5
6
7

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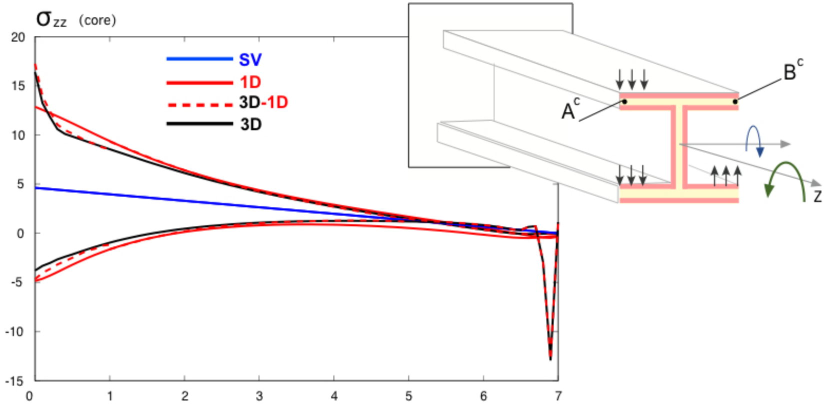
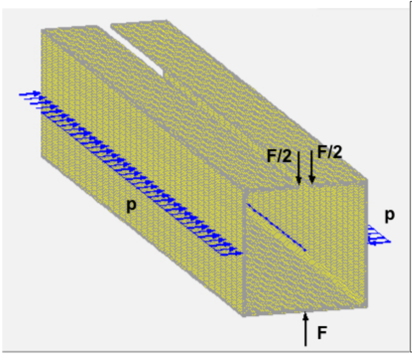
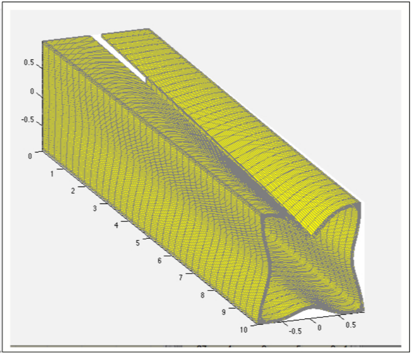
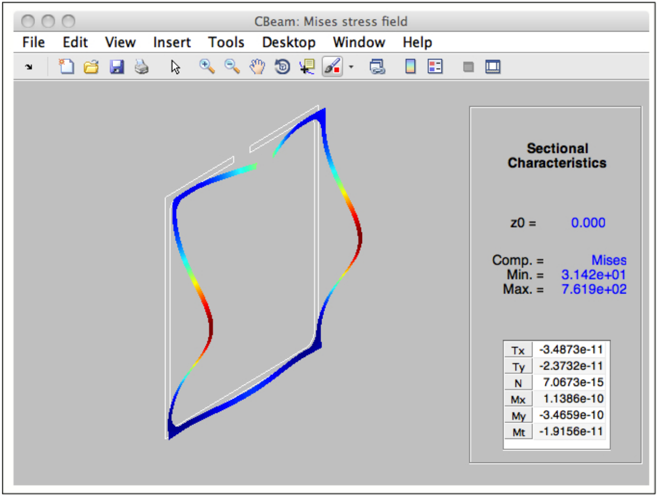


Loading with a zero resultants (force and moment)





Loading with a zero resultants (force and moment)



Conclusion

- **Key point** : the sectional modes
  - contains the physics of the section
  - but need first numerical **2D**-FEM computations
- **Advanced Beam Theory** (**ABT**)
  - the number of modes is as important as we want
  - 2 levels of computation
    - **1D** : for the 1D behavior and the 3D **central solution**
    - 3D-**1D** : to get more information on edge effects

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  - .... *dynamic* and *buckling* under development

Professional use

- Lots of applications

Academic use: **help** to ...

- **show/understand** the 1D-3D behavior of hom./comp. beams
- **discuss beam theories**: SV, Timoshenko, Vlasov, ...
- **introduce composite** structure
- analyse **tests** / composite materials (Lab.)