UPDATING METHODS OF DYNAMIC STRUCTURAL FINITE ELEMENT MODELS: DYNAMIC REACTION FORCES APPROACH AND ERROR IN CONSTITUTIVE EQUATION APPROACH

by

P. LADEVÈZE (*), M. REYNIER (*), H. BERGER (**), R. OHAYON (**), F. QUÉTIN (**) and L. BARTHE (**)

ABSTRACT

The purpose of the Finite Element Model adjustment is to control and improve the representative model of a real structure using a preliminary finite element modelling and dynamic data, obtained experimentally.

Over the last ten years, this problem has aroused a great interest in universities and industries and a European group has been conducted by ONERA to test the different methods through specific benchmarks.

In this paper, we present some studies improving the F.E. model by means of experimental modal results, and more precisely we describe two approaches:

– the updating method developed at ONERA, that is based on the variational expression of the dynamic equilibrium equations; the dynamic reaction forces due to the stiffness matrix and mass matrix imperfections of the finite element model are used to localize the error, using experimental frequencies and corresponding displacements;

– the approach developed by the LMT laboratory in Cachan assumes that the modelling errors are localized where the errors on the constitutive relation are the largest.

Keywords (NASA thesaurus): Finite element method – Dynamic structural analysis – Correlation analysis-tests – Adjusting – Dynamic reaction force – Error on the constitutive relation – Vibration tests.

(*) Laboratoire de Mécanique et Technologie (LMT), ENS of Cachan/CNRS, University Paris-VI, 61, avenue du Président-Wilson, 94325 Cachan Cedex.

(**) ONERA, B.P. 72, 92322 Châtillon Cedex.

INTRODUCTION

The updating of structural models from experimental data raises problems in the framework of a new computation/experimental dialog which is now appearing. The problems are inverse and the difficulties are multiple: theoretical, mathematical and mechanical as well as experimental and numerical. Although general approaches exist for describing and solving these problems, they in no way dispense with the need for considering aspects related to physics [2]. It is necessary to carefully choose the objective functions to be optimized and the experimental measurements to be used. This second aspect is perfectly illustrated by the model correction methods.

Qualification of structures, in particular space structures, by dynamic behavior increasingly involves numerical simulations in replacement of excessively costly tests. It is then necessary to have a representative model of reality. The most widely used mathematical model is that of free vibrations of a complex structure discretized by finite elements. Tests supply the modal characteristics (generally the first natural frequencies and the associated eigenmodes, incompletely known). The problem is then to **minimize the difference between computation and experiment** and **improve the F.E. model.**

Many adjustment strategies have been proposed:

– methods such as those of **Baruch** [3], **Berman** [4], **Chen** [5], **Caesar** [6], **Ewins** [7, 8], **Link** [9] who directly construct the corrected mass and stiffness matrices from the measured modal characteristics and the orthogonality relations;

– methods using sensitivity techniques: **Wei** [10], **Collins** [11], **Lallemant** [12], **Dascotte** [13] express the eigenvalues as a function of the design parameters by a Taylor series expansion. The correction coefficients appear directly from these methods before the elementary matrices. The most commonly used strategy is the least squares method (Bayesian estimate);

– **Cottin** [14], **Niedbal**, **Luber**, **Klusowski** [27] and **Nash** [28] attempt to improve the stiffness, mass and damping matrices by minimizing the errors on the inputs (experimental excitation forces-excitation forces calculated from the model and experimental displacements) or the errors on the outputs (experimental displacements-displacements calculated from the model and experimental excitation forces);

– **Boutin**, **Petiau** [15] minimize a "real structure-finite element model" error using the frequencies and modal deformations restricted to only the measured degrees of freedom verifying the dynamic equilibrium equations. The optimization problem amounts to a problem without constraints using a penalty method;

– the methods developed by **O'Callahan** [16] and **Leuridan** [17] use the experimental eigenforms to transform the stiffness and mass matrices of the finite element model in the mode base (Craig-Bampton form). Each term with a known experimental value can be expressed using a sum of elementary energy terms premultiplied by the correction coefficients. The solution is computed by the least squares method on the energy products for the measured modes.

The approach adopted at ONERA [18, 19, 20] is based on **calculation of the residues in the variational expression of the equilibrium equation**, and uses the experimental modes to solve linear static problems. The difference between the model and measurements results in nonzero reaction forces on the degrees of freedom of the structure. The calculation of the correction parameters is a quadratic optimization problem without constraints each iteration. This strategy was adopted in [21].

The approach suggested by LMT [22, 23, 24] and initiated with Aérospatiale [26] is based on the concept of **error in the constitutive equation** as described in [27]. The measured eigenvalues are considered exact but weighting is applied to the eigenmodes. They are considered simultaneously without imposing any order between them. The problem of localization is linear with a reasonable size and correction is a nonlinear problem with a dimension equal to the number of parameters to be corrected.

All these methods are **evolutionary**, *i.e.* they adapt continuously to the progress of experimentation and the numerical aspect.

LOCALIZATION OF MODEL ERRORS

DYNAMIC REACTION FORCES APPROACH

One of the major difficulties to be solved in the adjustment problem is that the data available are incomplete, the experimental displacements being known in only a few measurements points, much smaller in number than the number of degrees of freedom of the model. Different methods were used to construct a complete "experimental" vector. ONERA developed a methodology which proceeds by solving three consecutive static problems.

The first static problem, stated below in terms of local equilibrium equations, is solved:

Let Ω be a structure with boundary $\delta\Omega = \delta\Omega_1 \cup \delta\Omega_2$ where $\delta\Omega_1$ is the part of the boundary with known displacements (given boundary conditions and experimental displacements) and $\delta\Omega_2$ is the complementary part. It is attempted to solve

$$\operatorname{div}\sigma + \lambda_{\exp}\rho u = 0$$

in Ω where σ is the stress field.
u is the displacement field
ρ is the density
with the boundary conditions

$$u = 0 \quad \text{on } \delta\Omega_1'$$

$$u = U_{\exp} \quad \text{on } \delta\Omega_1'' \qquad \text{where} \quad \delta\Omega_1 = \delta\Omega_1' \cup \delta\Omega_1''$$

which leads to the solution

$$u = U_1 \quad \text{in } \Omega \cup \delta\Omega_2$$

and

$$\sigma \mathbf{n} = R \neq 0 \quad \text{on } \delta\Omega_1'' \quad (R \text{ reaction forces}).$$

For a discretized medium, the variational expression of these equations leads to the following static problem (1) after discretization by finite elements, defining the matrix operator T_k:

$$T_k = K_0 - \lambda_{\exp k} M_0 + \sum_1^n \beta_j K_j - \lambda_{\exp k} \sum_1^p \gamma_i M_i.$$

$K_0 (M_0)$ is the stiffness (mass) matrix of the initial model;

$K_j (M_i)$ is the elementary stiffness (mass) matrix of element $j(i)$ (with dimension extended to that of K_0;

$\beta_j (\gamma_i)$ is the relative error of the stiffness (mass) characteristics of element $j(i)$;

$\lambda_{\exp k}$ is the experimental eigenvalue of mode k;

$U_{\exp k}$ is the known part of the experimental displacement;

U_k is the unknown displacement;

β_j, γ_i can affect a group of elements.

Below, corrections concerning the stiffness matrix and the mass matrix are considered.

For a specific value β_j, γ_i ($\beta_j = \gamma_i = 0$ initially) and for each mode denoted k:

$$\begin{bmatrix} T_{k11} & T_{k12} \\ T_{k21} & T_{k22} \end{bmatrix} \begin{bmatrix} U_k \\ U_{\exp k} \end{bmatrix} = \begin{bmatrix} 0 \\ R_k \end{bmatrix} \qquad (1)$$

where zero reaction forces are imposed on all the unknown DOFs. Solving (1) supplies U_k and R_k.

In order to obtain nonzero reaction forces on all the degrees of freedom of the structure, the two following static problems are solved in succession:

$$\begin{bmatrix} T_{k11} & T_{k12} \\ T_{k21} & T_{k22} \end{bmatrix} \begin{bmatrix} U_k \\ U_k' \end{bmatrix} = \begin{bmatrix} R_k' \\ 0 \end{bmatrix} \qquad (2)$$

which supplies U_k' and R_k', and finally,

$$\begin{bmatrix} T_{k11} & T_{k12} \\ T_{k21} & T_{k22} \end{bmatrix} \begin{bmatrix} U_k'' \\ U_{\exp k} \end{bmatrix} = \begin{bmatrix} R_k' \\ R_k'' \end{bmatrix} \qquad (3)$$

which allows displacement $U_{\exp k}$ to be completed and the reaction forces to be obtained on all the DOFs of the structure.

Remark: Calculation of the complete displacement is possible if submatrices T_{k11} and T_{k22} are invertible, which is the case for low frequency problems with regularly distributed measured DOFs.

The error function is defined by

$$g_i = \frac{\sup_k \| R_{ik} \|}{\max_i (\sup_k \| R_{ik} \|)} \qquad (4)$$

where $\| R_{ik} \|$ is the norm of the reaction force in node i for mode k.

THE DISTRIBUTION OF THIS FUNCTION ON THE STRUCTURE DEFINES THE SUBSTRUCTURES TO BE CORRECTED

Error function g_i can be used to extract the nodes E_m with minimum error during the first iteration such that for $g_i < \varepsilon$ given, the completed displacement D_{E_m} in these nodes can be considered exact. For subsequent iterations, the complete displacement can be computed only on nodes E_M identified as those with maximum error by imposing values D_Γ extracted from D_{E_m} on the common boundary Γ between E_M and E_m (Figs. 1 and 2). The reaction forces of E_m on E_M must not be included in computation of the cost function. This technique, which consists of examining a local region in detail from the displacements known on the boundary of the substructure yields independent problems of a small size.

APPROACH BY THE ERROR IN CONSTITUTIVE EQUATION CONCEPT

Let us consider a domain Ω with boundary $\partial\Omega$, with two complementary parts of the boundary, $\partial\Omega_1$ and $\partial\Omega_2$, such that the displacements are imposed on $\partial\Omega_1$ and the loads on the complementary part, $\partial\Omega_2$.

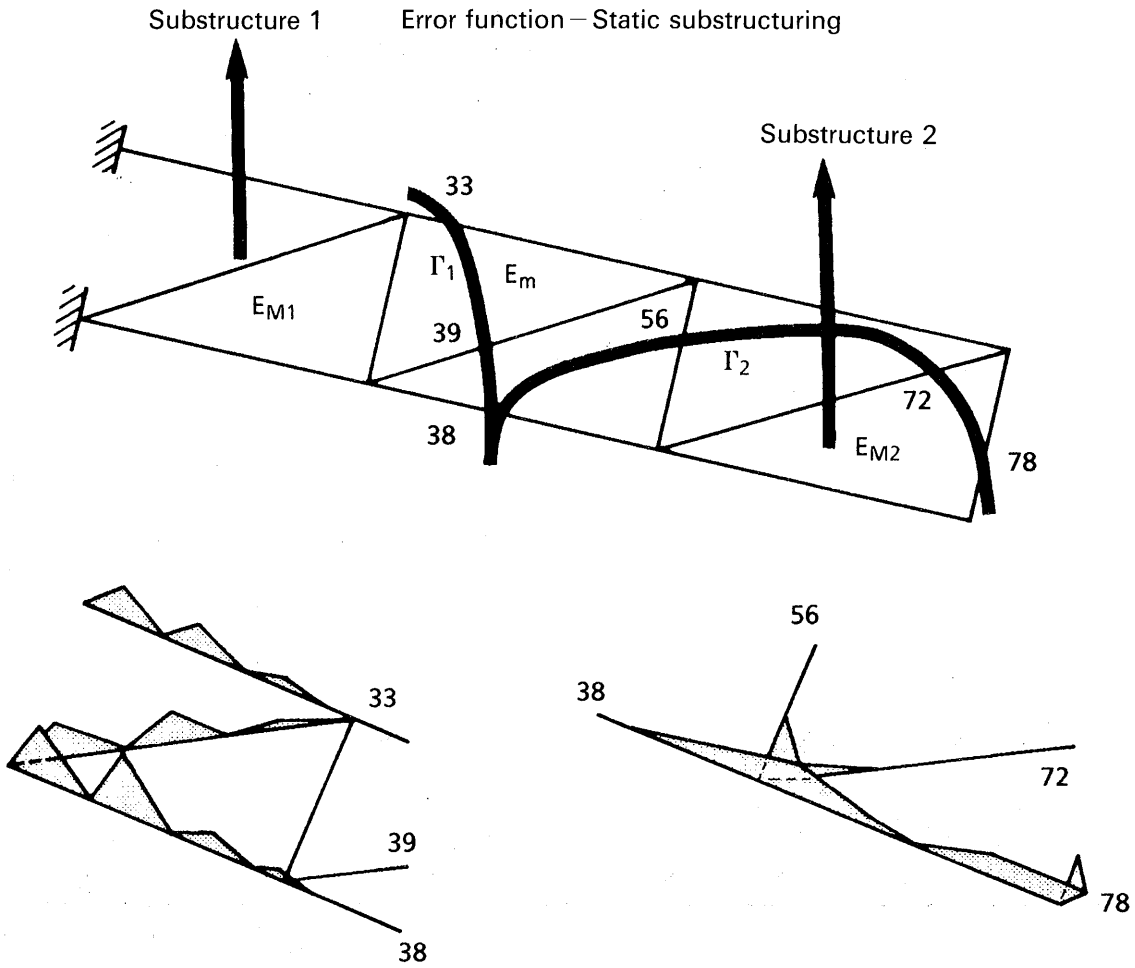


Fig. 1. – Error function. Static substructuring.

			$\varepsilon\% = \frac{\text{Exp. displ.} - \text{Completed displ.}}{\text{Exp. displ.}}$				
	Boundary Γ		Vector No.				
	Node		1	2	3	4	5
	33	u meas. v meas. θ_z	0 0 0.07	0 0 1.8	0 0 0.2	0 0 0.13	0 0 0.2
Γ_1	39	u v θ_z	0.7 0.003 0.02	0.0002 0.02 0.006	0.006 0.003 0.08	0.3 0.02 0.004	0.002 0.009 0.005
	38	u v θ_z	0 0.003 0.02	0.0002 0.02 0.14	0.004 0.002 0.06	0 0.003 0.02	0 0.02 0.002
	56	u v θ_z	0.0006 0 0.0008	0.0003 0.009 4.4	0.01 0 0.0003	0.002 0.0001 0.002	0.001 0.0001 0.05
Γ_2	72	u meas. v meas. θ_z	0 0 0.0001	0 0 0.04	0 0 0.002	0 0 0.006	0 0 0.02
	78	u meas. v meas. θ_z	0 0 0.002	0 0 1.9	0 0 0.05	0 0 0.2	0 0 0.4

Fig. 2. – Static substructuring.

Abscissa: index of elements s

Ordinate: error $\sum_{k=1}^{q} \Gamma_s^2$

$q=5$ given experimental modes

error $\sum_{k=1}^{q} \Gamma_s^2$

drawing on the structure

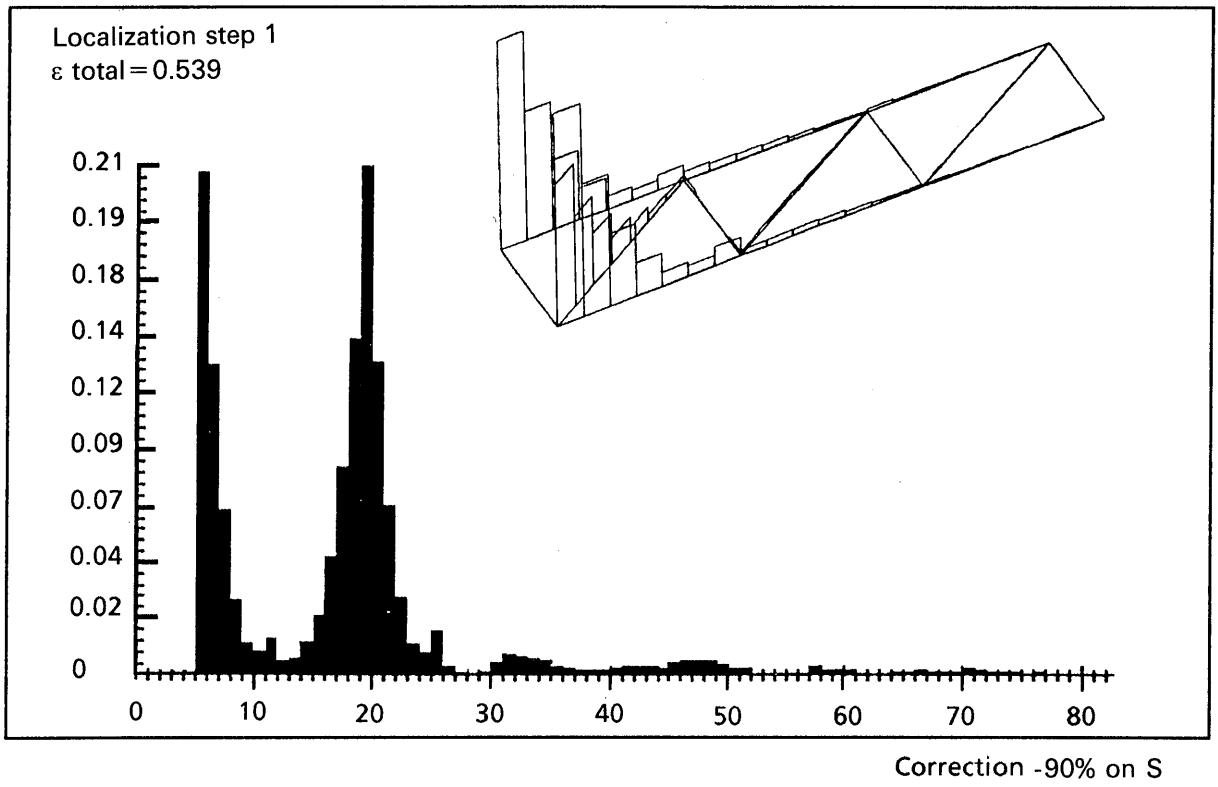


Fig. 3. – Adjustment of the model (LMT laboratory).

It is assumed that the exact eigenvalue γ is known. The associated modes are the solution of the problem:

Find a displacement field v and a stress field σ such that:

the pair v, σ is **admissible**, verifying (5) and (6)

$$v \,|\, \partial\Omega_1 = 0 \quad \text{and} \quad v \text{ regular} \qquad (5)$$

$$\forall v^* \in V$$

$$\int_\Omega \mathrm{Tr}\,(\sigma\varepsilon(v^*))\, d\Omega = \lambda \int_\Omega \rho\, v v^*\, d\Omega \qquad (6)$$

$V = \{ v,\ v \,|\, \partial\Omega_1 = 0,\ v \text{ regular} \}$ ρ is the density and such that:

(v, σ) verify the **constitutive equation**

$$\sigma = K\varepsilon(v)$$

where K is Hooke's tensor and ε is the strain field.

Quantities K and ρ are those of the finite element model.

Let us consider the experimental eigenvalue $\underline{\lambda}_k$ of mode k. A pair $(\underline{\sigma}, \underline{v})$ of admissible fields [verifying (5) and (6)] can be associated with it to minimize the error in the constitutive equation:

$$(v, \sigma) \to \| \sigma - K\varepsilon(v) \|^2$$

$$= \int_\Omega \mathrm{Tr}\,((\sigma - K\varepsilon(v))\, K^{-1}\, (\sigma - K\varepsilon(v)))\, d\Omega.$$

If $\underline{\lambda}_k$ is the exact eigenvalue for the (K, ρ) model, the error in the constitutive equation $\| \underline{\sigma} - K\varepsilon(\underline{v}) \|$ is zero.

This last problem can be rewritten by associating a displacement field u C.A. (kinematically admissible) which is a solution of the elastic problem with the stress σ S.A. (statically admissible):

$$\int_\Omega \mathrm{Tr}\,((K\varepsilon(u) - \sigma)\, \varepsilon(v^*))\, d\Omega = 0,$$

$$\forall v^* \in V$$

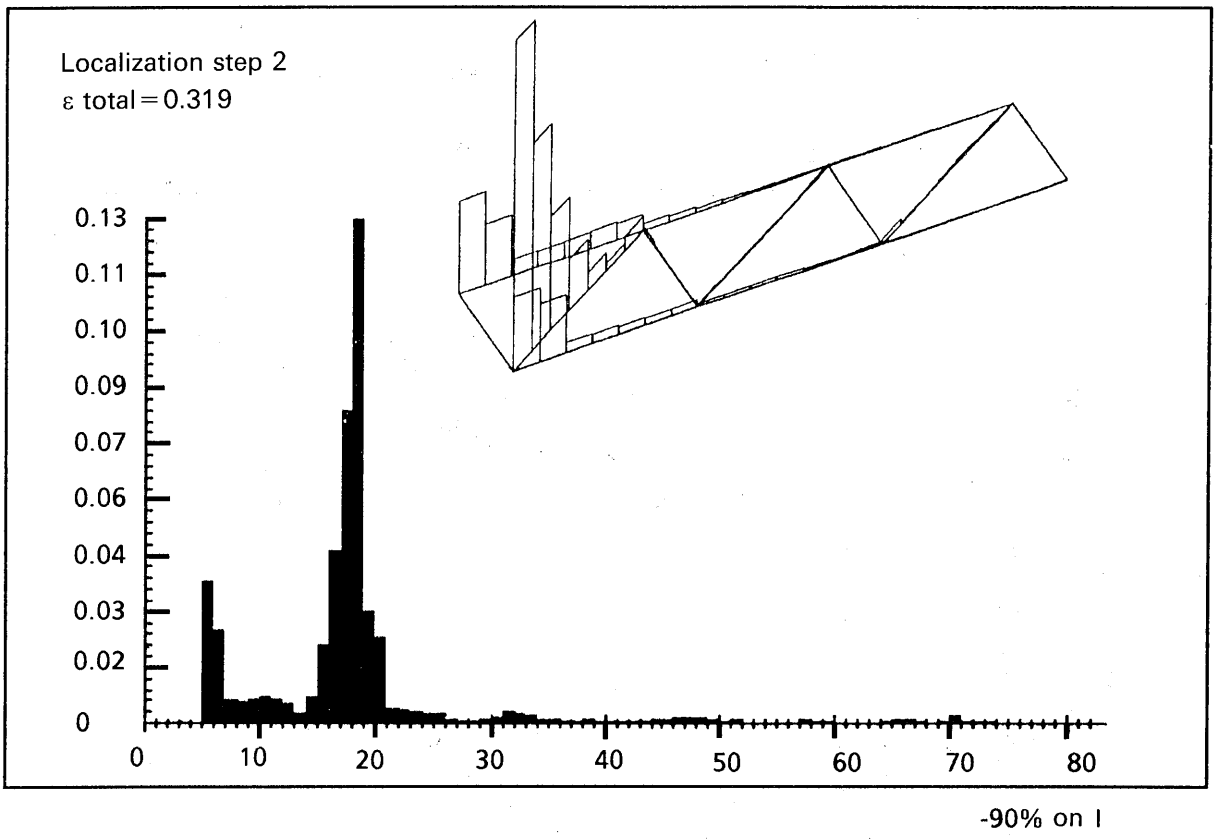


Fig. 4. – Adjustment of the model (LMT laboratory).

We then associate the error in the constitutive equation $\|u-v\|^2$ with (v, u), where (v, u) verify (5) and the following equilibrium equation:

$$u \in V, \qquad v \in V, \qquad \forall v^* \in V$$

$$\int_\Omega \mathrm{Tr}\,(K\varepsilon(u)\,\varepsilon(v^*))\,d\Omega = \underline{\lambda}_k \int_\Omega \rho\, v v^*\, d\Omega.$$

After discretization, this last problem is solved and the nodal values of u and v are also denoted (u, v) to simplify the notations.

Let $\Pi\,\underline{v}_k$ be the column of measured displacements associated with the measured eigenvalue $\underline{\lambda}_k$. We then associate with it the pair $(\underline{u}, \underline{v})$ such that $(\underline{u}, \underline{v})$ minimizes the error in the constitutive equation:

$$\mathrm{E}_k^2(u, v) = \|u-v\|^2 + \frac{r}{1-r}\,|||\Pi v - \Pi\,\underline{v}_k|||^2. \qquad (7)$$

Π is the given projection operator on the measured degrees of freedom and $|||\ \ |||$ is a norm which is described below.

For each given experimental mode k, the problem is therefore:

Find $(\underline{v}, \underline{u})$ C.A. at "0" minimizing:

$$E_k^2(u, v) = (u-v)^t\,\mathbb{K}\,(u-v) + \frac{r}{1-r}\,(\Pi v - \Pi\,\underline{v}_k)^t\,\mathbb{K}_r\,(\Pi v - \Pi\,\underline{v}_k) \qquad (8)$$

with the constraint $\mathbb{K}\,u = \underline{\lambda}_k\,\mathbb{M}\,v$.

$\mathbb{K}$, $\mathbb{M}$ are the symmetric stiffness and mass matrices respectively of the model with dimension n: the number of degrees of freedom of the model.

$\mathbb{K}_r$ is, for instance, the Guyan reduction of $\mathbb{K}$ on the measured degrees of freedom.

$|||\ldots|||$ then represents twice the strain energy given by the initial model in the static problem where the displacement field is obliged to take on values Πv in the measurement points.

r is a scalar expressing the level of confidence in the quality of the modes measured, $r \ll 1$ for unreliable experimental values and close to 1 otherwise. $r = 0.5$ is a common value.

(8) characterizes the distance between the predictions of the first model and the experimental mode $k(\underline{\lambda}_k, \Pi\,\underline{v}_k)$. If $E_k(\underline{u}, \underline{v})$ is zero, the experimental mode $k(\underline{\lambda}_k, \Pi\,\underline{v}_k)$ is perfectly represented by the theoretical model.

For all the q modes measured, the quality of prediction of the theoretical model is defined by

$$E_t = \left(\sum_{k=1}^{q} E_k^2(\underline{u}, \underline{v})\right)^{1/2}$$

For a given experimental mode k, $(\underline{\lambda}_k, \Pi\,\underline{v}_k)$, the measure of the error in the constitutive equation is then:

$$\underline{e} = \|\underline{u} - \underline{v}\|$$

Abscissa: index of elements s

Ordinate: error $\sum_{k=1}^{q} \Gamma_s^2$ error $\sum_{k=1}^{q} \Gamma_s^2$

$q=5$ given experimental modes drawing on the structure

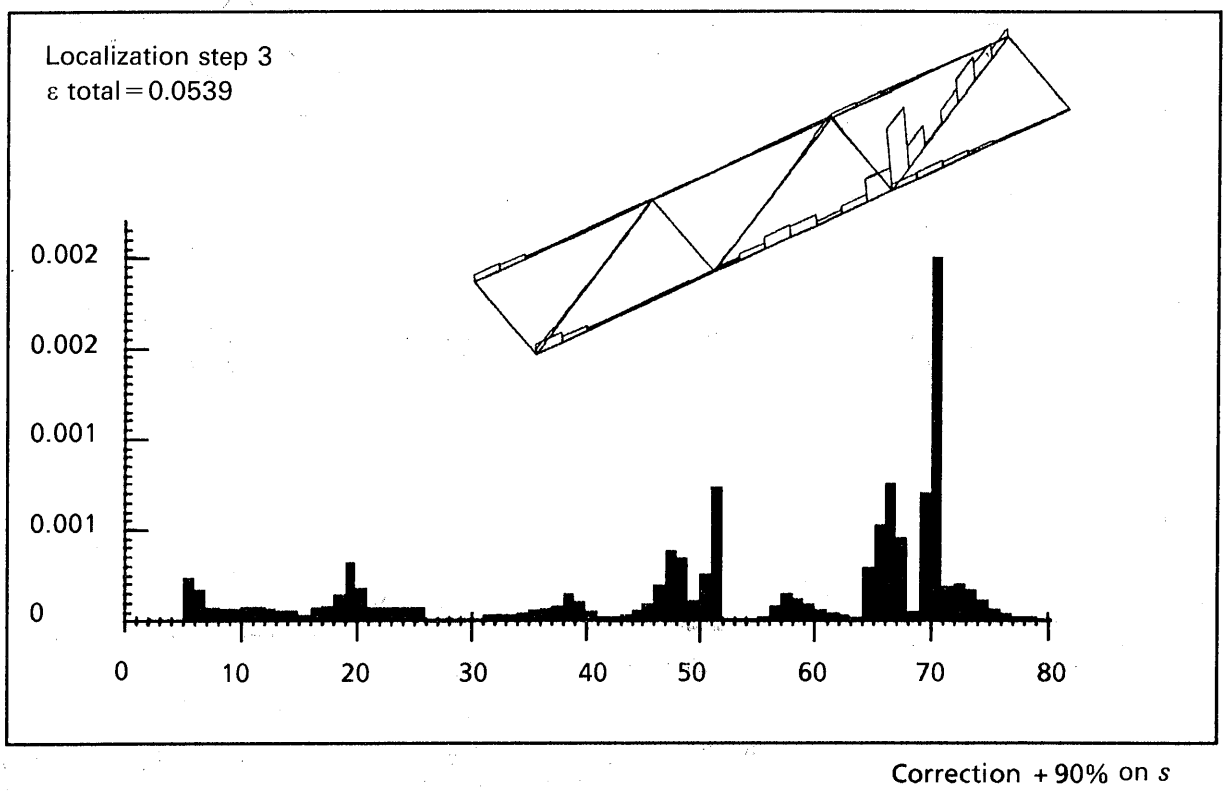


Fig. 5. – Adjustment of the model (LMT laboratory).

and in its relative form

$$\underline{\underline{\varepsilon}}_k^2 = \frac{\| \underline{u} - \underline{v} \|^2}{1/2 (\| \underline{u} \|^2 + \| \underline{v} \|^2)}.$$

When the structure considered is divided into substructures (s), $\underline{\underline{\varepsilon}}_k^2$ is the sum of the contributions of each substructure:

$$\underline{\underline{\varepsilon}}_k^2 = \sum_s \underline{\underline{\varepsilon}}_k^{2s} \quad \text{avec} \quad \underline{\underline{\varepsilon}}_k^{2s} = \frac{\| \underline{u} - \underline{v} \|_{(s)}^2}{1/2 (\| \underline{u} \|^2 + \| \underline{v} \|^2)_{(s)}}.$$

On all the available q modes measured, two indicators Γ_s^2 and η_s^2 are defined for each substructure (s)

$$\Gamma_s^2 = \sum_{k=1}^{q} \underline{\underline{\varepsilon}}_k^{2s},$$

$$\eta_s^2 = \sum_{k=1}^{q} \frac{\| \underline{u} - \underline{v} \|_{(s)}^2}{1/2 (\| \underline{u} \|_{(s)}^2 + \| \underline{v} \|_{(s)}^2)}.$$

These indicators correspond to the relative errors made on the substructures. Indicators η_s^2 are thus used **to localize** poorly modeled regions and indicators Γ_s^2 characterize the intensity of the corrections required (Figs. 3 to 6).

PARAMETRIC CORRECTION

DYNAMIC REACTION FORCES APPROACH

Let $D_k^t = [U_k'^t \, U_{\text{exp}k}^t]$ be the completed displacement and $\alpha^t = [\beta_1, \beta_2, \ldots, \beta_n, \gamma_1, \gamma_2, \ldots, \gamma_p]$ the vector of the m correction parameters ($m = n + p$) corresponding to the identified substructures.

If α is a solution:

$$F_0(\alpha) + F(\alpha)\,\alpha = 0 \tag{9}$$

where

$$F_0(\alpha)^t = [((K_0 - \lambda_{\text{exp}^1} M_0) D_1)^t, ((K_0 - \lambda_{\text{exp}^2} M_0) D_2)^t, \ldots, ((K_0 - \lambda_{\text{exp}^q} M_0) D_q)^t]$$

which is the column vector of the nodal forces corresponding to the q measured modes, $F(\alpha)$ is the matrix

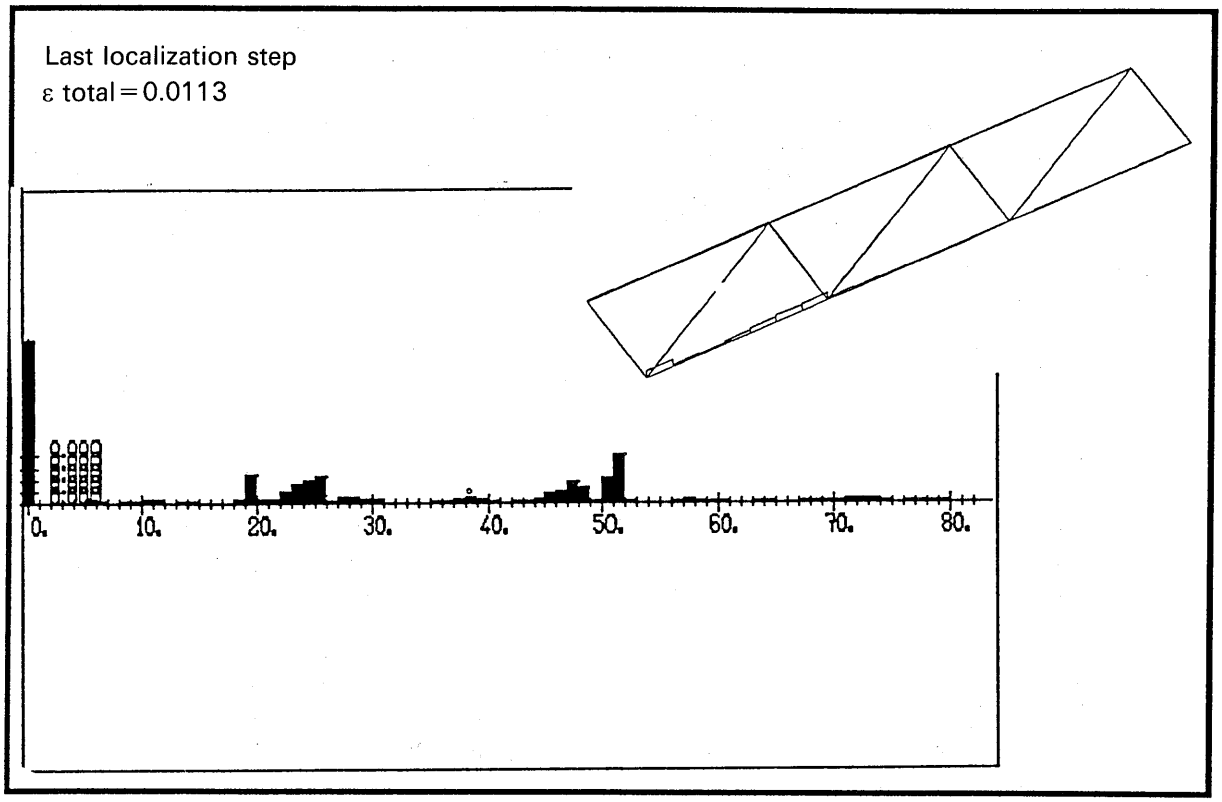


Fig. 6. – Adjustment of the model (LMT laboratory).

of elementary nodal forces for all the q modes measured. It consists of subcolumns of the type:

$$K_j D_k, \quad -\lambda_{expk} M_i D_k \qquad (j=1, n;\ i=1, p;\ k=1, q).$$

System (9) is linear with variable coefficients.

By application of the fixed point theorem, we attempt to find α_{p+1} in iteration $p+1$ such that:

$$F_0(\alpha_p) + F(\alpha_p)\,\alpha_{p+1} = 0. \tag{10}$$

At iteration p, $F_0(\alpha_p) + F(\alpha_p)\,\alpha_p = R(\alpha_p)$ yielding

$$F(\alpha_p)\,\Delta\alpha_p + R(\alpha_p) = 0 \qquad \text{where} \quad \Delta\alpha_p = \alpha_{p+1} - \alpha_p. \tag{11}$$

(11) is generally overdetermined and is solved in the sense of the least squares.

Setting $\varepsilon_p = F(\alpha_p)\,\Delta\alpha_p + R(\alpha_p)$, and solving (10) in the sense of the least squares amounts to minimizing function $f_p = \varepsilon_p^t \varepsilon_p$ each iteration, which leads to solving $(F^t F)_p \Delta\alpha_p = -(F^t R)_p$ where $(F^t F)$ represents an approximation of the Hessian of f_p. We thus arrive at a **problem of optimization without constraints** (solved by a Gauss-Newton method) when R is non-zero on all the DOFs of the structure.

ERROR IN THE CONSTITUTIVE EQUATION APPROACH

It is considered that the structure is defined by a number of structural parameters $p \in \mathbb{P}$. Matrices $\mathbb{K}$ and $\mathbb{M}$ are therefore a function of p. The correction problem is written:

Find $p \in \mathbb{P}$ such that $\mathbb{K}(p)$ and $\mathbb{M}(p)$ minimize:

$$\underline{\underline{E}}^2(p) = \sum_{k=1}^{q} \left\{ \| \underline{\underline{u}}_k - \underline{\underline{v}}_k \|^2 + \frac{r}{1-r} \| \Pi\, \underline{\underline{v}}_k - \Pi\, \underline{\underline{v}}_k \|^2 \right\}$$

for q measured modes.

This **nonlinear problem** is solved by a **conjugate gradient** algorithm

More specifically, we set $\underline{\underline{\mathbb{K}}} = \mathbb{K} + \Delta\mathbb{K}$, $\underline{\underline{\mathbb{M}}} = \mathbb{M} + \Delta\mathbb{M}$ and, in the case of the test examples discussed:

$$\Delta\mathbb{K} = \sum_{c=1}^{L} C_c \Delta p_c \qquad \text{and} \qquad \Delta\mathbb{M} = \sum_{c=1}^{L} D_c \Delta p_c$$

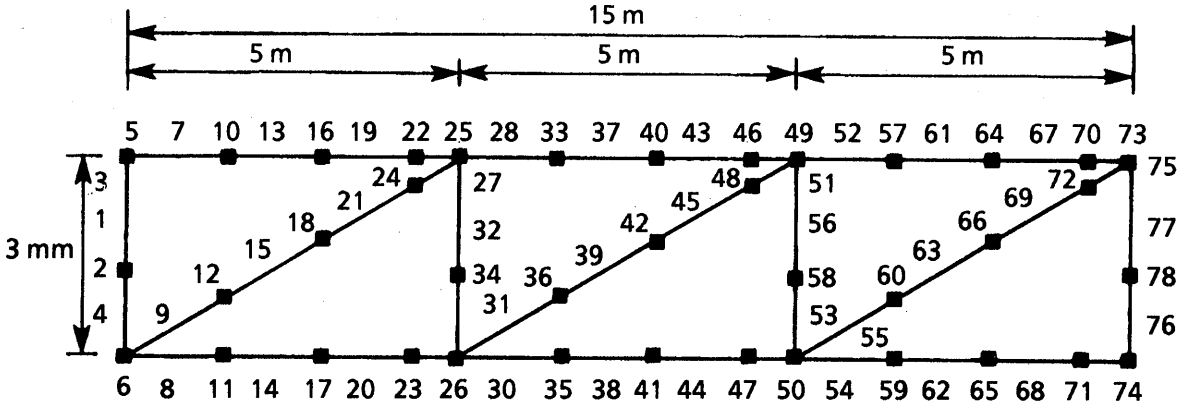
where L is the number of structural parameters to be modified. We then solve the following problem:

Find $(\underline{\underline{u}}_k, \underline{\underline{v}}_k, \Delta p_c)$, $c \in (1, L)$ $k \in (1, q)$ such that they minimize

$$F(u_k, v_k, \Delta p_c) = \sum_{k=1}^{q} \| u_k - v_k \|^2 + \frac{r}{1-r} \| \Pi\, v_k - \Pi\, \underline{\underline{v}}_k \|^2 \tag{12}$$

where

$$\underline{\underline{\mathbb{K}}}\, u_k = \underline{\underline{\lambda}}_k\, \mathbb{M}\, v_k \tag{13}$$



Left edge clamped

■ Measurement points (two degrees of freedom *x*, *y* measured)

Young's modulus	:	$E = 0.75\ 10^{11}$ Pa	Vertical elements	:	$S = 0.6\ 10^{-2}$ m²
Density	:	$p = 2800$ kg m²	Horizontal elements	:	$S = 0.4\ 10^{-2}$ m²
Inertia	:	$I = 0.756\ 10^{-1}$ m⁴	Diagonal elements	:	$S = 0.3\ 10^{-2}$ m²

Fig. 7.

	Real relative perturbations		Calculated relative perturbations	
	Stiffness	Mass	Stiffness	Mass
Elements	$\frac{\Delta\alpha}{\alpha}$	$\frac{\Delta\beta}{\beta}$	$\frac{\Delta\alpha}{\alpha}$	$\frac{\Delta\beta}{\beta}$
5-7	−0.9	0	−0.9	0
7-10	−0.9	0	−0.9	0
6-9	−0.9	0	−0.9	0
9-12	−0.9	0	−0.9	0
12-15	−0.9	0	−0.9	0
6-8	−0.9	0	−0.9	0
8-11	−0.9	0	−0.9	0
11-14	+0.1	0	+0.105	0
14-17	+1	0	+1	0
17-20	+1	0	+1	0
47-50	0	+1	0	+1
50-55	0	+1	0	+1
55-60	0	+1	0	+1
65-68	+1	0	+1	0
68-71	+1	0	+0.81	0
71-74	0	0	+0.09	0

Fig. 8. – Truss clamped at one end and free at the other.

The solution in v_k leads to:

$$F(\underline{v}_k, \Delta p_c) = \sum_{k=1}^{q} \left(\left(\frac{r}{1-r} \right) (\Pi \underline{v}_k)^t \mathbb{K}_r \Pi (\underline{v}_k - \underline{v}_k) \right)$$

$$= \sum_{k=1}^{q} \left(\left(\frac{r}{1-r} \right)^2 (\mathbb{K}_r \Pi \underline{v}_k)^t \left(A_k^t \mathbb{K} A_k + \frac{r}{1-r} \mathbb{K} \right)^{-1} \times (\mathbb{K}_r \Pi \underline{v}_k) + \left(\frac{r}{1-r} \right) \Pi \underline{v}_{k^t} \mathbb{K}_r \Pi \underline{v}_k \right)$$

with $u - v = A_k v$ where A_k is a function of Δp_c.

This last problem can thus be reduced to the following nonlinear problem:

Find Δp_c, $c \in (1, L)$ such that they minimize:

$$H(\Delta p_c) = F(\underline{v}_k, \Delta p_c)$$

which is a problem with L scalar variables.

GARTEUR EXAMPLE [1]

Let us consider the truss structure discretized by finite elements: bar and beam bending in the plane. The structure is clamped at one end and free at the other (Fig. 7). The mechanical properties were selected so as to obtain *overall* modal deformations. The behavior of the real structure was numerically simulated by disturbing the sections and the inertias, thereby causing **stiffness and mass** errors on the model. The first five frequencies of the disturbed structure and part of the degrees of freedom were given. The following upper and lower limits were set for the relative errors: $[-0.9, +1]$ for section inertias, $[-0.5, +1]$ for bar sections. Figures 8 and 9 illustrate the udpating results for the two methods developed above.

CONCLUSION

The strategies adopted for solving the inverse problem, *i.e.* the problem of updating models from vibration tests, are differentiated by the degree of integration of the physical context. Generally,

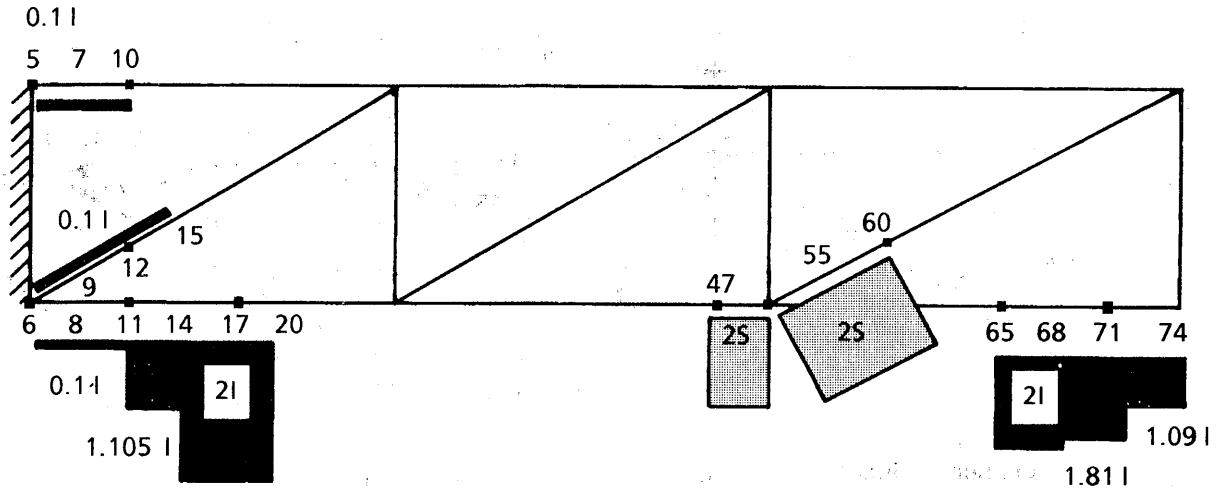


Fig. 9. – GARTEUR: Second test case.
Vibrations in the plane of an undamped disturbed truss, clamped and free.

the more mechanical the approach, the more efficient the reconstruction of the data. The completely automatic approaches proposed amount to minimizing a cost function that has several solutions, certain of which are not compatible with the physics of the problem. It is not easy to sort the solutions taking into account the quality of the experimental results, especially since the quality is itself difficult to evaluate. It is not reasonable to introduce all the parametric variables of the problem in a functional to be minimized. Although solving an optimization problem of a large size is not always an obstacle with today's computers, it remains true that the problem is poorly stated as regards evaluation of the quality of the measurements. The approach we recommend consists of identifying the poorly modeled regions and then correcting the model *taking the physical context into account.*

Manuscript submitted July 9, 1990; *accepted June* 5, 1991.

REFERENCES

[1] GARTEUR-AG 11. – Group for aeronautical research and technology in Europe, action group on parametric updating of finite element models using experimental simulations.

[2] TIKHONOV A. and ARSENINE V. – *Méthodes de résolution de problèmes mal posés,* Édition Mir, Moscou, (1974).

[3] BARUCH M. – *Optimal correction for mass matrix and stiffness matrix using measuring modes,* A.I.A.A. 82-4265, (1982).

[4] BERMAN A. and FLANNELLY O. – *Theory of incomplete models of dynamic structures,* AIAA journal, vol. 9/8, (1971), p. 1481-1487.

[5] CHEN J. C. and GARBA J. A. – *Analytical model improvement using modal test results,* AIAA Paper 79-0831, vol. 18, n° 6, Avril 1989.

[6] CAESAR B. – *Correlation and update of dynamic mathematical models,* 4th F.E. world congress, Interlaken, Septembre 1984.

[7] HE J. and EWINS D. J. – *Analytical stiffness matrix correction using measured vibration modes,* Int. journ. analytical & experimental modal analysis, vol. 1, n° 3, juillet 1986.

[8] EWINS D. J. and HE J. – *A review of the error matrix method for structural dynamic modes comparison,* Proc. Int. Conf. on Spacecraft Structures Testing, ESA-SP-289, (1988).

[9] LINK M., WEILEND M. and BARRAGAN J. M. – *Direct physical matrix identification as compared to phase resonance testing. An assessment based on practical application,* Proc., IMAC, London, (1987).

[10] WEI J. J. C. and ALLEMANG R. J. – *Correction of finite element model via selected physical parameters,* Proceedings, IMAC, p. 1231-1238, Las Vegas, (1989).

[11] COLLINS J. D., HART G. C., HASSELMAN J. K. and KENNEDY B. – *Statical identification of structures,* AIAA journal, vol. 12/2, (1974), p. 185-190.

[12] ZHANG W., LALLEMENT G., FILLOD R. and PIRANDA J. – *Parametric identification of conservative self-adjoint structures,* Proc. Int. Conf. on Spacecraft Structures Testing, ESA-SP-289, (1988).

[13] DASCOTTE E. and VANHONECKER P. – *Development of an automatic mathematical model updating program,* Proc., IMAC, p. 1183-1190, London, (1987).

[14] COTTIN N., FELGENHAUER H. P. and NATKE H. G. – *On the parameter identification of elastomechanical systems using input and output residuals,* Ingenieur Archiv. 54, p. 378-387, Spring Verlag, (1984).

[15] BOUTIN P. and PETIAU M. – *Identification automatique de structures dynamiques sur modèles d'éléments finis,* AMD/BA, Saint-Cloud, Doc. DGT n° 23351, août 1983.

[16] O'CALLAHAN J. C. and CHOU C. M. – *Localization of test/analysis structural model error,* AIAA Paper 89-1244-CP, p. 748-752, (1989).

[17] LEURIDAN J. M., BROWN D. L. and ALLEMANG R. J. – *Direct system identification of mechanical structures with application to modal analysis,* AIAA Paper 82-0767, p. 548-556.

[18] BERGER H., BARTHE L. and OHAYON R., *Parametric updating of a finite element model from experimental modal characteristics,* Proc. European Forum on Aeroelasticity and Structural Dynamics, 1989, Aachan April 1989.

[19] BERGER H., CHAQUIN J. P. and OHAYON R. – *Finite element model adjustment using experimental data,* Proc. IMAC-2, Orlando, Florida, (1984).

[20] BERGER H., BARTHE L. and OHAYON R. – *Recalage d'un modèle par éléments finis à partir de données expérimentales du type vibratoire. Concept de localisation,* R.T. DRET n° 7/3313 RY 070 R., mai 1987.

[21] FISSETTE E., STAVRINIDIS C. and IBRAHIM S. – *Error location and updating of analytical dynamic models using a force balance method,* Proceedings, IMAC, (1988), p. 1063-1070.

[22] LADEVEZE P. and REYNIER M. – *Recalage de modélisation par éléments finis en dynamique,* Rapport Interne du Laboratoire de Mécanique et Technologie, n° 97, avril 1989, LMT, 61, avenue du Président-Wilson, 94235 Cachan.

[23] LADEVEZE P. and REYNIER M. – *A localization method of stiffness errors for the adjustment of F.E. models,* Proceedings, 12th ASME Mechanical Vibration and Noise Conference, Montreal, septembre 1989.

[24] REYNIER M. – *Sur le contrôle des modélisations par éléments finis : Recalage à partir d'essais dynamiques,* Thèse de Doctorat d'Université Paris-VI, 17 décembre 1990, Laboratoire de Mécanique et Technologie, 61, avenue du Président-Wilson, 94235 Cachan.

[25] LADEVEZE P., *Recalage de modélisations des structures complexes,* Note technique n° 33.11.01.4. Aérospatiale, Les Mureaux, (1983).

[26] LADEVEZE P., COFFIGNAL G. and PELLE J. P. – *Accuracy of elastoplastic and dynamic analysis* in *Accuracy estimates and adaptative refinements in finite element computation,* Chap. 11, p. 181-203, Babuska, Zienkiewicz, Gago, Oliveira, Editor J. Wiley, (1986).

[27] NIEDBAL N., KLUSOWSKI E. and LUBER W. – *Updating of F.E. models by means of normal modes,* Proc. ESA-Congress, ESA-SP-289, (1988).

[28] NASH M. – *An approach to the correction of finite element and model test studies,* CADCAM ans Vibration Measurements, (1988).